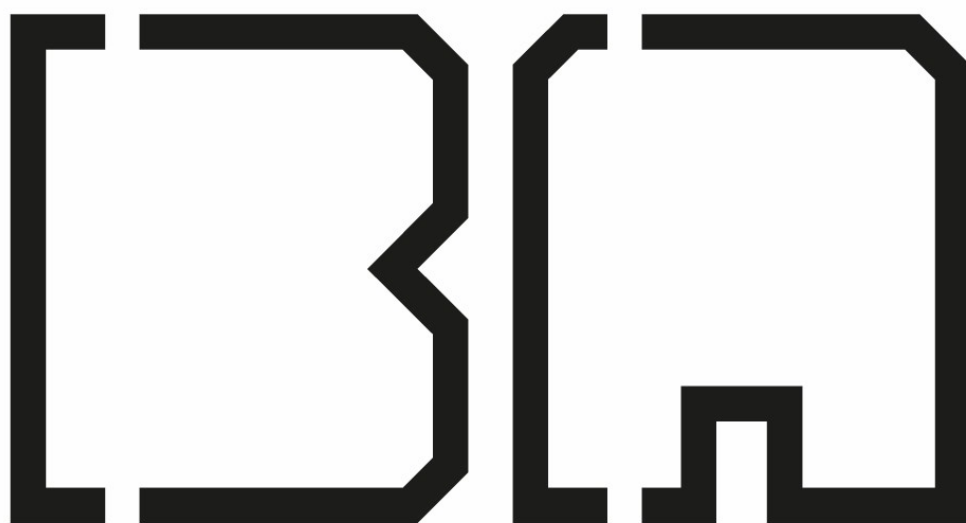


Politechnika Lubelska

Wydział Budownictwa i Architektury



Mechanika Budowli I

Projekt 2: Metoda Sił

Wykonał:

.....

Sprawdził:

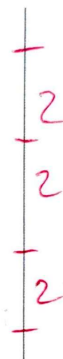
dr inż. Jakub Gontarz

Student

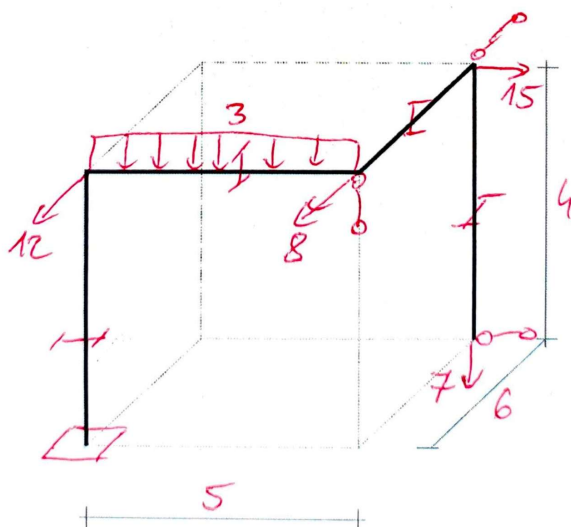
Grupa

Rok akademicki

Dane ustroje statycznie niewyznaczalne rozwiązać metodą sił



$$\alpha_1 = 1,2 \cdot 10^{-5} \text{ 1/K}$$

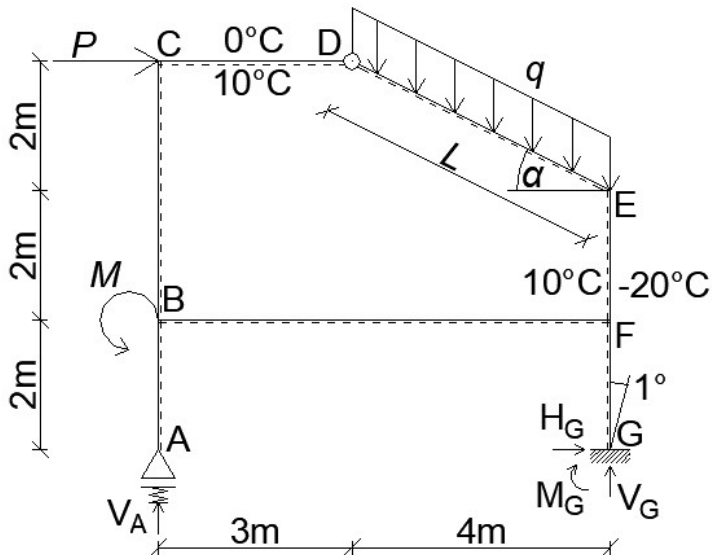
Przekrój: IPN 120

$$\alpha_1 = 1,2 \cdot 10^{-5} \text{ 1/K}$$

Przekrój: IPE 180

$$\text{kNm} := \text{kN} \cdot \text{m}$$

Zadanie 1: Rama 2D



$$E := 210 \text{ GPa}$$

$$\alpha_t := 1.2 \cdot 10^{-5} \frac{1}{\text{K}}$$

Przekrój: IPN 120

$$J := 328 \text{ cm}^4$$

$$EJ := E \cdot J = 688.8 \text{ kN} \cdot \text{m}^2$$

$$h := 120 \text{ mm}$$

Długość pręta ukośnego:

$$L := \sqrt{(2 \text{ m})^2 + (4 \text{ m})^2} = 4.472 \text{ m}$$

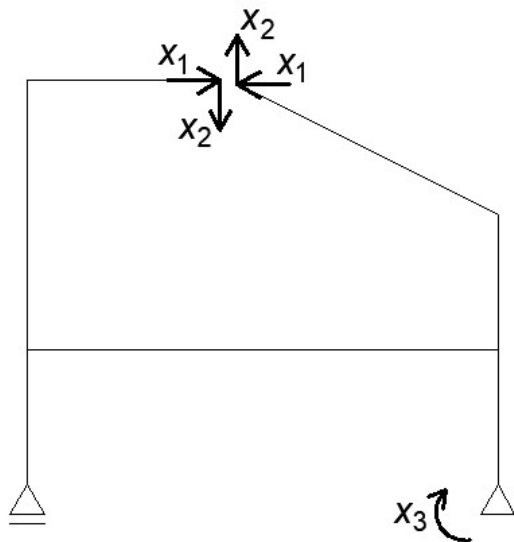
$$\sin \alpha := \frac{2 \text{ m}}{L} = 0.447 \quad \cos \alpha := \frac{4 \text{ m}}{L} = 0.894$$

$$P := 6 \text{ kN}$$

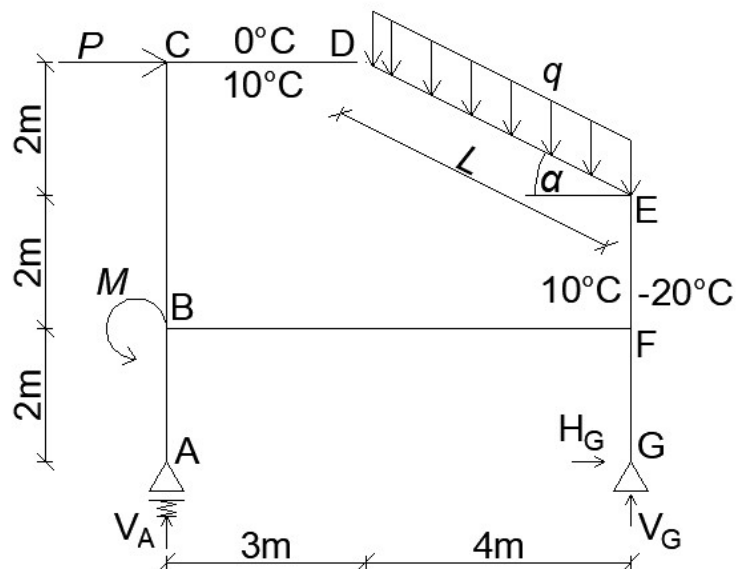
$$M := 10 \text{ kNm}$$

$$q := 2 \frac{\text{kN}}{\text{m}}$$

UPMS



Stan p



Reakcje:

$$\Sigma M_A = P \cdot 6 - M - q \cdot L \cdot 5 - V_G \cdot 7 = 0$$

$$\Sigma X = H_G + 6 = 0$$

$$\Sigma Y = V_A + V_G - q \cdot L = 0$$

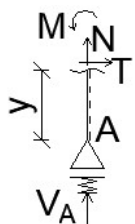
$$V_G := \frac{P \cdot 6 - M + q \cdot L \cdot 5}{7} = 10.103 \text{ kN}$$

$$H_G := -6 \text{ kN}$$

$$V_A := -V_G + q \cdot L = -1.159 \text{ kN}$$

Pręt AB

$$y \in (0; 2) \text{ m}$$



$$N_{AB} := -V_A = 1.159 \text{ kN}$$

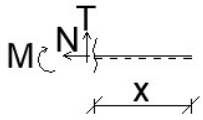
$$T_{AB} := 0 \text{ kN}$$

$$M_{AB}(y) := 0 \text{ kNm}$$

$$M_{AB}(0 \text{ m}) = 0 \text{ kNm} \quad M_{AB}(2 \text{ m}) = 0 \text{ kNm}$$

Pręt CD

$x \in (0;3)\text{m}$



$$N_{CD} := 0 \text{ kN}$$

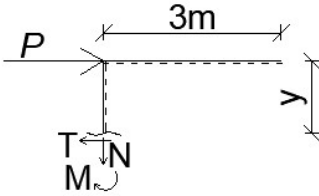
$$T_{CD} := 0 \text{ kN}$$

$$M_{CD}(x) := 0 \text{ kNm}$$

$$M_{CD}(0 \text{ m}) = 0 \text{ kNm} \quad M_{CD}(3 \text{ m}) = 0 \text{ kNm}$$

Pręt BC

$y \in (0;4)\text{m}$



$$N_{BC} := 0 \text{ kN}$$

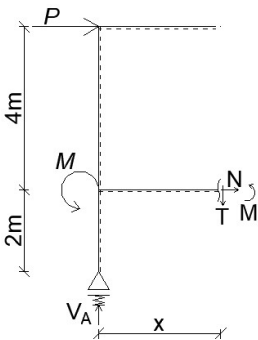
$$T_{BC} := P = 6 \text{ kN}$$

$$M_{BC}(y) := -P \cdot y$$

$$M_{BC}(0 \text{ m}) = 0 \text{ kNm} \quad M_{BC}(4 \text{ m}) = -24 \text{ kNm}$$

Pręt BF

$x \in (0;7)\text{m}$



$$N_{BF} := -P = -6 \text{ kN}$$

$$T_{BF} := V_A = -1.159 \text{ kN}$$

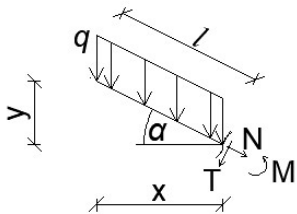
$$M_{BF}(x) := P \cdot 4 \text{ m} - M + V_A \cdot x$$

$$M_{BF}(0 \text{ m}) = 14 \text{ kNm} \quad M_{BF}(7 \text{ m}) = 5.889 \text{ kNm}$$

Pręt DE

$x \in (0;4)\text{m}$

$$y(x) := 0.5 x \quad l(x) := \sqrt{y(x)^2 + x^2}$$



$$N_{DE}(x) := -q \cdot l(x) \cdot \sin \alpha$$

$$T_{DE}(x) := -q \cdot l(x) \cdot \cos \alpha$$

$$M_{DE}(x) := -q \cdot l(x) \cdot \frac{x}{2}$$

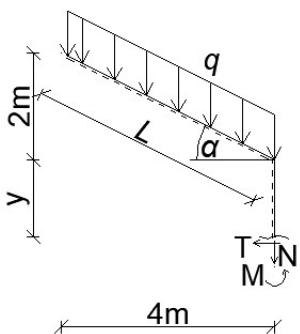
$$N_{DE}(0 \text{ m}) = 0 \text{ kN} \quad N_{DE}(4 \text{ m}) = -4 \text{ kN}$$

$$T_{DE}(0 \text{ m}) = 0 \text{ kN} \quad T_{DE}(4 \text{ m}) = -8 \text{ kN}$$

$$M_{DE}(0 \text{ m}) = 0 \text{ kNm} \quad M_{DE}(4 \text{ m}) = -17.889 \text{ kNm}$$

Pręt EF

$y \in (0;2)\text{m}$



$$N_{EF} := -q \cdot L = -8.944 \text{ kN}$$

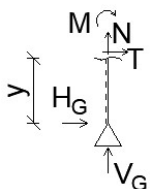
$$T_{EF} := 0 \text{ kN}$$

$$M_{EF}(y) := -q \cdot L \cdot 2 \text{ m}$$

$$M_{EF}(0 \text{ m}) = -17.889 \text{ kNm} \quad M_{EF}(7 \text{ m}) = -17.889 \text{ kNm}$$

Pręt FG

$y \in (0;2)\text{m}$

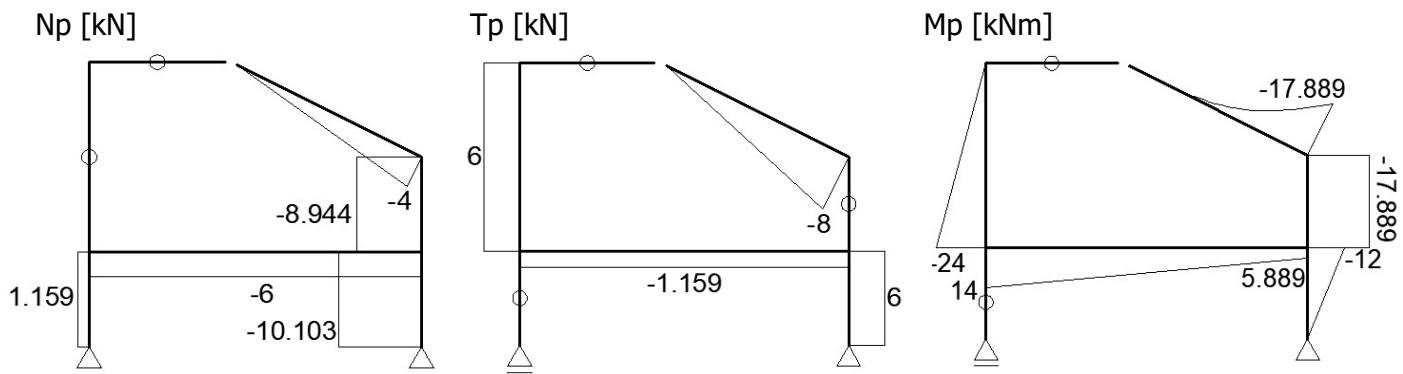


$$N_{FG} := -V_G = -10.103 \text{ kN}$$

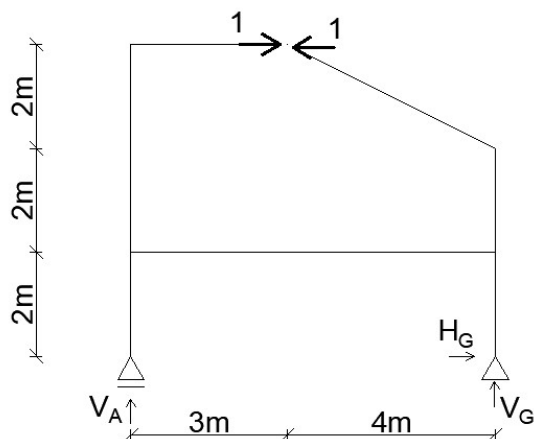
$$T_{FG} := -H_G = 6 \text{ kN}$$

$$M_{FG}(y) := H_G \cdot y$$

$$M_{FG}(0 \text{ m}) = 0 \text{ kNm} \quad M_{FG}(2 \text{ m}) = -12 \text{ kNm}$$



Stan x1=1



Reakcje:

$$\Sigma M_A = 1 \cdot 6 - 1 \cdot 6 - V_G \cdot 7 = 0$$

$$V_G := \frac{1 \cdot 6 \text{ m} - 1 \cdot 6 \text{ m}}{7 \text{ m}} = 0$$

$$\Sigma X = H_G + 1 - 1 = 0$$

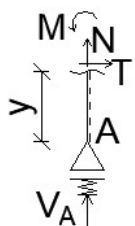
$$H_G := -1 + 1 = 0$$

$$\Sigma Y = V_A + V_G = 0$$

$$V_A := -V_G = 0$$

Pręt AB

$$y \in (0; 2) \text{ m}$$



$$N_{AB} := -V_A = 0$$

$$T_{AB} := 0$$

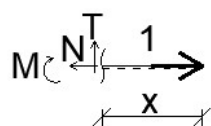
$$M_{AB}(y) := 0 \text{ m}$$

$$M_{AB}(0 \text{ m}) = 0 \text{ m}$$

$$M_{AB}(2 \text{ m}) = 0 \text{ m}$$

Pręt CD

$$x \in (0; 3) \text{ m}$$



$$N_{CD} := 1$$

$$T_{CD} := 0$$

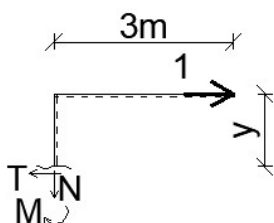
$$M_{CD}(x) := 0 \text{ m}$$

$$M_{CD}(0 \text{ m}) = 0 \text{ m}$$

$$M_{CD}(3 \text{ m}) = 0 \text{ m}$$

Pręt BC

$$y \in (0; 4) \text{ m}$$



$$N_{BC} := 0$$

$$T_{BC} := 1$$

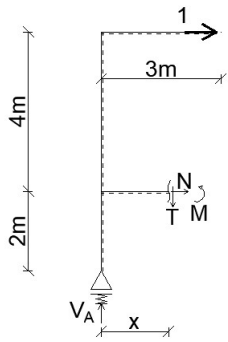
$$M_{BC}(y) := -1 \cdot y$$

$$M_{BC}(0 \text{ m}) = 0 \text{ m}$$

$$M_{BC}(4 \text{ m}) = -4 \text{ m}$$

Pręt BF

$x \in (0; 7) \text{ m}$



$$N_{BF} := -1$$

$$T_{BF} := V_A = 0$$

$$M_{BF}(x) := V_A \cdot x + 1 \cdot 4 \text{ m}$$

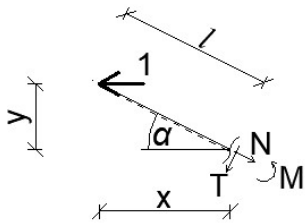
$$M_{BF}(0 \text{ m}) = 4 \text{ m} \quad M_{BF}(7 \text{ m}) = 4 \text{ m}$$

Pręt DE

$x \in (0; 4) \text{ m}$

$$y(x) := 0.5 x$$

$$l(x) := \sqrt{y(x)^2 + x^2}$$



$$N_{DE} := 1 \cdot \cos \alpha = 0.894$$

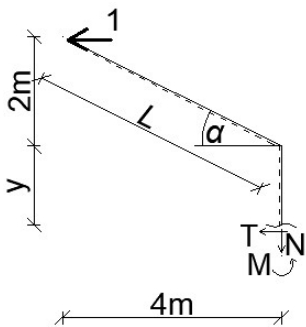
$$T_{DE} := -1 \cdot \sin \alpha = -0.447$$

$$M_{DE}(x) := -1 \cdot y(x)$$

$$M_{DE}(0 \text{ m}) = 0 \text{ m} \quad M_{DE}(4 \text{ m}) = -2 \text{ m}$$

Pręt EF

$y \in (0; 2) \text{ m}$



$$N_{EF} := 0$$

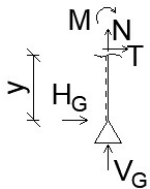
$$T_{EF} := -1$$

$$M_{EF}(y) := -1 \cdot (2 \text{ m} + y)$$

$$M_{EF}(0 \text{ m}) = -2 \text{ m} \quad M_{EF}(7 \text{ m}) = -9 \text{ m}$$

Pręt FG

$y \in (0; 2) \text{ m}$



$$N_{FG} := -V_G = 0$$

$$T_{FG} := -H_G = 0$$

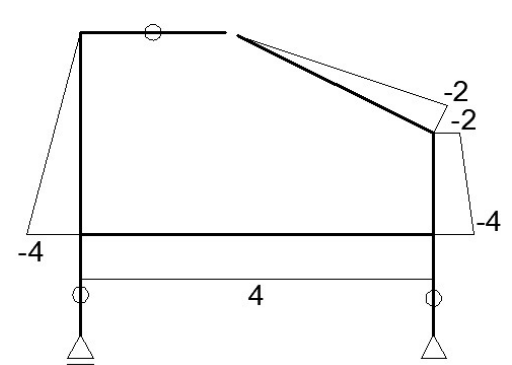
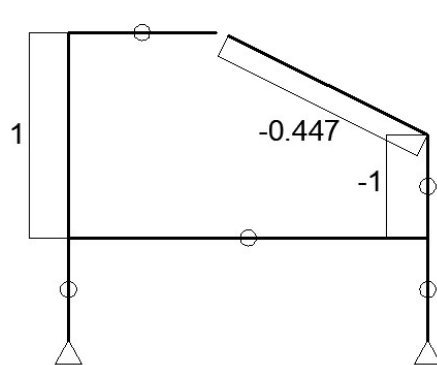
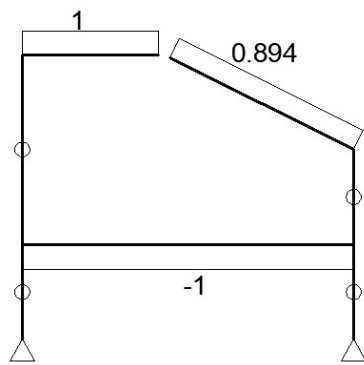
$$M_{FG}(y) := H_G \cdot y$$

$$M_{FG}(0 \text{ m}) = 0 \text{ m} \quad M_{FG}(2 \text{ m}) = 0 \text{ m}$$

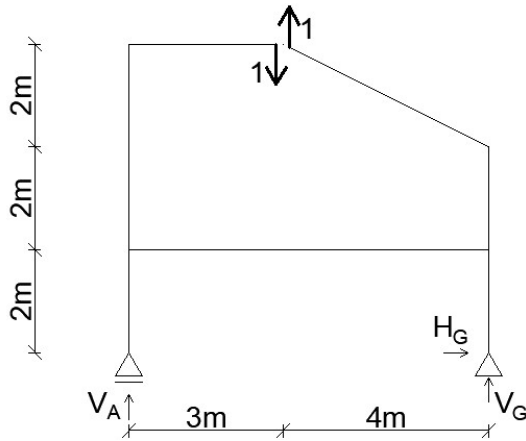
N1 [-]

T1 [-]

M1 [m]



Stan x2=1



Reakcje:

$$\Sigma M_A = 1 \cdot 3 - 1 \cdot 3 - V_G \cdot 7 = 0$$

$$V_G := \frac{1 \cdot 3 \text{ m} - 1 \cdot 3 \text{ m}}{7 \text{ m}} = 0$$

$$\Sigma X = H_G = 0$$

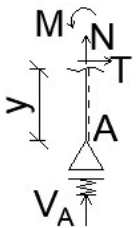
$$H_G := 0$$

$$\Sigma Y = V_A + V_G + 1 - 1 = 0$$

$$V_A := -V_G - 1 + 1 = 0$$

Pręt AB

$$y \in (0; 2) \text{ m}$$



$$N_{AB} := -V_A = 0$$

$$T_{AB} := 0$$

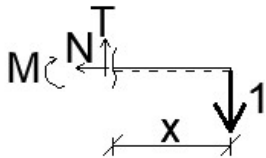
$$M_{AB}(y) := 0 \text{ m}$$

$$M_{AB}(0 \text{ m}) = 0 \text{ m}$$

$$M_{AB}(2 \text{ m}) = 0 \text{ m}$$

Pręt CD

$$x \in (0; 3) \text{ m}$$



$$N_{CD} := 0$$

$$T_{CD} := 1$$

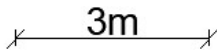
$$M_{CD}(x) := -1 \cdot x$$

$$M_{CD}(0 \text{ m}) = 0 \text{ m}$$

$$M_{CD}(3 \text{ m}) = -3 \text{ m}$$

Pręt BC

$$y \in (0; 4) \text{ m}$$



$$N_{BC} := -1$$

$$T_{BC} := 0$$

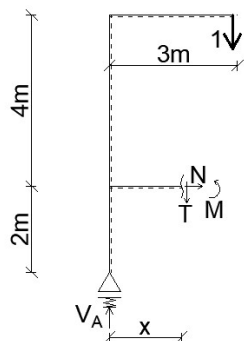
$$M_{BC}(y) := -1 \cdot 3 \text{ m}$$

$$M_{BC}(0 \text{ m}) = -3 \text{ m}$$

$$M_{BC}(4 \text{ m}) = -3 \text{ m}$$

Pręt BF

$$x \in (0; 7) \text{ m}$$



$$N_{BF} := 0$$

$$T_{BF} := V_A - 1 = -1$$

$$M_{BF}(x) := V_A \cdot x - 1 \cdot (x - 3 \text{ m})$$

$$M_{BF}(0 \text{ m}) = 3 \text{ m}$$

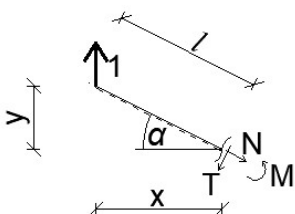
$$M_{BF}(7 \text{ m}) = -4 \text{ m}$$

Pręt DE

$$x \in (0; 4) \text{ m}$$

$$y(x) := 0.5 x$$

$$l(x) := \sqrt{y(x)^2 + x^2}$$



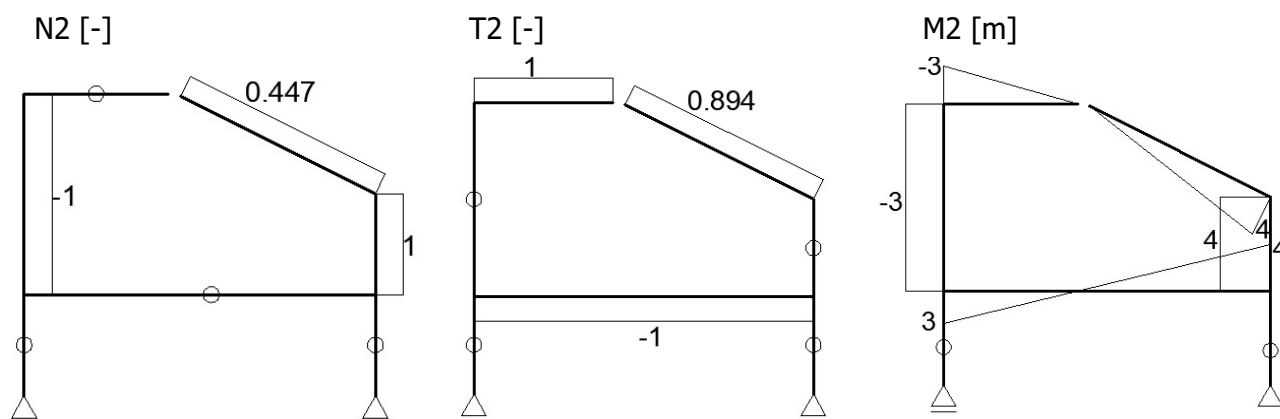
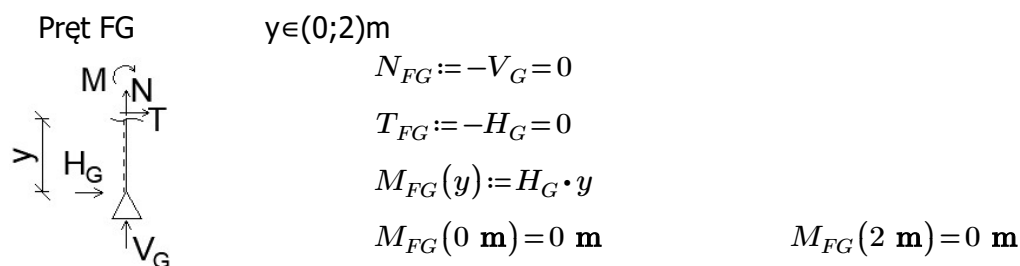
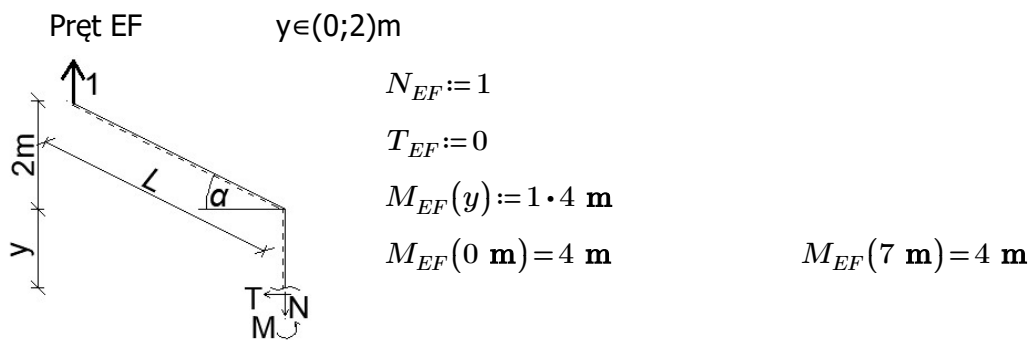
$$N_{DE} := 1 \cdot \sin \alpha = 0.447$$

$$T_{DE} := 1 \cdot \cos \alpha = 0.894$$

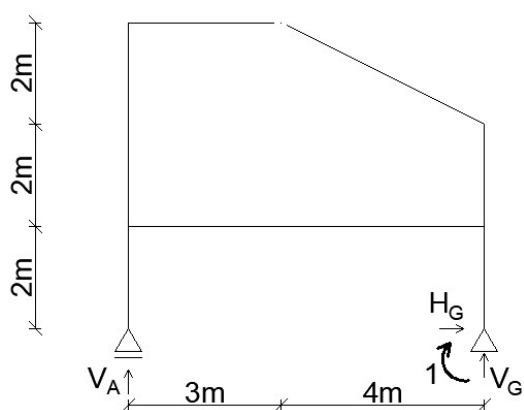
$$M_{DE}(x) := 1 \cdot x$$

$$M_{DE}(0 \text{ m}) = 0 \text{ m}$$

$$M_{DE}(4 \text{ m}) = 4 \text{ m}$$



Stan x3=1



Reakcje:

$$\Sigma M_A = 1 - V_G \cdot 7 = 0$$

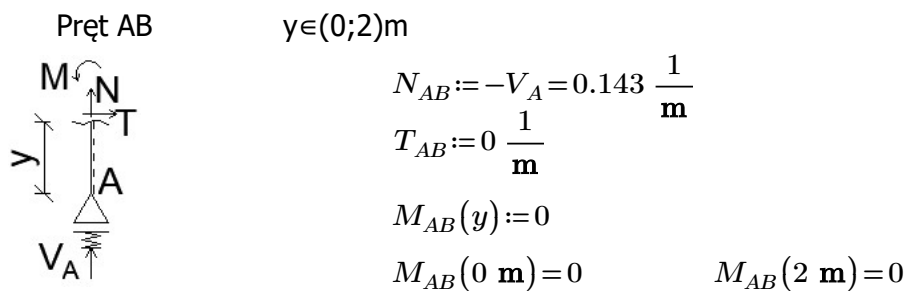
$$V_G := \frac{1}{7 \text{ m}} = 0.143 \frac{1}{\text{m}}$$

$$\Sigma X = H_G = 0$$

$$H_G := 0 \frac{1}{\text{m}}$$

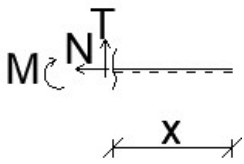
$$\Sigma Y = V_A + V_G = 0$$

$$V_A := -V_G = -0.143 \frac{1}{\text{m}}$$



Pręt CD

$$x \in (0; 3) \text{ m}$$



$$N_{CD} := 0 \quad \frac{1}{\text{m}} \quad T_{CD} := 0 \quad \frac{1}{\text{m}}$$

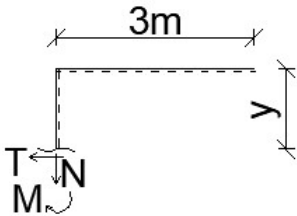
$$M_{CD}(x) := 0$$

$$M_{CD}(0 \text{ m}) = 0$$

$$M_{CD}(3 \text{ m}) = 0$$

Pręt BC

$$y \in (0; 4) \text{ m}$$



$$N_{BC} := 0 \quad \frac{1}{\text{m}} \quad T_{BC} := 0 \quad \frac{1}{\text{m}}$$

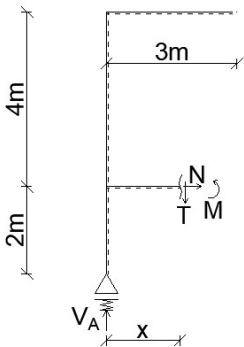
$$M_{BC}(y) := 0$$

$$M_{BC}(0 \text{ m}) = 0$$

$$M_{BC}(4 \text{ m}) = 0$$

Pręt BF

$$x \in (0; 7) \text{ m}$$



$$N_{BF} := 0 \quad \frac{1}{\text{m}}$$

$$T_{BF} := V_A = -0.143 \quad \frac{1}{\text{m}}$$

$$M_{BF}(x) := V_A \cdot x$$

$$M_{BF}(0 \text{ m}) = 0$$

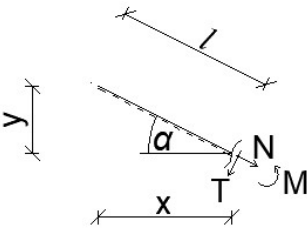
$$M_{BF}(7 \text{ m}) = -1$$

Pręt DE

$$x \in (0; 4) \text{ m}$$

$$y(x) := 0.5 x$$

$$l(x) := \sqrt{y(x)^2 + x^2}$$



$$N_{DE} := 0 \quad \frac{1}{\text{m}} \quad T_{DE} := 0 \quad \frac{1}{\text{m}}$$

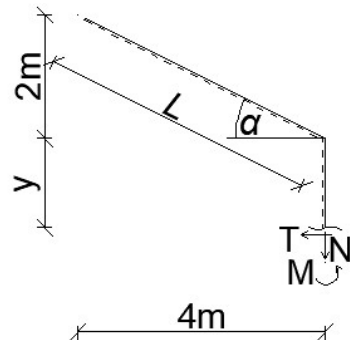
$$M_{DE}(x) := 0$$

$$M_{DE}(0 \text{ m}) = 0$$

$$M_{DE}(4 \text{ m}) = 0$$

Pręt EF

$$y \in (0; 2) \text{ m}$$



$$N_{EF} := 0 \quad \frac{1}{\text{m}} \quad T_{EF} := 0 \quad \frac{1}{\text{m}}$$

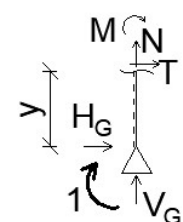
$$M_{EF}(y) := 0$$

$$M_{EF}(0 \text{ m}) = 0$$

$$M_{EF}(7 \text{ m}) = 0$$

Pręt FG

$$y \in (0; 2) \text{ m}$$



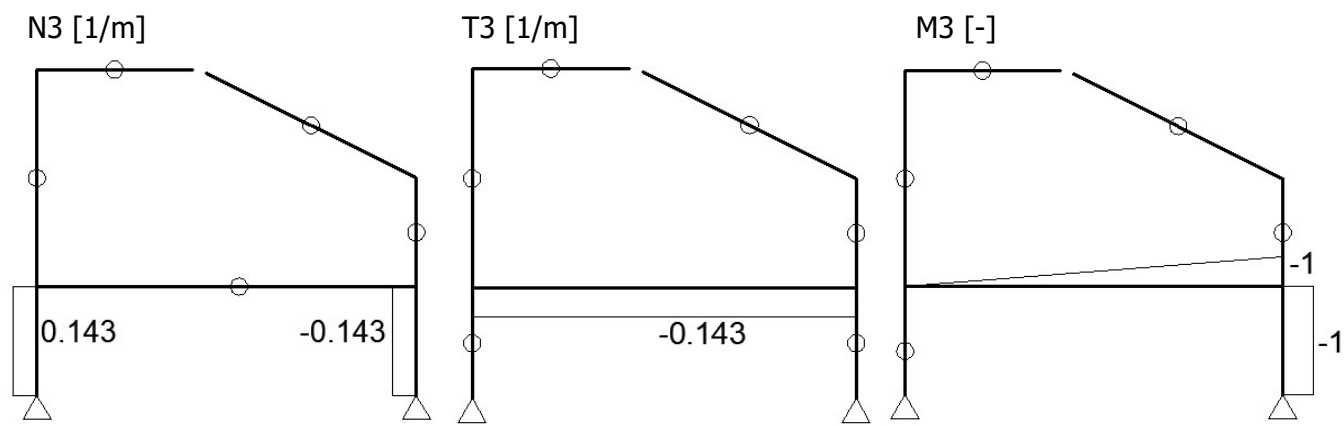
$$N_{FG} := -V_G = -0.143 \quad \frac{1}{\text{m}}$$

$$T_{FG} := -H_G = 0 \quad \frac{1}{\text{m}}$$

$$M_{FG}(y) := H_G \cdot y - 1$$

$$M_{FG}(0 \text{ m}) = -1$$

$$M_{FG}(2 \text{ m}) = -1$$



Obliczenie przemieszczeń

$$\delta_{11} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right]$$

$$\delta_{11} := \frac{\text{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 + 4 \cdot 7 \cdot 4 + \frac{1}{2} \cdot 2 \cdot 4 \cdot 4.472 \cdot \frac{2}{3} \cdot 2 + \frac{1}{2} \cdot 2 \cdot 2 \cdot \left(\frac{2}{3} \cdot 2 + \frac{1}{3} \cdot 4 \right) + \frac{1}{2} \cdot 2 \cdot 4 \cdot \left(\frac{1}{3} \cdot 2 + \frac{2}{3} \cdot 4 \right) \right)$$

$$\delta_{11} = 0.22933 \frac{\text{m}}{\text{kN}}$$

$$\delta_{12} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right]$$

$$\delta_{12} := \frac{\text{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 4 \cdot 4 \cdot 3 + 4 \cdot 7 \cdot \frac{3-4}{2} - \frac{1}{2} \cdot 2 \cdot 4 \cdot 4.472 \cdot \frac{2}{3} \cdot 4 - \frac{2+4}{2} \cdot 2 \cdot 4 \right) = -0.03764 \frac{\text{m}}{\text{kN}} \quad \delta_{21} := \delta_{12}$$

$$\delta_{13} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]$$

$$\delta_{13} := \frac{\text{m}^2}{EJ} \cdot \left(-4 \cdot 7 \cdot \frac{1}{2} \cdot 1 \right) = -0.02033 \frac{1}{\text{kN}} \quad \delta_{31} := \delta_{13}$$

$$\delta_{22} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \\ \text{Diagram 5} \end{array} \right]$$

$$\delta_{22} := \frac{\text{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 3 \cdot 3 \cdot \frac{2}{3} \cdot 3 + 3 \cdot 4 \cdot 3 + \frac{1}{2} \cdot 3 \cdot 7 \cdot \left(\frac{2}{3} \cdot 3 - \frac{1}{3} \cdot 4 \right) + \frac{1}{2} \cdot 4 \cdot 7 \cdot \left(-\frac{1}{3} \cdot 3 + \frac{2}{3} \cdot 4 \right) + \frac{1}{2} \cdot 4 \cdot 4 \cdot 4.472 \cdot \frac{2}{3} \cdot 4 + 4 \cdot 2 \cdot 4 \right)$$

$$\delta_{22} = 0.19045 \frac{\text{m}}{\text{kN}}$$

$$\delta_{23} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]$$

$$\delta_{23} := \frac{\text{m}^2}{EJ} \cdot \left(\frac{1}{2} \cdot 1 \cdot 7 \cdot \left(\frac{2}{3} \cdot 4 - \frac{1}{3} \cdot 3 \right) \right) = 0.00847 \frac{1}{\text{kN}} \quad \delta_{32} := \delta_{23}$$

$$\delta_{33} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1: Triangle with base 7, height 1} \\ \text{Diagram 2: Rectangle with width 1, height 2} \\ \text{Diagram 3: Rectangle with width 1, height 1} \end{array} \right]$$

$$\delta_{33} := \frac{\mathbf{m}}{EJ} \cdot \left(\frac{1}{2} \cdot 1 \cdot 7 \cdot \frac{2}{3} \cdot 1 + 1 \cdot 2 \cdot 1 \right) = 0.00629 \frac{1}{\mathbf{kNm}}$$

$$\delta_{1p} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1: Triangle with base 24, height 4} \\ \text{Diagram 2: Triangle with base 4, height 4} \\ \text{Diagram 3: Trapezoid with bases 14 and 7, height 5.889} \\ \text{Diagram 4: Trapezoid with bases 4.472 and 17.889, height 2} \\ \text{Diagram 5: Rectangle with width 17.889, height 2} \\ \text{Diagram 6: Triangle with base 2, height 4} \end{array} \right]$$

Pole parabolii: $A_p := \frac{2}{3} \cdot \frac{q \cdot \cos \alpha \cdot L^2}{8} \cdot L \cdot \frac{1}{\mathbf{kN} \cdot \mathbf{m}^2} = 13.333$

$$\delta_{1p} := \frac{\mathbf{kN} \cdot \mathbf{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 24 \cdot 4 \cdot \frac{2}{3} \cdot 4 + 4 \cdot 7 \cdot \frac{14 + 5.889}{2} + \frac{1}{2} \cdot 17.889 \cdot 4.472 \cdot \frac{2}{3} \cdot 2 - A_p \cdot \frac{1}{2} \cdot 2 + \frac{2 + 4}{2} \cdot 2 \cdot 17.889 \right) = 0.80398 \mathbf{m}$$

$$\delta_{2p} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1: Triangle with base 24, height 4} \\ \text{Diagram 2: Rectangle with width 3, height 4} \\ \text{Diagram 3: Trapezoid with bases 14 and 7, height 5.889} \\ \text{Diagram 4: Trapezoid with bases 4.472 and 17.889, height 4} \\ \text{Diagram 5: Rectangle with width 17.889, height 4} \\ \text{Diagram 6: Rectangle with width 2, height 4} \end{array} \right]$$

$$\delta_{2p1} := \frac{\mathbf{kN} \cdot \mathbf{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 24 \cdot 4 \cdot 3 + \frac{1}{2} \cdot 14 \cdot 7 \cdot \left(\frac{2}{3} \cdot 3 - \frac{1}{3} \cdot 4 \right) + \frac{1}{2} \cdot 5.889 \cdot 7 \cdot \left(\frac{1}{3} \cdot 3 - \frac{2}{3} \cdot 4 \right) \right)$$

$$\delta_{2p2} := \frac{\mathbf{kN} \cdot \mathbf{m}^3}{EJ} \cdot \left(-\frac{1}{2} \cdot 17.889 \cdot 4.472 \cdot \frac{2}{3} \cdot 4 + A_p \cdot \frac{1}{2} \cdot 4 - 17.889 \cdot 2 \cdot 4 \right)$$

$$\delta_{2p} := \delta_{2p1} + \delta_{2p2} = -0.1173 \mathbf{m}$$

$$\delta_{3p} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1: Trapezoid with bases 14 and 7, height 5.889} \\ \text{Diagram 2: Triangle with base 12, height 1} \\ \text{Diagram 3: Rectangle with width 2, height 1} \end{array} \right]$$

$$\delta_{3p} := \frac{\mathbf{kN} \cdot \mathbf{m}^2}{EJ} \cdot \left(\frac{-1}{2} \cdot 1 \cdot 7 \cdot \left(\frac{2}{3} \cdot 5.889 + \frac{1}{3} \cdot 14 \right) + \frac{1}{2} \cdot 12 \cdot 2 \cdot 1 \right) = -0.02624$$

Temperatura średnia:

$$t_{01} := \frac{0 + 10}{2} \mathbf{K} = 5 \mathbf{K} \quad t_{02} := \frac{10 - 20}{2} \mathbf{K} = -5 \mathbf{K}$$

$$\delta_{1t0} := 1 \cdot 3 \mathbf{m} \cdot t_{01} \cdot \alpha_t + 0 \cdot 2 \mathbf{m} \cdot t_{02} \cdot \alpha_t = 0.00018 \mathbf{m}$$

$$\delta_{2t0} := 0 \cdot 3 \mathbf{m} \cdot t_{01} \cdot \alpha_t + 1 \cdot 2 \mathbf{m} \cdot t_{02} \cdot \alpha_t = -0.00012 \mathbf{m}$$

$$\delta_{3t0} := 0 \frac{1}{\mathbf{m}} \cdot 3 \mathbf{m} \cdot t_{01} \cdot \alpha_t + 0 \frac{1}{\mathbf{m}} \cdot 2 \mathbf{m} \cdot t_{02} \cdot \alpha_t = 0$$

Różnica temperatur:

$$\Delta t_1 := (10 - 0) \mathbf{K} = 10 \mathbf{K} \quad \Delta t_2 := (10 - -20) \mathbf{K} = 30 \mathbf{K}$$

$$\delta_{1\Delta t} := 0 \mathbf{m} \cdot 3 \mathbf{m} \cdot \frac{\Delta t_1 \cdot \alpha_t}{h} - \frac{2 + 4}{2} \mathbf{m} \cdot 2 \mathbf{m} \cdot \frac{\Delta t_2 \cdot \alpha_t}{h} = -0.018 \mathbf{m}$$

$$\delta_{2\Delta t} := \frac{-1}{2} \cdot 3 \mathbf{m} \cdot 3 \mathbf{m} \cdot \frac{\Delta t_1 \cdot \alpha_t}{h} + 4 \mathbf{m} \cdot 2 \mathbf{m} \cdot \frac{\Delta t_2 \cdot \alpha_t}{h} = 0.0195 \mathbf{m}$$

$$\delta_{3\Delta t} := 0 \cdot 3 \mathbf{m} \cdot \frac{\Delta t_1 \cdot \alpha_t}{h} + 0 \cdot 2 \mathbf{m} \cdot \frac{\Delta t_2 \cdot \alpha_t}{h} = 0$$

Wymuszenie w podporze:

$$R_1 := 0 \text{ m} \quad R_2 := 0 \text{ m} \quad R_3 := 1 \quad \Delta := 1^\circ = 0.01745$$

$$\delta_{1\Delta} := -R_1 \cdot \Delta = 0 \text{ m} \quad \delta_{2\Delta} := -R_2 \cdot \Delta = 0 \text{ m} \quad \delta_{3\Delta} := -R_3 \cdot \Delta = -0.01745$$

Podpora sprężysta:

$$R_p := -1.159 \text{ kN} \quad R_1 := 0 \quad R_2 := 0 \quad R_3 := -0.143 \frac{1}{\text{m}} \quad k := 500 \frac{\text{kN}}{\text{m}}$$

$$\delta_{11k} := \frac{R_1 \cdot R_1}{k} = 0 \frac{\text{m}}{\text{kN}} \quad \delta_{12k} := \frac{R_1 \cdot R_2}{k} = 0 \frac{\text{m}}{\text{kN}} \quad \delta_{13k} := \frac{R_1 \cdot R_3}{k} = 0 \frac{1}{\text{kN}}$$

$$\delta_{21k} := \frac{R_2 \cdot R_1}{k} = 0 \frac{\text{m}}{\text{kN}} \quad \delta_{22k} := \frac{R_2 \cdot R_2}{k} = 0 \frac{\text{m}}{\text{kN}} \quad \delta_{23k} := \frac{R_2 \cdot R_3}{k} = 0 \frac{1}{\text{kN}}$$

$$\delta_{31k} := \frac{R_3 \cdot R_1}{k} = 0 \frac{1}{\text{kN}} \quad \delta_{32k} := \frac{R_3 \cdot R_2}{k} = 0 \frac{1}{\text{kN}} \quad \delta_{33k} := \frac{R_3 \cdot R_3}{k} = 0.00004 \frac{1}{\text{kNm}}$$

$$\delta_{1k} := \frac{R_1 \cdot R_p}{k} = 0 \text{ m} \quad \delta_{2k} := \frac{R_2 \cdot R_p}{k} = 0 \text{ m} \quad \delta_{3k} := \frac{R_3 \cdot R_p}{k} = 0.00033$$

Sumaryczne przemieszczenia:

$$\delta_{1P} := \delta_{1p} + \delta_{1t0} + \delta_{1\Delta t} + \delta_{1\Delta} + \delta_{1k} = 0.78616 \text{ m}$$

$$\delta_{2P} := \delta_{2p} + \delta_{2t0} + \delta_{2\Delta t} + \delta_{2\Delta} + \delta_{2k} = -0.09792 \text{ m}$$

$$\delta_{3P} := \delta_{3p} + \delta_{3t0} + \delta_{3\Delta t} + \delta_{3\Delta} + \delta_{3k} = -0.04336$$

Rozwiązanie układu równań

$$(\delta_{11} + \delta_{11k}) x_1 + (\delta_{12} + \delta_{12k}) x_2 + (\delta_{13} + \delta_{13k}) x_3 + \delta_{1P} = 0$$

$$(\delta_{21} + \delta_{21k}) x_1 + (\delta_{22} + \delta_{22k}) x_2 + (\delta_{23} + \delta_{23k}) x_3 + \delta_{2P} = 0$$

$$(\delta_{31} + \delta_{31k}) x_1 + (\delta_{32} + \delta_{32k}) x_2 + (\delta_{33} + \delta_{33k}) x_3 + \delta_{3P} = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} \delta_{11} + \delta_{11k} & \delta_{12} + \delta_{12k} & \delta_{13} + \delta_{13k} \\ \delta_{21} + \delta_{21k} & \delta_{22} + \delta_{22k} & \delta_{23} + \delta_{23k} \\ \delta_{31} + \delta_{31k} & \delta_{32} + \delta_{32k} & \delta_{33} + \delta_{33k} \end{bmatrix}^{-1} \cdot - \begin{bmatrix} \delta_{1P} \\ \delta_{2P} \\ \delta_{3P} \end{bmatrix}$$

$$x_1 = -3.943 \text{ kN} \quad x_2 = -0.007 \text{ kN} \quad x_3 = -5.8 \text{ kNm}$$

Wykresy ostateczne

$$R_{ost} = R_1 \cdot x_1 + R_2 \cdot x_2 + R_3 \cdot x_3 + R_p$$

$$V_A := 0 x_1 + 0 x_2 + \left(-0.143 \cdot \frac{1}{\text{m}} \right) x_3 + (-1.159 \text{ kN}) = -0.33 \text{ kN}$$

$$V_G := 0 x_1 + 0 x_2 + \left(0.143 \cdot \frac{1}{\text{m}} \right) x_3 + (10.103 \text{ kN}) = 9.274 \text{ kN}$$

$$H_G := 0 x_1 + 0 x_2 + \left(0 \cdot \frac{1}{\text{m}} \right) x_3 + (-6 \text{ kN}) = -6 \text{ kN}$$

$$M_G := 0 \text{ m} x_1 + 0 \text{ m} x_2 + (1) x_3 + (0 \text{ kNm}) = -5.8 \text{ kNm}$$

$$N_{ost} = N_1 \cdot x_1 + N_2 \cdot x_2 + N_3 \cdot x_3 + N_p$$

$$N_{ost} := \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ -1 & -1 \\ 0.894 & 0.894 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot x_1 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & -1 \\ 0 & 0 \\ 0.447 & 0.447 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \cdot x_2 + \begin{bmatrix} 0.143 & 0.143 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -0.143 & -0.143 \end{bmatrix} \frac{1}{\mathbf{m}} x_3 + \begin{bmatrix} 1.159 & 1.159 \\ 0 & 0 \\ 0 & 0 \\ -6 & -6 \\ 0 & -4 \\ -8.944 & -8.944 \\ -10.103 & -10.103 \end{bmatrix} \text{ kN}$$

$$T_{ost} = T_1 \cdot x_1 + T_2 \cdot x_2 + T_3 \cdot x_3 + T_p$$

$$T_{ost} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ -0.447 & -0.447 \\ -1 & -1 \\ 0 & 0 \end{bmatrix} \cdot x_1 + \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ -1 & -1 \\ 0.894 & 0.894 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot x_2 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -0.143 & -0.143 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{\mathbf{m}} x_3 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 6 & 6 \\ -1.159 & -1.159 \\ 0 & -8 \\ 0 & 0 \\ 6 & 6 \end{bmatrix} \text{ kN}$$

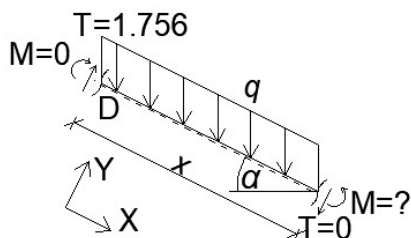
$$M_{ost} = M_1 \cdot x_1 + M_2 \cdot x_2 + M_3 \cdot x_3 + M_p$$

$$M_{ost} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -4 \\ 4 & 4 \\ 0 & -2 \\ -2 & -4 \\ 0 & 0 \end{bmatrix} \mathbf{m} \cdot x_1 + \begin{bmatrix} 0 & 0 \\ 0 & -3 \\ -3 & -3 \\ 3 & -4 \\ 0 & 4 \\ 4 & 4 \\ 0 & 0 \end{bmatrix} \mathbf{m} \cdot x_2 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ -1 & -1 \end{bmatrix} x_3 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -24 \\ 14 & 5.889 \\ 0 & -17.889 \\ -17.889 & -17.889 \\ -12 & 0 \end{bmatrix} \text{ kNm}$$

Wyniki:

$$N_{ost} = \begin{bmatrix} 0.33 & 0.33 \\ -3.943 & -3.943 \\ 0.007 & 0.007 \\ -2.057 & -2.057 \\ -3.529 & -7.529 \\ -8.951 & -8.951 \\ -9.274 & -9.274 \end{bmatrix} \text{ kN} \quad T_{ost} = \begin{bmatrix} 0 & 0 \\ -0.007 & -0.007 \\ 2.057 & 2.057 \\ -0.322 & -0.322 \\ 1.756 & -6.244 \\ 3.943 & 3.943 \\ 6 & 6 \end{bmatrix} \text{ kN} \quad M_{ost} = \begin{bmatrix} 0 & 0 \\ 0 & 0.022 \\ 0.022 & -8.205 \\ -1.795 & -4.055 \\ 0 & -10.031 \\ -10.031 & -2.145 \\ -6.2 & 5.8 \end{bmatrix} \text{ kNm}$$

Obliczenie ekstremum paraboli



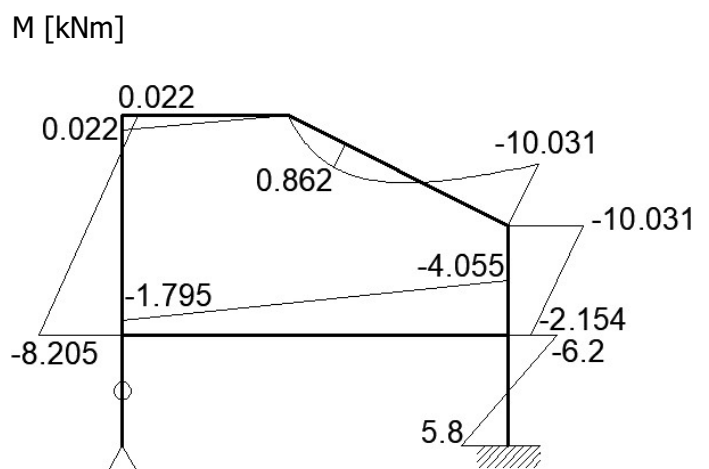
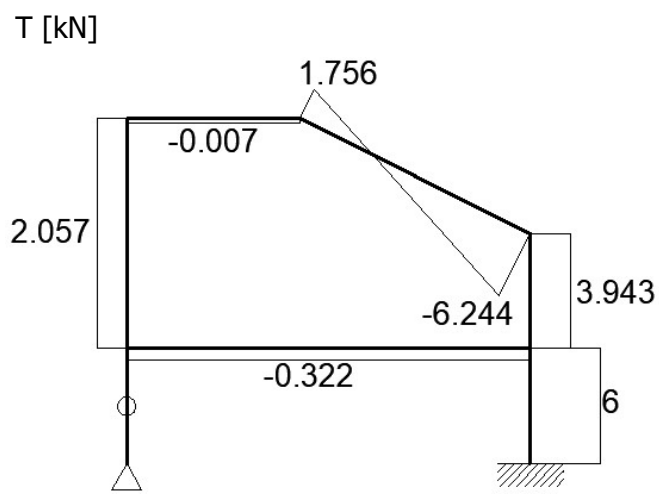
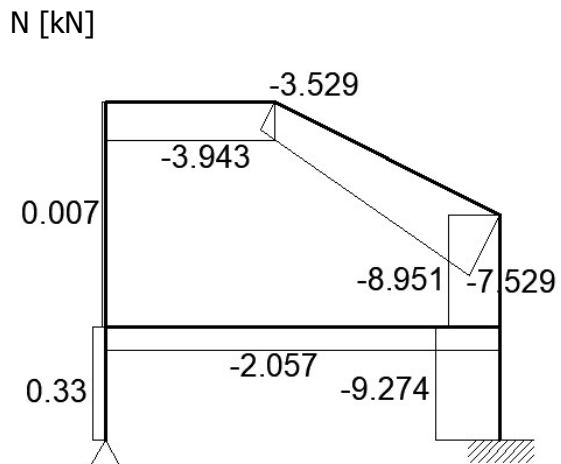
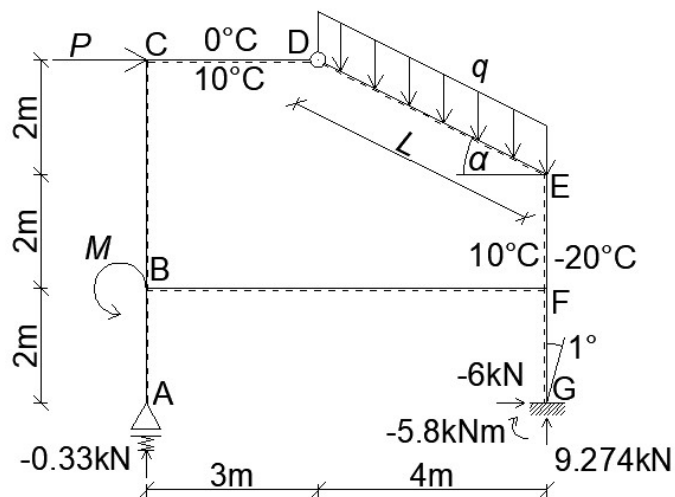
$$\Sigma Y = 1.756 - 0 - q \cdot \cos \alpha \cdot x = 0$$

$$x := \frac{1.756 \text{ kN} - 0 \text{ kN}}{q \cdot \cos \alpha} = 0.982 \text{ m}$$

$$\Sigma M_D = 0 - M_{max} + q \cdot x \cdot \frac{x \cdot \cos \alpha}{2} = 0$$

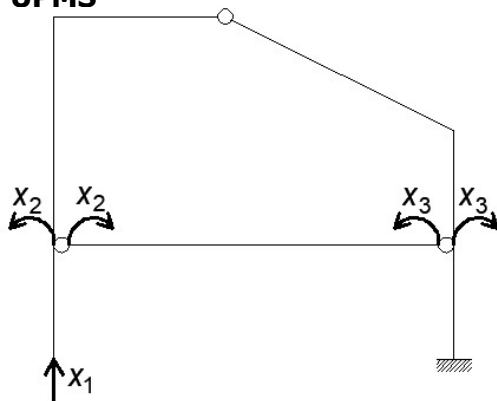
$$M_{max} := q \cdot x \cdot \frac{x \cdot \cos \alpha}{2} = 0.862 \text{ kNm}$$

Wykresy ostateczne



Sprawdzenie kinematyczne

UPMS*



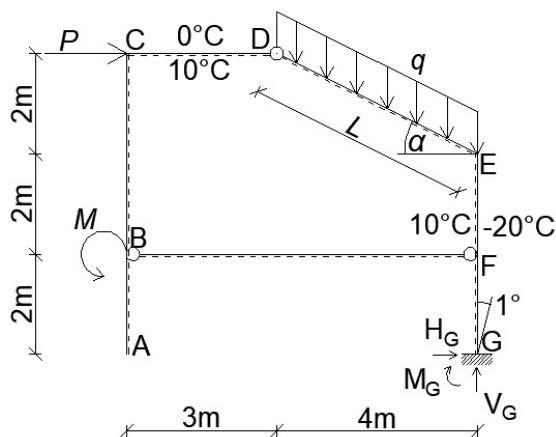
Reakcje:

$$\Sigma M_G = P \cdot 6 - M - q \cdot L \cdot 2 + M_G = 0$$

$$\Sigma X = H_G + 6 = 0$$

$$\Sigma Y = V_G - q \cdot L = 0$$

stan p*

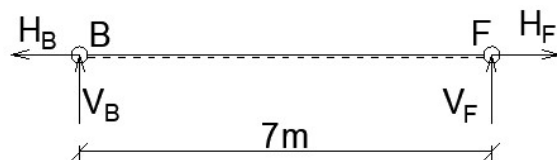
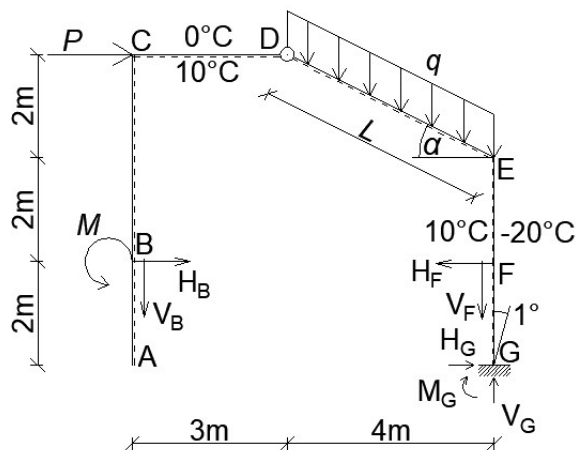


$$M_G := -P \cdot 6 \text{ m} + M + q \cdot L \cdot 2 \text{ m} = -8.111 \text{ kNm}$$

$$H_G := -6 \text{ kN}$$

$$V_G := q \cdot L = 8.944 \text{ kN}$$

Siły w ściągu:



Ściąg: $\Sigma M_B = -V_F \cdot 7 = 0 \quad V_F := 0 \text{ kN}$

Ściąg: $\Sigma Y = V_B + V_F = 0 \quad V_B := -V_F = 0 \text{ kN}$

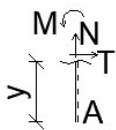
Rama: $\Sigma M_D^L = -M - H_B \cdot 4 - V_B \cdot 3 = 0$

$$H_B := \frac{-M - V_B \cdot 3 \text{ m}}{4 \text{ m}} = -2.5 \text{ kN}$$

Ściąg: $\Sigma X = -H_B + H_F = 0 \quad H_F := H_B = -2.5 \text{ kN}$

Pręt AB

$$y \in (0; 2) \text{ m}$$



$$N_{AB} := 0 \text{ kN}$$

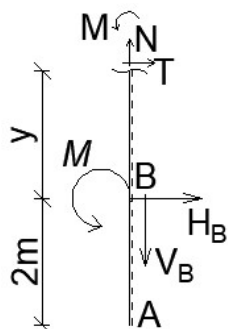
$$T_{AB} := 0 \text{ kN}$$

$$M_{AB}(y) := 0 \text{ kNm}$$

$$M_{AB}(0 \text{ m}) = 0 \text{ kNm} \quad M_{AB}(2 \text{ m}) = 0 \text{ kNm}$$

Pręt BC

$$y \in (0; 4) \text{ m}$$



$$N_{BC} := V_B = 0 \text{ kN}$$

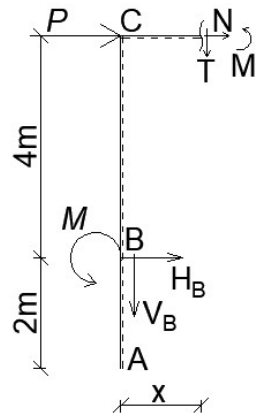
$$T_{BC} := -H_B = 2.5 \text{ kN}$$

$$M_{BC}(y) := -M - H_B \cdot y$$

$$M_{BC}(0 \text{ m}) = -10 \text{ kNm} \quad M_{BC}(4 \text{ m}) = 0 \text{ kNm}$$

Pręt CD

$$x \in (0; 3) \text{ m}$$



$$N_{CD} := -H_B - P = -3.5 \text{ kN}$$

$$T_{CD} := -V_B = 0 \text{ kN}$$

$$M_{CD}(x) := -M - H_B \cdot 4 \text{ m} - V_B \cdot x$$

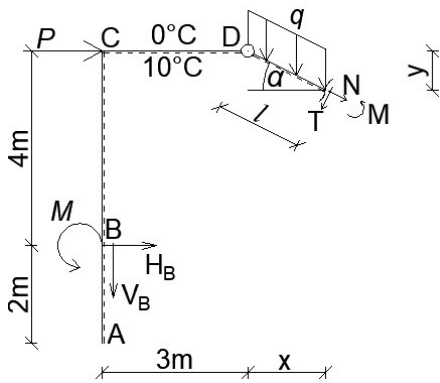
$$M_{CD}(0 \text{ m}) = 0 \text{ kNm} \quad M_{CD}(3 \text{ m}) = 0 \text{ kNm}$$

Pręt DE

$x \in (0; 4) \text{ m}$

$$y(x) := 0.5 x$$

$$l(x) := \sqrt{y(x)^2 + x^2}$$



$$N_{DE}(x) := -H_B \cdot \cos \alpha - V_B \cdot \sin \alpha - P \cdot \cos \alpha - q \cdot l(x) \cdot \sin \alpha$$

$$T_{DE}(x) := H_B \cdot \sin \alpha - V_B \cdot \cos \alpha + P \cdot \sin \alpha - q \cdot l(x) \cdot \cos \alpha$$

$$M_{DE}(x) := -M - H_B \cdot (4 \text{ m} - y(x)) - V_B \cdot (3 \text{ m} + x) + P \cdot y(x) - q \cdot l(x) \cdot \frac{x}{2}$$

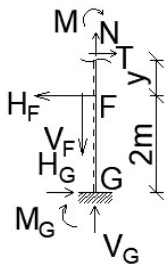
$$N_{DE}(0 \text{ m}) = -3.13 \text{ kN} \quad N_{DE}(4 \text{ m}) = -7.13 \text{ kN}$$

$$T_{DE}(0 \text{ m}) = 1.565 \text{ kN} \quad T_{DE}(4 \text{ m}) = -6.435 \text{ kN}$$

$$M_{DE}(0 \text{ m}) = 0 \text{ kNm} \quad M_{DE}(4 \text{ m}) = -10.889 \text{ kNm}$$

Pręt EF

$y \in (0; 2) \text{ m}$



$$N_{EF} := V_F - V_G = -8.944 \text{ kN}$$

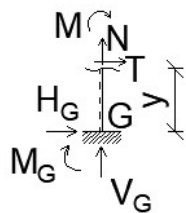
$$T_{EF} := H_F - H_G = 3.5 \text{ kN}$$

$$M_{EF}(y) := -H_F \cdot y + H_G \cdot (2 \text{ m} + y) - M_G$$

$$M_{EF}(0 \text{ m}) = -3.889 \text{ kNm} \quad M_{EF}(2 \text{ m}) = -10.889 \text{ kNm}$$

Pręt FG

$y \in (0; 2) \text{ m}$



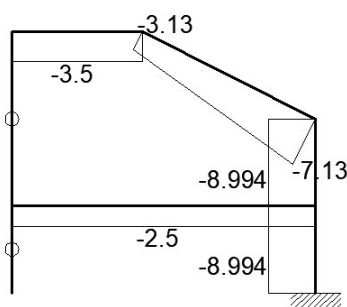
$$N_{FG} := -V_G = -8.944 \text{ kN}$$

$$T_{FG} := -H_G = 6 \text{ kN}$$

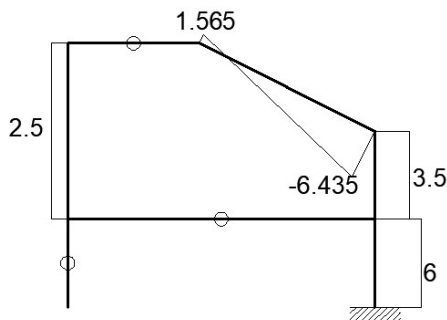
$$M_{FG}(y) := H_G \cdot y - M_G$$

$$M_{FG}(0 \text{ m}) = 8.111 \text{ kNm} \quad M_{FG}(2 \text{ m}) = -3.889 \text{ kNm}$$

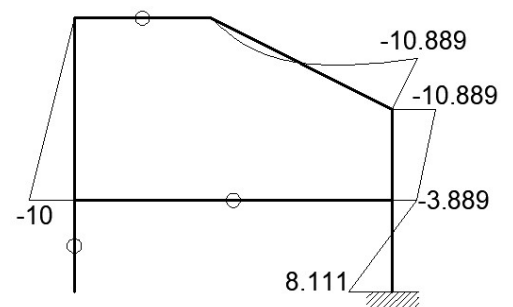
$N_p^* \text{ [kN]}$



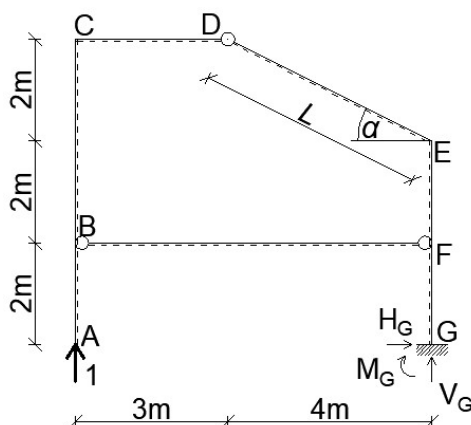
$T_p^* \text{ [kN]}$



$M_p^* \text{ [kNm]}$



Stan $x_1^* = 1$



Reakcje:

$$\Sigma M_G = 1 \cdot 7 + M_G = 0$$

$$M_G := -1 \cdot 7 \text{ m} = -7 \text{ m}$$

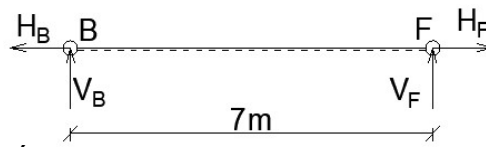
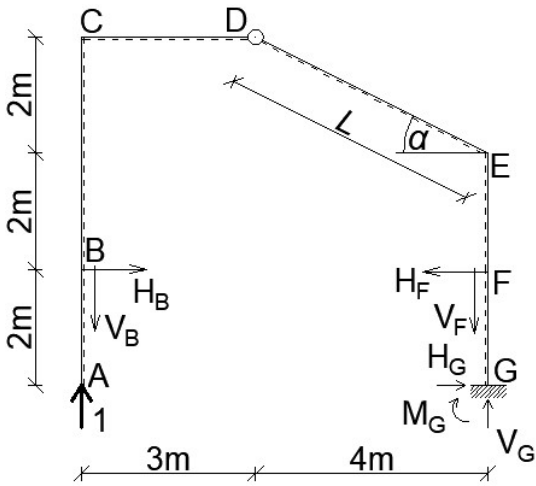
$$\Sigma X = H_G = 0$$

$$H_G := 0$$

$$\Sigma Y = V_G + 1 = 0$$

$$V_G := -1$$

Siły w ściąg:



$$\text{Ściąg: } \Sigma M_B = -V_F \cdot 7 = 0 \quad V_F := 0$$

$$\text{Ściąg: } \Sigma Y = V_B + V_F = 0 \quad V_B := -V_F = 0$$

$$\text{Rama: } \Sigma M_D^L = 1 \cdot 3 - H_B \cdot 4 - V_B \cdot 3 = 0$$

$$H_B := \frac{1 \cdot 3 \text{ m} - V_B \cdot 3 \text{ m}}{4 \text{ m}} = 0.75$$

$$\text{Ściąg: } \Sigma X = -H_B + H_F = 0 \quad H_F := H_B = 0.75$$

Pręt AB

$$y \in (0; 2) \text{ m}$$

$$N_{AB} := -1$$

$$T_{AB} := 0$$

$$M_{AB}(y) := 0 \text{ m}$$

$$M_{AB}(0 \text{ m}) = 0 \text{ m} \quad M_{AB}(2 \text{ m}) = 0 \text{ m}$$

Pręt BC

$$y \in (0; 4) \text{ m}$$

$$N_{BC} := -1 + V_B = -1$$

$$T_{BC} := -H_B = -0.75$$

$$M_{BC}(y) := -H_B \cdot y$$

$$M_{BC}(0 \text{ m}) = 0 \text{ m} \quad M_{BC}(4 \text{ m}) = -3 \text{ m}$$

Pręt CD

$$x \in (0; 3) \text{ m}$$

$$N_{CD} := -H_B = -0.75$$

$$T_{CD} := 1 - V_B = 1$$

$$M_{CD}(x) := 1 \cdot x - H_B \cdot 4 \text{ m} - V_B \cdot x$$

$$M_{CD}(0 \text{ m}) = -3 \text{ m} \quad M_{CD}(3 \text{ m}) = 0 \text{ m}$$

Pręt DE

$$x \in (0; 4) \text{ m}$$

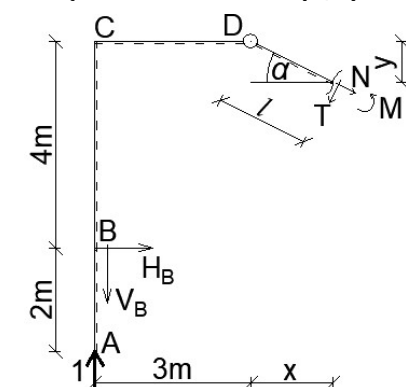
$$y(x) := 0.5 x \quad l(x) := \sqrt{y(x)^2 + x^2}$$

$$N_{DE} := 1 \cdot \sin \alpha - H_B \cdot \cos \alpha - V_B \cdot \sin \alpha = -0.224$$

$$T_{DE} := 1 \cdot \cos \alpha + H_B \cdot \sin \alpha - V_B \cdot \cos \alpha = 1.23$$

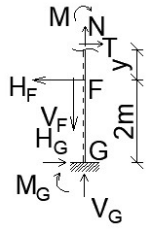
$$M_{DE}(x) := 1 \cdot (3 \text{ m} + x) - H_B \cdot (4 \text{ m} - y(x)) - V_B \cdot (3 \text{ m} + x)$$

$$M_{DE}(0 \text{ m}) = 0 \text{ m} \quad M_{DE}(4 \text{ m}) = 5.5 \text{ m}$$



Pręt EF

$y \in (0; 2) \text{ m}$



$$N_{EF} := V_F - V_G = 1$$

$$T_{EF} := H_F - H_G = 0.75$$

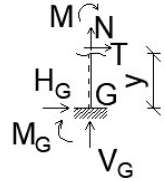
$$M_{EF}(y) := -H_F \cdot y + H_G \cdot (2 \text{ m} + y) - M_G$$

$$M_{EF}(0 \text{ m}) = 7 \text{ m}$$

$$M_{EF}(2 \text{ m}) = 5.5 \text{ m}$$

Pręt FG

$y \in (0; 2) \text{ m}$



$$N_{FG} := -V_G = 1$$

$$T_{FG} := -H_G = 0$$

$$M_{FG}(y) := H_G \cdot y - M_G$$

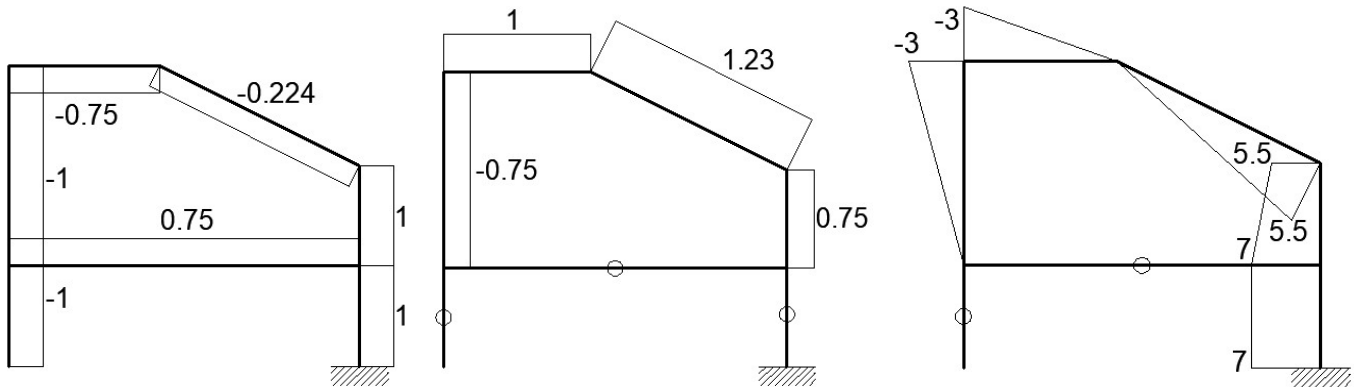
$$M_{FG}(0 \text{ m}) = 7 \text{ m}$$

$$M_{FG}(2 \text{ m}) = 7 \text{ m}$$

$N1^* [-]$

$T1^* [-]$

$M1^* [\text{m}]$



Obliczenie przemieszczeń

$$\delta_{lost} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1: } \frac{1}{2} \cdot 3 \cdot 4 \cdot \left(\frac{1}{3} \cdot 8.205 - \frac{2}{3} \cdot 0.022 \right) \\ \text{Diagram 2: } -\frac{1}{2} \cdot 0.022 \cdot 3 \cdot \frac{2}{3} \cdot 3 \\ \text{Diagram 3: } -\frac{1}{2} \cdot 10.031 \cdot 4.472 \cdot \frac{2}{3} \cdot 5.5 \\ \text{Diagram 4: } \frac{1}{2} \cdot 2.154 \cdot 2 \cdot \left(\frac{1}{3} \cdot 5.5 + \frac{2}{3} \cdot 7 \right) \\ \text{Diagram 5: } 7 \cdot 2 \cdot \frac{5.8 - 6.2}{2} \end{array} \right]$$

$$\delta_{lost1} := \frac{\text{kN} \cdot \text{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 3 \cdot 4 \cdot \left(\frac{1}{3} \cdot 8.205 - \frac{2}{3} \cdot 0.022 \right) - \frac{1}{2} \cdot 0.022 \cdot 3 \cdot \frac{2}{3} \cdot 3 - \frac{1}{2} \cdot 10.031 \cdot 4.472 \cdot \frac{2}{3} \cdot 5.5 \right)$$

$$\delta_{lost2} := \frac{\text{kN} \cdot \text{m}^3}{EJ} \cdot \left(A_p \cdot \frac{1}{2} \cdot 5.5 - \frac{1}{2} \cdot 10.031 \cdot 2 \cdot \left(\frac{2}{3} \cdot 5.5 + \frac{1}{3} \cdot 7 \right) - \frac{1}{2} \cdot 2.154 \cdot 2 \cdot \left(\frac{1}{3} \cdot 5.5 + \frac{2}{3} \cdot 7 \right) + 7 \cdot 2 \cdot \frac{5.8 - 6.2}{2} \right)$$

$$\delta_{lost} := \delta_{lost1} + \delta_{lost2} = -0.15433 \text{ m}$$

$$\delta_{1t0} := -0.75 \cdot 3 \text{ m} \cdot t_{01} \cdot \alpha_t + 1 \cdot 2 \text{ m} \cdot t_{02} \cdot \alpha_t = -0.00026 \text{ m}$$

$$\delta_{1\Delta t} := -\frac{1}{2} \cdot 3 \text{ m} \cdot 3 \text{ m} \cdot \frac{\Delta t_1 \cdot \alpha_t}{h} + \frac{5.5 + 7}{2} \text{ m} \cdot 2 \text{ m} \cdot \frac{\Delta t_2 \cdot \alpha_t}{h} = 0.033 \text{ m}$$

$$\delta_{1\Delta} := -(-7 \text{ m}) \cdot \Delta = 0.12217 \text{ m}$$

$$\delta_{1k} = \frac{R_1 \cdot R_{ost}}{k} \quad \delta_{1k} := \frac{1 \cdot -0.33 \text{ kN}}{k} = -0.00066 \text{ m}$$

$$\delta_1 := \delta_{lost} + \delta_{1t0} + \delta_{1\Delta t} + \delta_{1\Delta} + \delta_{1k} = -0.00008 \text{ m}$$

Sprawdzenie względne:

$$\delta_{1p} = \frac{1}{EJ} \left[\begin{array}{c} \text{Diagram 1: Triangle with base 10, height 4, area 20} \\ \text{Diagram 2: Trapezoid with top 3, bottom 10.889, height 5.5, area 38.89} \\ \text{Diagram 3: Triangle with base 7, height 2, area 7} \\ \text{Diagram 4: Rectangle with width 7, height 2, area 14} \end{array} \right]$$

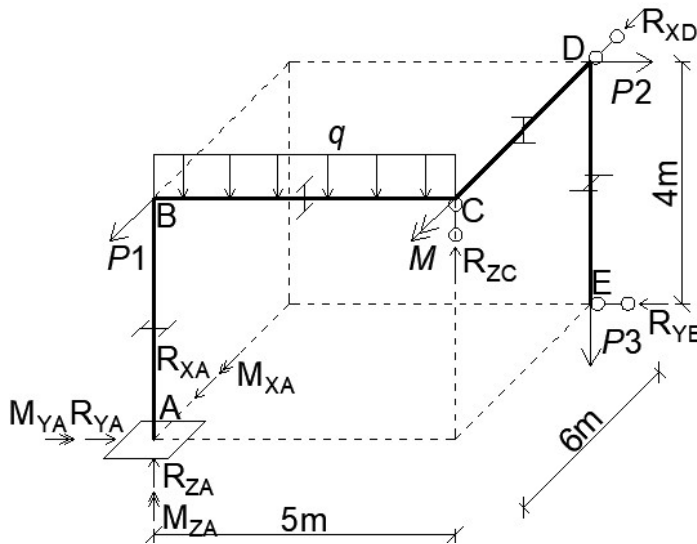
$$\delta_{1p1} := \frac{\text{kN} \cdot \text{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 10 \cdot 4 \cdot \frac{1}{3 \cdot 3} - \frac{1}{2} \cdot 10.889 \cdot 4.472 \cdot \frac{2}{3} \cdot 5.5 + A_p \cdot \frac{1}{2} \cdot 5.5 \right)$$

$$\delta_{1p2} := \frac{\text{kN} \cdot \text{m}^3}{EJ} \cdot \left(-\frac{1}{2} \cdot 10.889 \cdot 2 \cdot \left(\frac{2}{3} \cdot 5.5 + \frac{1}{3} \cdot 7 \right) - \frac{1}{2} \cdot 3.889 \cdot 2 \cdot \left(\frac{1}{3} \cdot 5.5 + \frac{2}{3} \cdot 7 \right) + 7 \cdot 2 \cdot \frac{8.111 - 3.889}{2} \right)$$

$$\delta_{1p} := \delta_{1p1} + \delta_{1p2} = -0.1618 \text{ m}$$

$$\frac{\delta_1}{\delta_{1p}} = 0.047\% < 5\% \quad \text{Warunek spełniony}$$

Zadanie 2: Rama 3D



$$E := 210 \text{ GPa} \quad \nu := 0.3$$

$$G := \frac{E}{2 \cdot (1 + \nu)} = 80.769 \text{ GPa}$$

Przekrój: IPE 180

$$J_2 := 1320 \text{ cm}^4$$

$$EJ_2 := E \cdot J_2 = 2772 \text{ kN} \cdot \text{m}^2$$

$$J_3 := 101 \text{ cm}^4$$

$$EJ_3 := E \cdot J_3 = 212.1 \text{ kN} \cdot \text{m}^2$$

$$J_0 := 4.79 \text{ cm}^4$$

$$GJ_0 := G \cdot J_0 = 3.869 \text{ kN} \cdot \text{m}^2$$

$$P1 := 12 \text{ kN}$$

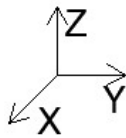
$$P2 := 15 \text{ kN}$$

$$P3 := 7 \text{ kN}$$

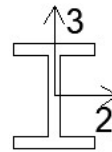
$$M := 8 \text{ kNm}$$

$$q := 3 \frac{\text{kN}}{\text{m}}$$

Układ współrzędnych globalnych:

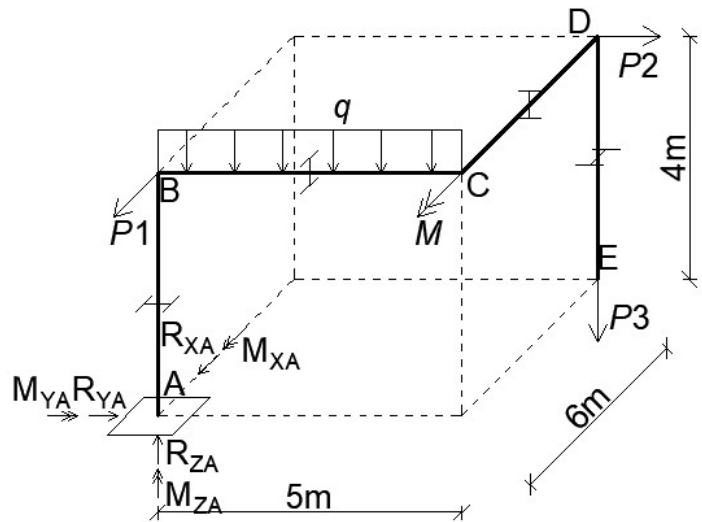
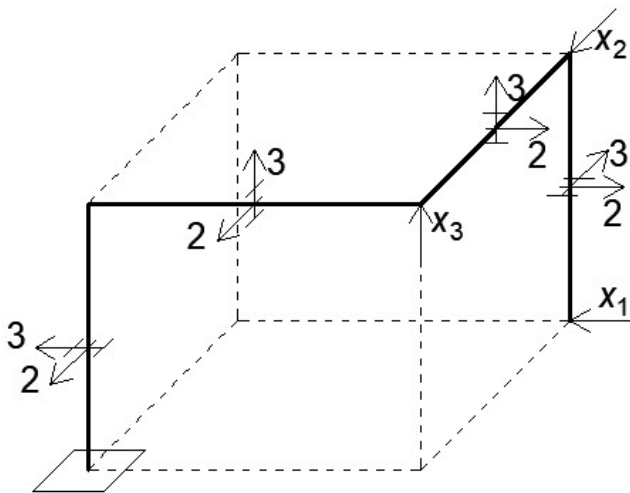


Układ współrzędnych lokalnych:



UPMS i przyjęcie układów lokalnych

Stan p



Reakcje:

$$\sum X = R_{XA} + P1 = 0$$

$$R_{XA} := -P1 = -12 \text{ kN}$$

$$\sum Y = R_{YA} + P2 = 0$$

$$R_{YA} := -P2 = -15 \text{ kN}$$

$$\sum Z = R_{ZA} - P3 - q \cdot 5 = 0$$

$$R_{ZA} := P3 + q \cdot 5 \text{ m} = 22 \text{ kN}$$

$$\sum M_{AX} = M_{XA} + M - q \cdot 5 \cdot 2.5 - P2 \cdot 4 - P3 \cdot 5 = 0$$

$$M_{XA} := -M + q \cdot 5 \text{ m} \cdot 2.5 \text{ m} + P2 \cdot 4 \text{ m} + P3 \cdot 5 \text{ m} = 124.5 \text{ kNm}$$

$$\sum M_{AY} = M_{YA} + P1 \cdot 4 - P3 \cdot 6 = 0$$

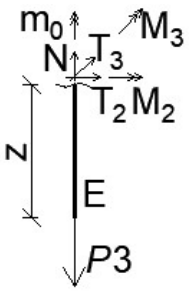
$$M_{YA} := -P1 \cdot 4 \text{ m} + P3 \cdot 6 \text{ m} = -6 \text{ kNm}$$

$$\sum M_{AZ} = M_{ZA} - P2 \cdot 6 = 0$$

$$M_{ZA} := P2 \cdot 6 \text{ m} = 90 \text{ kNm}$$

Pręt ED

$z \in (0;4)\text{m}$



$$N := P3 = 7 \text{ kN}$$

$$T_2 := 0 \text{ kN}$$

$$T_3 := 0 \text{ kN}$$

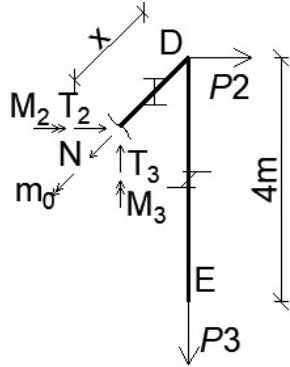
$$m_0 := 0 \text{ kNm}$$

$$M_2 := 0 \text{ kNm}$$

$$M_3 := 0 \text{ kNm}$$

Pręt DC

$x \in (0;6)\text{m}$



$$N := 0 \text{ kN}$$

$$T_2 := -P2 = -15 \text{ kN}$$

$$T_3 := P3 = 7 \text{ kN}$$

$$m_0 := 0 \text{ kNm}$$

$$M_2(x) := P3 \cdot x$$

$$M_3(x) := P2 \cdot x$$

$$M_2(0 \text{ m}) = 0 \text{ kNm}$$

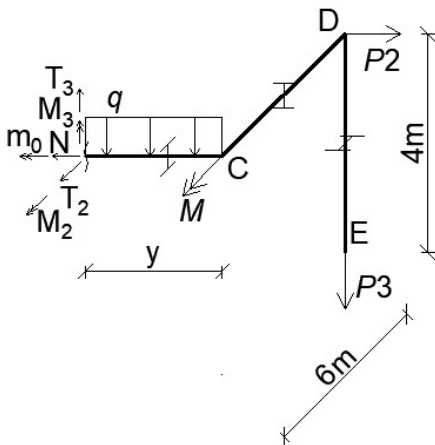
$$M_3(0 \text{ m}) = 0 \text{ kNm}$$

$$M_2(6 \text{ m}) = 42 \text{ kNm}$$

$$M_3(6 \text{ m}) = 90 \text{ kNm}$$

Pręt CB

$y \in (0;5)\text{m}$



$$N := P2 = 15 \text{ kN}$$

$$T_2 := 0 \text{ kN}$$

$$T_3(y) := P3 + q \cdot y$$

$$m_0 := -P3 \cdot 6 \text{ m} = -42 \text{ kNm}$$

$$M_2(y) := P3 \cdot y - M + q \cdot y \cdot \frac{y}{2}$$

$$M_3 := P2 \cdot 6 \text{ m} = 90 \text{ kNm}$$

$$M_2(0 \text{ m}) = -8 \text{ kNm}$$

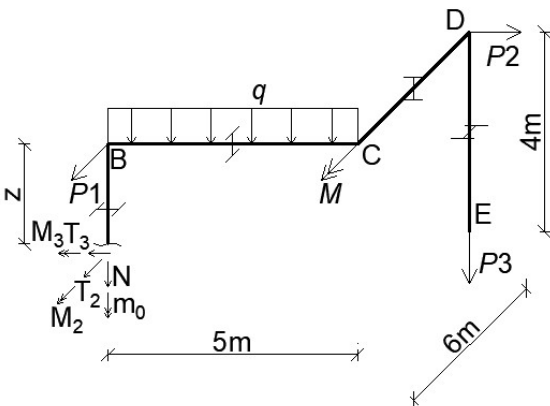
$$T_3(0 \text{ m}) = 7 \text{ kN}$$

$$M_2(5 \text{ m}) = 64.5 \text{ kNm}$$

$$T_3(5 \text{ m}) = 22 \text{ kN}$$

Pręt BA

$z \in (0;4)\text{m}$



$$N := -P3 - q \cdot 5 \text{ m} = -22 \text{ kN}$$

$$T_2 := -P1 = -12 \text{ kN}$$

$$T_3 := P2 = 15 \text{ kN}$$

$$m_0 := -P2 \cdot 6 \text{ m} = -90 \text{ kNm}$$

$$M_2(z) := -M + q \cdot 5 \text{ m} \cdot 2.5 \text{ m} + P2 \cdot z + P3 \cdot 5 \text{ m}$$

$$M_3(z) := P1 \cdot z - P3 \cdot 6 \text{ m}$$

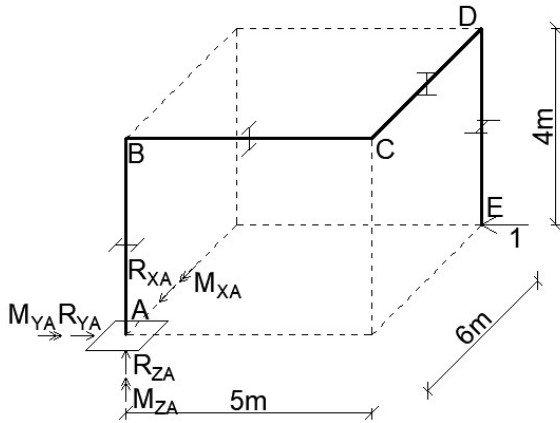
$$M_2(0 \text{ m}) = 64.5 \text{ kNm}$$

$$M_3(0 \text{ m}) = -42 \text{ kNm}$$

$$M_2(4 \text{ m}) = 124.5 \text{ kNm}$$

$$M_3(4 \text{ m}) = 6 \text{ kNm}$$

Stan x1=1



Reakcje:

$$\Sigma X = R_{XA} = 0$$

$$R_{XA} := 0$$

$$\Sigma Y = R_{YA} - 1 = 0$$

$$R_{YA} := 1$$

$$\Sigma Z = R_{ZA} = 0$$

$$R_{ZA} := 0$$

$$\Sigma M_{AX} = M_{XA} = 0$$

$$M_{XA} := 0 \text{ m}$$

$$\Sigma M_{AY} = M_{YA} = 0$$

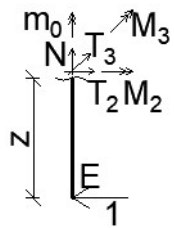
$$M_{YA} := 0 \text{ m}$$

$$\Sigma M_{AZ} = M_{ZA} + 1 \cdot 6 = 0$$

$$M_{ZA} := -1 \cdot 6 \text{ m} = -6 \text{ m}$$

Pręt ED

$$z \in (0; 4) \text{ m}$$



$$N := 0$$

$$T_2 := 1$$

$$T_3 := 0$$

$$m_0 := 0 \text{ m}$$

$$M_2 := 0 \text{ m}$$

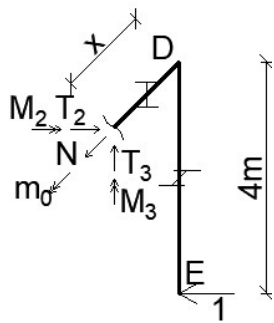
$$M_3(z) := -1 \cdot z$$

$$M_3(0 \text{ m}) = 0 \text{ m}$$

$$M_3(4 \text{ m}) = -4 \text{ m}$$

Pręt DC

$$x \in (0; 6) \text{ m}$$



$$N := 0$$

$$T_2 := 1$$

$$T_3 := 0$$

$$m_0 := 1 \cdot 4 \text{ m} = 4 \text{ m}$$

$$M_2 := 0 \text{ m}$$

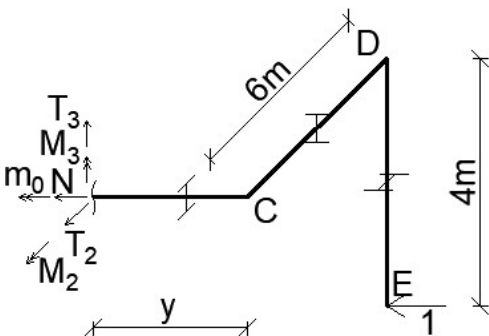
$$M_3(x) := -1 \cdot x$$

$$M_3(0 \text{ m}) = 0 \text{ m}$$

$$M_3(6 \text{ m}) = -6 \text{ m}$$

Pręt CB

$$y \in (0; 5) \text{ m}$$



$$N := -1$$

$$T_2 := 0$$

$$T_3 := 0$$

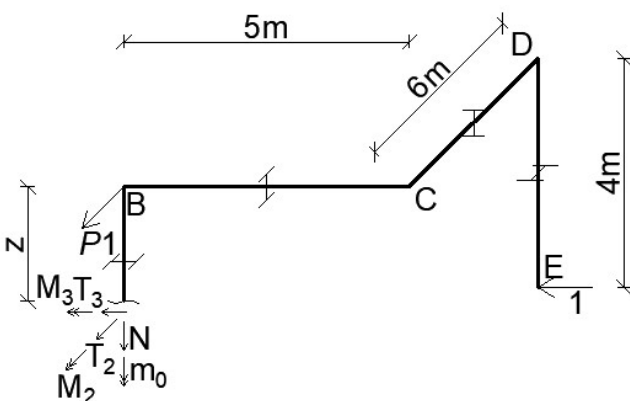
$$m_0 := 0 \text{ m}$$

$$M_2 := 1 \cdot 4 \text{ m} = 4 \text{ m}$$

$$M_3 := -1 \cdot 6 \text{ m} = -6 \text{ m}$$

Pręt BA

$$z \in (0; 4) \text{ m}$$



$$N := 0$$

$$T_2 := 0$$

$$T_3 := -1$$

$$m_0 := 1 \cdot 6 \text{ m} = 6 \text{ m}$$

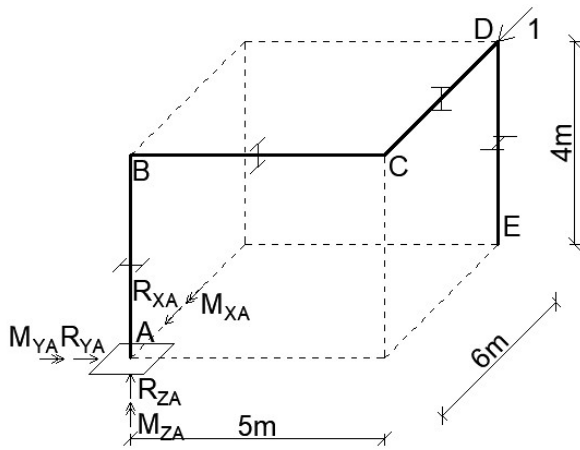
$$M_2(z) := 1 \cdot (4 \text{ m} - z)$$

$$M_3 := 0$$

$$M_2(0 \text{ m}) = 4 \text{ m}$$

$$M_2(4 \text{ m}) = 0 \text{ m}$$

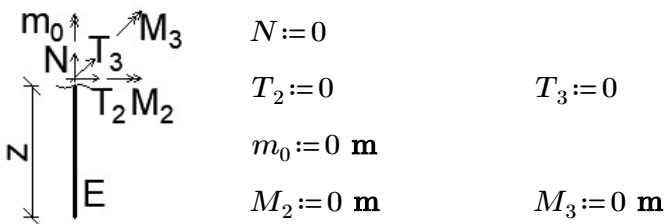
Stan x2=1



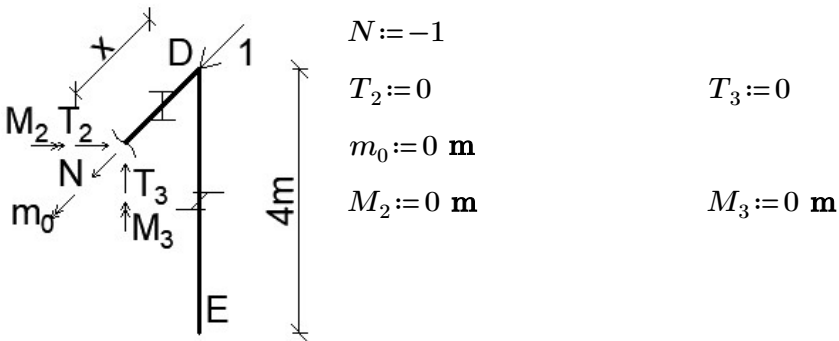
Reakcje:

$$\begin{aligned}\Sigma X = R_{XA} + 1 &= 0 & R_{XA} &:= -1 \\ \Sigma Y = R_{YA} &= 0 & R_{YA} &:= 0 \\ \Sigma Z = R_{ZA} &= 0 & R_{ZA} &:= 0 \\ \Sigma M_{AX} = M_{XA} &= 0 & M_{XA} &:= 0 \text{ m} \\ \Sigma M_{AY} = M_{YA} + 1 \cdot 4 &= 0 & M_{YA} &:= -1 \cdot 4 \text{ m} = -4 \text{ m} \\ \Sigma M_{AZ} = M_{ZA} - 1 \cdot 5 &= 0 & M_{ZA} &:= 1 \cdot 5 \text{ m} = 5 \text{ m}\end{aligned}$$

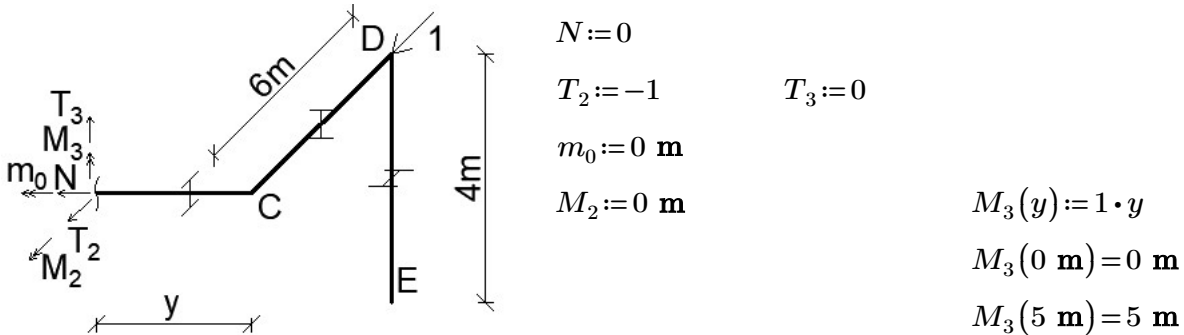
Pręt ED $z \in (0;4)\text{m}$



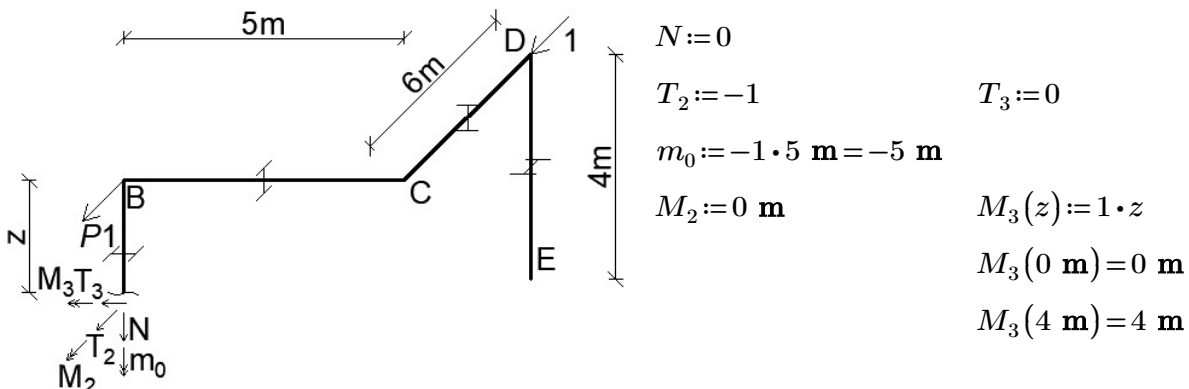
Pręt DC $x \in (0;6)\text{m}$



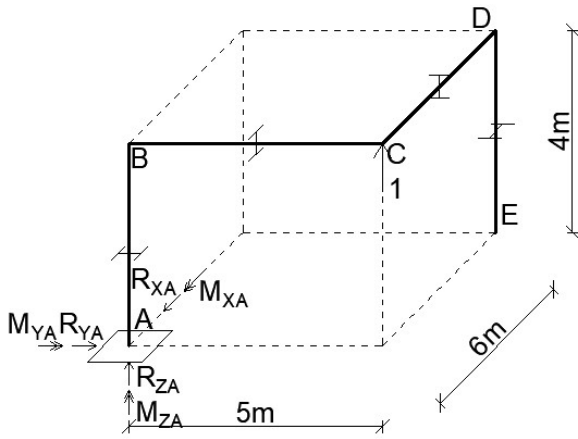
Pręt CB $y \in (0;5)\text{m}$



Pręt BA $z \in (0;4)\text{m}$



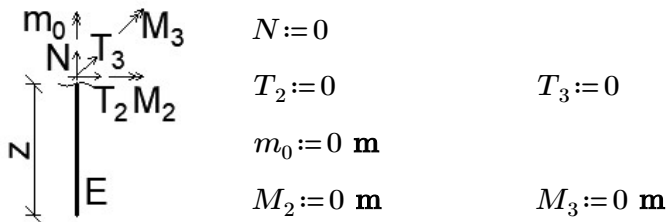
Stan x3=1



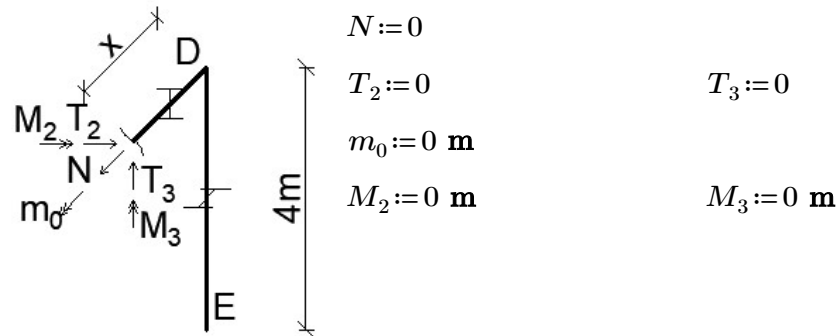
Reakcje:

$$\begin{aligned}\Sigma X = R_{XA} &= 0 & R_{XA} &:= 0 \\ \Sigma Y = R_{YA} &= 0 & R_{YA} &:= 0 \\ \Sigma Z = R_{ZA} + 1 &= 0 & R_{ZA} &:= -1 \\ \Sigma M_{AX} = M_{XA} + 1 \cdot 5 &= 0 & M_{XA} &:= -1 \cdot 5 \text{ m} \\ \Sigma M_{AY} = M_{YA} &= 0 & M_{YA} &:= 0 \text{ m} \\ \Sigma M_{AZ} = M_{ZA} &= 0 & M_{ZA} &:= 0 \text{ m}\end{aligned}$$

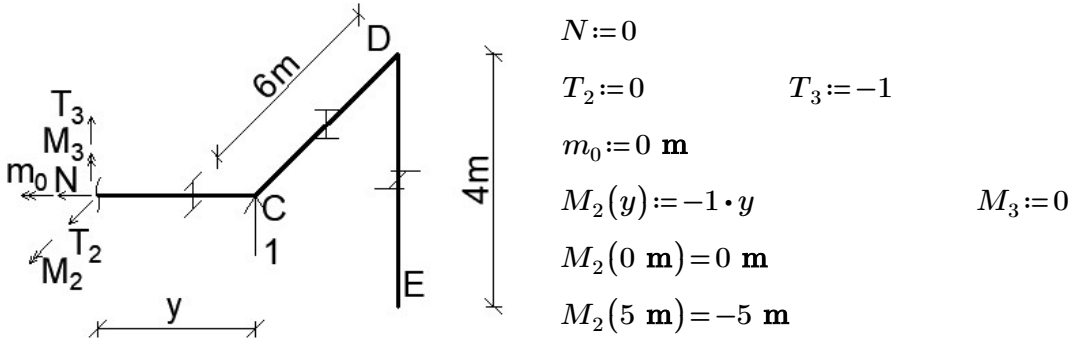
Pręt ED $z \in (0;4)\text{m}$



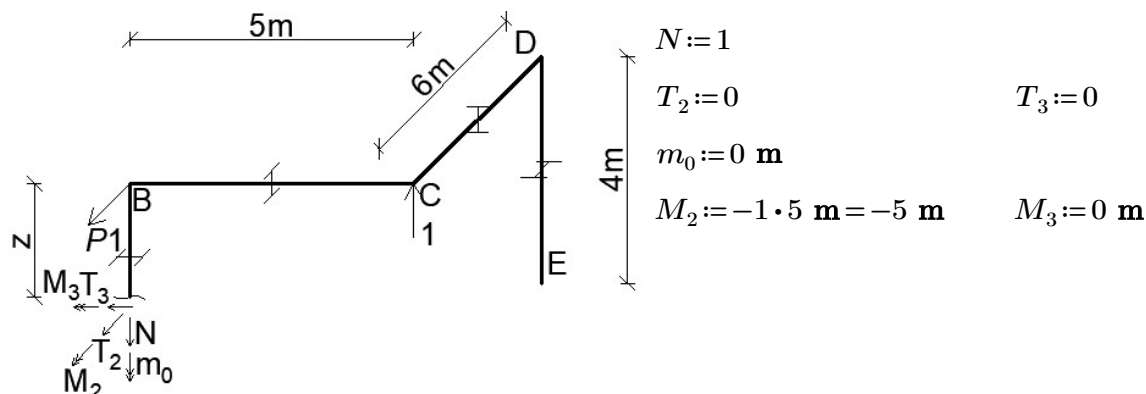
Pręt DC $x \in (0;6)\text{m}$



Pręt CB $y \in (0;5)\text{m}$

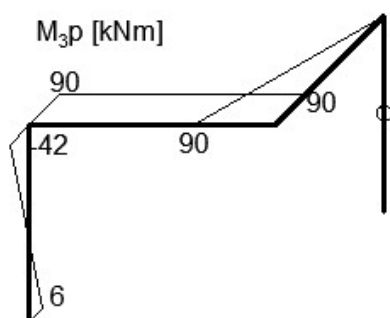
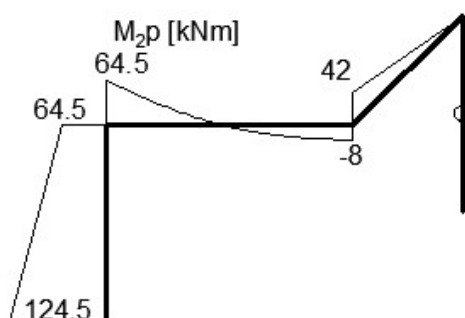
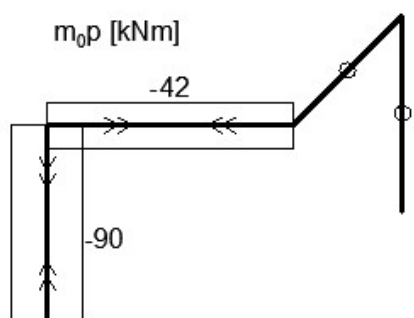
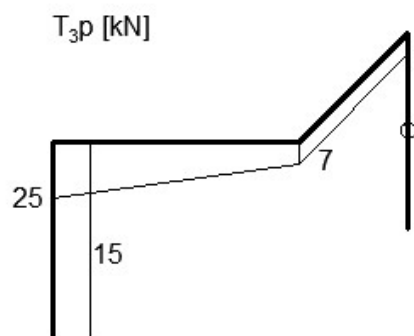
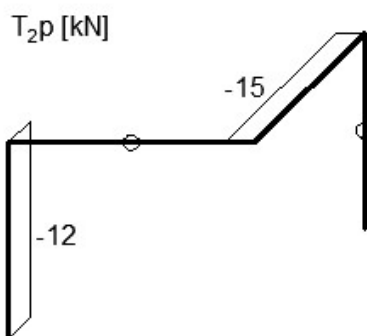
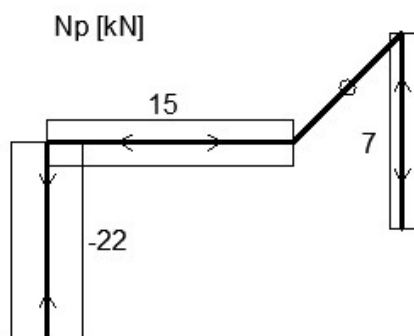


Pręt BA $z \in (0;4)\text{m}$

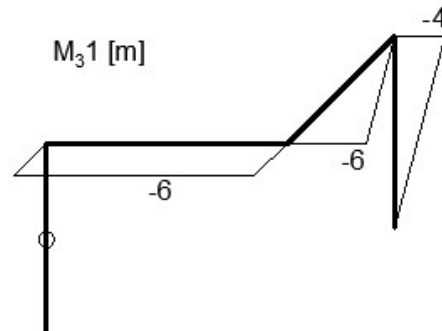
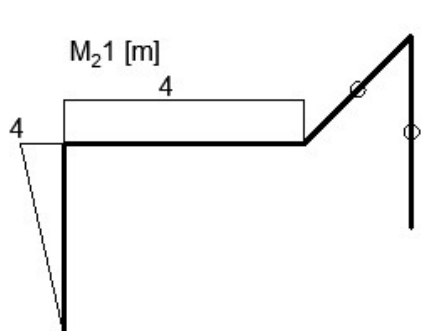
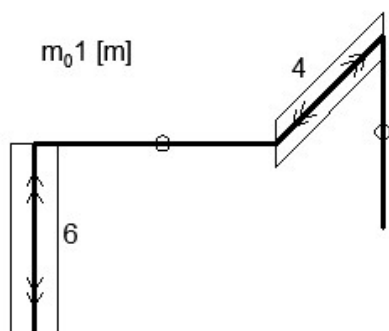
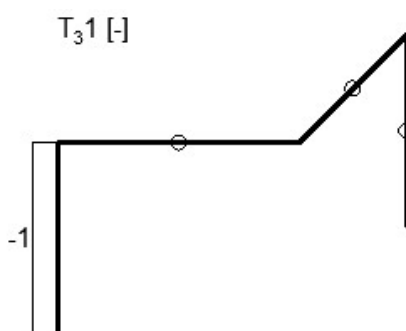
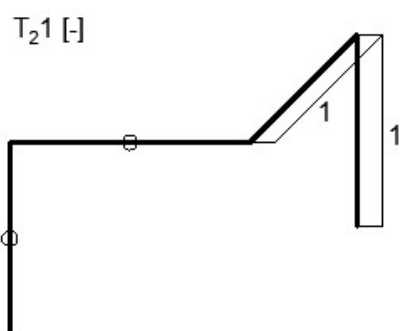
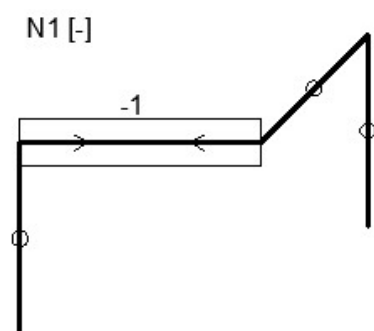


Wykresy

Stan p

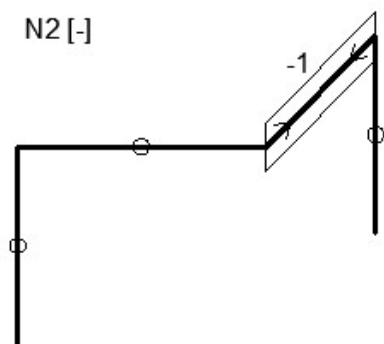


Stan 1

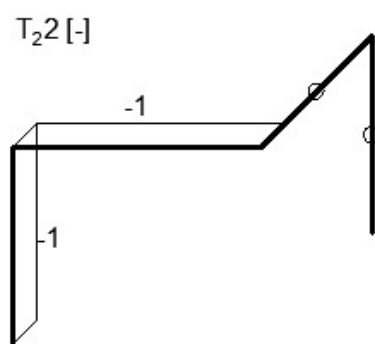


Stan 2

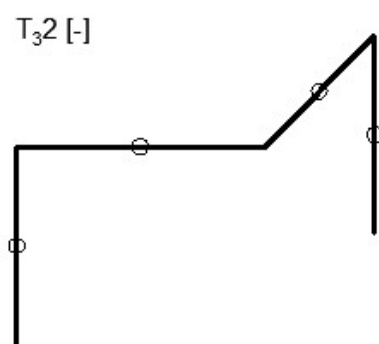
$N_2 [-]$



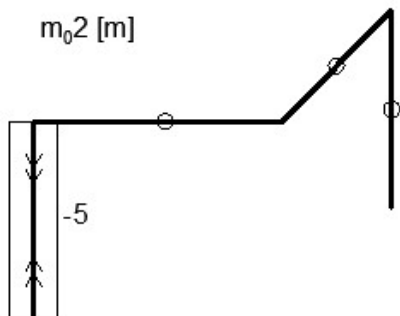
$T_{22} [-]$



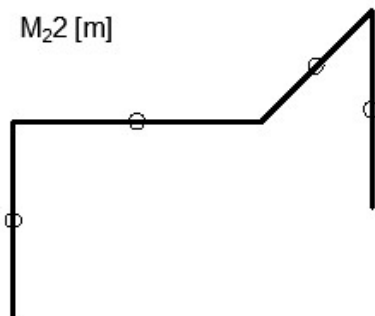
$T_{32} [-]$



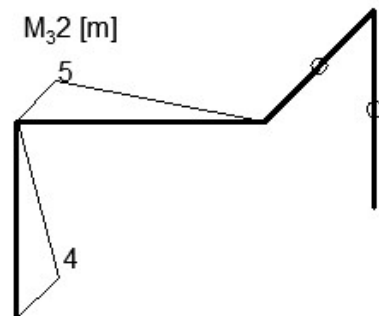
$m_{02} [m]$



$M_{22} [m]$

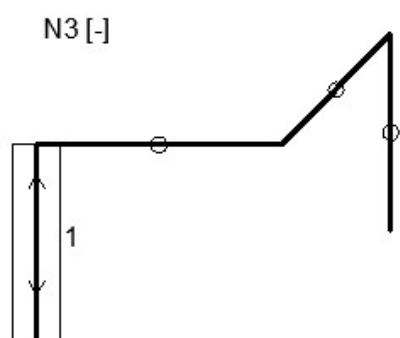


$M_{32} [m]$

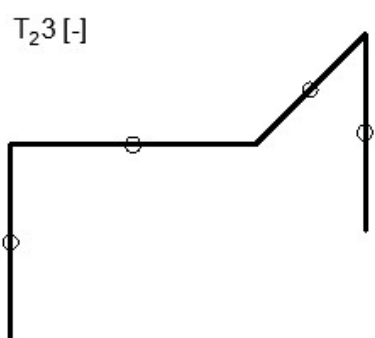


Stan 3

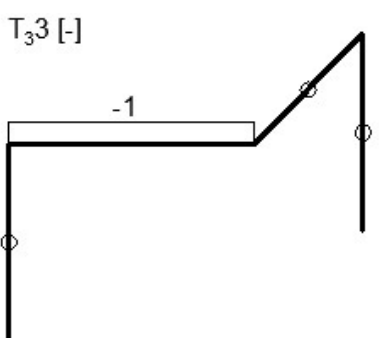
$N_3 [-]$



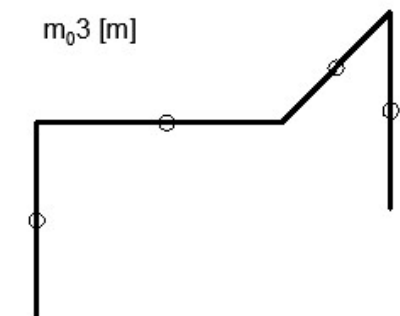
$T_{23} [-]$



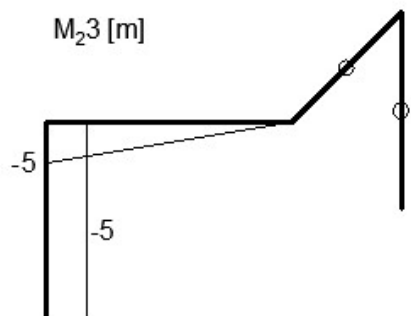
$T_{33} [-]$



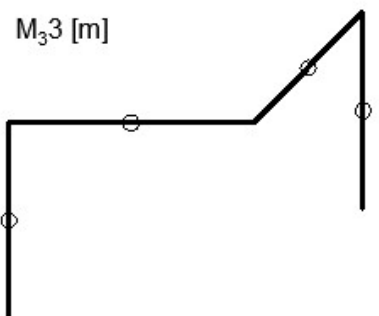
$m_{03} [m]$



$M_{23} [m]$



$M_{33} [m]$



Przemieszczenia

$$\delta_{11} = \frac{1}{EJ_2} \left[\begin{array}{c} 4 \\ \text{---} \\ 5 \\ \text{---} \\ 4 \end{array} + \begin{array}{c} 4 \\ \diagdown \\ 4 \\ \diagup \\ 4 \end{array} \right] + \frac{1}{EJ_3} \left[\begin{array}{c} 4 \\ \diagdown \\ 4 \\ \diagup \\ 6 \end{array} + \begin{array}{c} 4 \\ \diagup \\ 6 \\ \diagdown \\ 6 \end{array} + \begin{array}{c} 6 \\ \text{---} \\ 5 \\ \text{---} \\ 6 \end{array} \right] + \frac{1}{GJ_0} \left[\begin{array}{c} 4 \\ \diagdown \\ 6 \\ \diagup \\ 4 \end{array} + \begin{array}{c} 6 \\ \text{---} \\ 4 \\ \text{---} \\ 6 \end{array} \right]$$

$$\delta_{11} := \frac{\mathbf{m}^3}{EJ_2} \left(4 \cdot 5 \cdot 4 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 \right) + \frac{\mathbf{m}^3}{EJ_3} \left(\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 + \frac{1}{2} \cdot 6 \cdot 6 \cdot \frac{2}{3} \cdot 6 + 6 \cdot 5 \cdot 6 \right) + \frac{\mathbf{m}^3}{GJ_0} (4 \cdot 6 \cdot 4 + 6 \cdot 4 \cdot 6)$$

$$\delta_{11} = 63.359 \frac{\mathbf{m}}{\mathbf{kN}}$$

$$\delta_{12} = \frac{1}{EJ_3} \left[\begin{array}{c} 6 \\ \text{---} \\ 5 \\ \text{---} \\ 5 \end{array} \right] + \frac{1}{GJ_0} \left[\begin{array}{c} 6 \\ \text{---} \\ 4 \\ \text{---} \\ -5 \end{array} \right]$$

$$\delta_{12} := \frac{\mathbf{m}^3}{EJ_2} \cdot 0 + \frac{\mathbf{m}^3}{EJ_3} \left(-6 \cdot 5 \cdot \frac{1}{2} \cdot 5 \right) + \frac{\mathbf{m}^3}{GJ_0} (6 \cdot 4 \cdot -5) = -31.371 \frac{\mathbf{m}}{\mathbf{kN}} \quad \delta_{21} := \delta_{12}$$

$$\delta_{13} = \frac{1}{EJ_2} \left[\begin{array}{c} 4 \\ \text{---} \\ 5 \\ \text{---} \\ 5 \end{array} + \begin{array}{c} 4 \\ \diagdown \\ 4 \\ \diagup \\ 5 \end{array} \right]$$

$$\delta_{13} := \frac{\mathbf{m}^3}{EJ_2} \left(-4 \cdot 5 \cdot \frac{1}{2} \cdot 5 - \frac{1}{2} \cdot 4 \cdot 4 \cdot 5 \right) + \frac{\mathbf{m}^3}{EJ_3} \cdot 0 + \frac{\mathbf{m}^3}{GJ_0} \cdot 0 \quad \delta_{31} := \delta_{13}$$

$$\delta_{22} = \frac{1}{EJ_3} \left[\begin{array}{c} 5 \\ \text{---} \\ 5 \\ \text{---} \\ 5 \end{array} + \begin{array}{c} 4 \\ \diagdown \\ 4 \\ \diagup \\ 4 \end{array} \right] + \frac{1}{GJ_0} \left[\begin{array}{c} -5 \\ \text{---} \\ 4 \\ \text{---} \\ -5 \end{array} \right]$$

$$\delta_{22} := \frac{\mathbf{m}^3}{EJ_2} \cdot 0 + \frac{\mathbf{m}^3}{EJ_3} \left(\frac{1}{2} \cdot 5 \cdot 5 \cdot \frac{2}{3} \cdot 5 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 \right) + \frac{\mathbf{m}^3}{GJ_0} (-5 \cdot 4 \cdot -5) = 26.145 \frac{\mathbf{m}}{\mathbf{kN}}$$

$$\delta_{23} := \frac{\mathbf{m}^3}{EJ_2} \cdot 0 + \frac{\mathbf{m}^3}{EJ_3} \cdot 0 + \frac{\mathbf{m}^3}{GJ_0} \cdot 0 \quad \delta_{32} := \delta_{23}$$

$$\delta_{33} = \frac{1}{EJ_2} \left[\begin{array}{c} 5 \\ \text{---} \\ 5 \\ \text{---} \\ 5 \end{array} + \begin{array}{c} 5 \\ \text{---} \\ 4 \\ \text{---} \\ 5 \end{array} \right]$$

$$\delta_{33} := \frac{\mathbf{m}^3}{EJ_2} \left(\frac{1}{2} \cdot 5 \cdot 5 \cdot \frac{2}{3} \cdot 5 + 5 \cdot 4 \cdot 5 \right) = 0.051 \frac{\mathbf{m}}{\mathbf{kN}}$$

$$\delta_{1p} = \frac{1}{EJ_2} \left[\begin{array}{c} 64.5 \\ \text{---} \\ 5 \\ \text{---} \\ 4 \end{array} + \begin{array}{c} 64.5 \\ \diagdown \\ 4 \\ \diagup \\ 124.5 \end{array} \right] + \frac{1}{EJ_3} \left[\begin{array}{c} 90 \\ \diagdown \\ 6 \\ \diagup \\ 6 \end{array} + \begin{array}{c} 90 \\ \text{---} \\ 5 \\ \text{---} \\ 6 \end{array} \right] + \frac{1}{GJ_0} \left[\begin{array}{c} -90 \\ \text{---} \\ 4 \\ \text{---} \\ 6 \end{array} \right]$$

$$\text{Pole paraboli: } A_p := \frac{2}{3} \cdot \frac{q \cdot (5 \mathbf{m})^2}{8} \cdot 5 \mathbf{m} \cdot \frac{1}{\mathbf{kN} \cdot \mathbf{m}^2} = 31.25$$

$$\delta_{1p2} := 4 \cdot 5 \cdot \frac{64.5 - 8}{2} - A_p \cdot 4 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \left(\frac{2}{3} \cdot 64.5 + \frac{1}{3} \cdot 124.5 \right) = 1116$$

$$\delta_{1p3} := -\frac{1}{2} \cdot 90 \cdot 6 \cdot \frac{2}{3} \cdot 6 - 90 \cdot 5 \cdot 6 = -3780$$

$$\delta_{1p0} := -90 \cdot 4 \cdot 6 = -2160$$

$$\delta_{1p} := \frac{\text{kN} \cdot \text{m}^3}{EJ_2} \cdot \delta_{1p2} + \frac{\text{kN} \cdot \text{m}^3}{EJ_3} \cdot \delta_{1p3} + \frac{\text{kN} \cdot \text{m}^3}{GJ_0} \cdot \delta_{1p0} = -575.725 \text{ m}$$

$$\delta_{2p} = \frac{1}{EJ_3} \left[\begin{array}{c} 90 \\ 5 \end{array} \begin{array}{c} 5 \\ 5 \end{array} + \begin{array}{c} 42 \\ 6 \end{array} \begin{array}{c} 4 \\ 4 \end{array} \right] + \frac{1}{GJ_0} \left[\begin{array}{c} -90 \\ 4 \end{array} \begin{array}{c} -5 \\ 4 \end{array} \right]$$

$$\delta_{2p2} := 0$$

$$\delta_{2p3} := 90 \cdot 5 \cdot \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \left(\frac{2}{3} \cdot 6 - \frac{1}{3} \cdot 42 \right) = 1045$$

$$\delta_{2p0} := -90 \cdot 4 \cdot -5 = 1800$$

$$\delta_{2p} := \frac{\text{kN} \cdot \text{m}^3}{EJ_2} \cdot \delta_{2p2} + \frac{\text{kN} \cdot \text{m}^3}{EJ_3} \cdot \delta_{2p3} + \frac{\text{kN} \cdot \text{m}^3}{GJ_0} \cdot \delta_{2p0} = 470.182 \text{ m}$$

$$\delta_{3p} = \frac{1}{EJ_2} \left[\begin{array}{c} 64.5 \\ 5 \end{array} \begin{array}{c} 8 \\ 5 \end{array} + \begin{array}{c} 64.5 \\ 124.5 \end{array} \begin{array}{c} 5 \\ 4 \end{array} \right]$$

$$\delta_{3p2} := \frac{1}{2} \cdot 5 \cdot 5 \cdot \left(\frac{1}{3} \cdot 8 - \frac{2}{3} \cdot 64.5 \right) + A_p \cdot \frac{1}{2} \cdot 5 - 5 \cdot 4 \cdot \frac{64.5 + 124.5}{2} = -2316.042$$

$$\delta_{3p3} := 0$$

$$\delta_{3p0} := 0$$

$$\delta_{3p} := \frac{\text{kN} \cdot \text{m}^3}{EJ_2} \cdot \delta_{3p2} + \frac{\text{kN} \cdot \text{m}^3}{EJ_3} \cdot \delta_{3p3} + \frac{\text{kN} \cdot \text{m}^3}{GJ_0} \cdot \delta_{3p0} = -0.836 \text{ m}$$

Rozwiązanie układu równań

$$\delta_{11} x_1 + \delta_{12} x_2 + \delta_{13} x_3 + \delta_{1p} = 0$$

$$\delta_{21} x_1 + \delta_{22} x_2 + \delta_{23} x_3 + \delta_{2p} = 0$$

$$\delta_{31} x_1 + \delta_{32} x_2 + \delta_{33} x_3 + \delta_{3p} = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix}^{-1} \cdot - \begin{bmatrix} \delta_{1p} \\ \delta_{2p} \\ \delta_{3p} \end{bmatrix}$$

$$x_1 = 0.47 \text{ kN} \quad x_2 = -17.419 \text{ kN} \quad x_3 = 16.647 \text{ kN}$$

Reakcje ostateczne

$$R_{ost} = R_1 \cdot x_1 + R_2 \cdot x_2 + R_3 \cdot x_3 + R_p$$

$$R_{XA} := 0 x_1 + -1 x_2 + 0 x_3 + -12 \text{ kN} = 5.419 \text{ kN}$$

$$R_{YA} := 1 x_1 + 0 x_2 + 0 x_3 + -15 \text{ kN} = -14.53 \text{ kN}$$

$$R_{ZA} := 0 x_1 + 0 x_2 + -1 x_3 + 22 \text{ kN} = 5.353 \text{ kN}$$

$$M_{XA} := 0 \text{ m} x_1 + 0 \text{ m} x_2 + -5 \text{ m} x_3 + 124.5 \text{ kNm} = 41.263 \text{ kNm}$$

$$M_{YA} := 0 \text{ m} x_1 + -4 \text{ m} x_2 + 0 \text{ m} x_3 + -6 \text{ kNm} = 63.678 \text{ kNm}$$

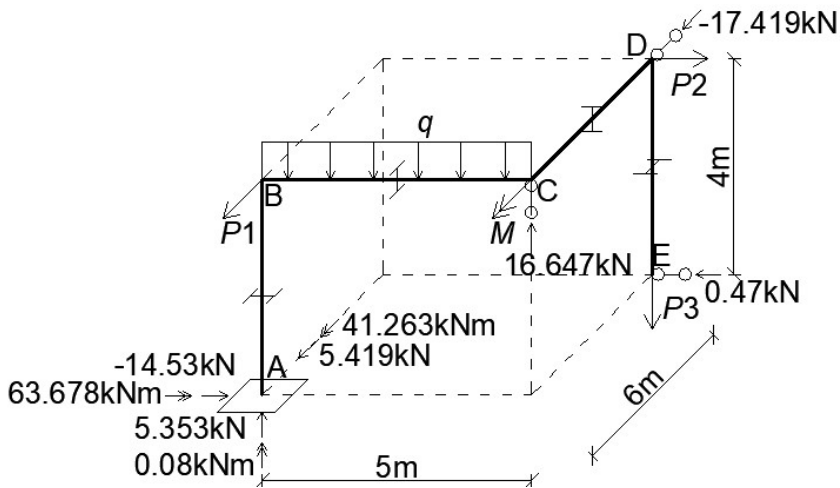
$$M_{ZA} := -6 \text{ m} x_1 + 5 \text{ m} x_2 + 0 \text{ m} x_3 + 90 \text{ kNm} = 0.08 \text{ kNm}$$

$$R_{YE} := x_1 = 0.47 \text{ kN}$$

$$R_{XD} := x_2 = -17.419 \text{ kN}$$

$$R_{ZC} := x_3 = 16.647 \text{ kN}$$

Reakcje ostateczne



Sprawdzenie statyczne

$$\Sigma X := R_{XA} + R_{XD} + P1 = 0 \text{ kN}$$

$$\Sigma Y := R_{YA} - R_{YE} + P2 = 0 \text{ kN}$$

$$\Sigma Z := R_{ZA} + R_{ZC} - P3 - q \cdot 5 \text{ m} = 0 \text{ kN}$$

$$\Sigma M_{AX} := M_{XA} + R_{ZC} \cdot 5 \text{ m} + M - q \cdot 5 \text{ m} \cdot 2.5 \text{ m} - P2 \cdot 4 \text{ m} - P3 \cdot 5 \text{ m} = 0 \text{ kNm}$$

$$\Sigma M_{AY} := M_{YA} + R_{XD} \cdot 4 \text{ m} + P1 \cdot 4 \text{ m} - P3 \cdot 6 \text{ m} = 0 \text{ kNm}$$

$$\Sigma M_{AZ} := M_{ZA} - R_{XD} \cdot 5 \text{ m} + R_{YE} \cdot 6 \text{ m} - P2 \cdot 6 \text{ m} = 0 \text{ kNm}$$

Wykresy ostateczne

$$N_{ost} = N_1 \cdot x_1 + N_2 \cdot x_2 + N_3 \cdot x_3 + N_p$$

$$N_{ost} := \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_3 + \begin{bmatrix} 7 \\ 0 \\ 15 \\ -22 \end{bmatrix} \text{ kN} = \begin{bmatrix} 7 \\ 17.419 \\ 14.53 \\ -5.353 \end{bmatrix} \text{ kN}$$

$$T_{ost} = T_1 \cdot x_1 + T_2 \cdot x_2 + T_3 \cdot x_3 + T_p$$

$$T_{ost} := \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ -15 \\ 0 \\ -12 \end{bmatrix} \text{ kN} = \begin{bmatrix} 0.47 \\ -14.53 \\ 17.419 \\ 5.419 \end{bmatrix} \text{ kN}$$

$$T_{3ost} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & -1 \end{bmatrix} x_1 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & -1 \\ 0 & 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 & 0 \\ 7 & 7 \\ 7 & 25 \\ 15 & 15 \end{bmatrix} \text{ kN} = \begin{bmatrix} 0 & 0 \\ 7 & 7 \\ -9.647 & 8.353 \\ 14.53 & 14.53 \end{bmatrix} \text{ kN}$$

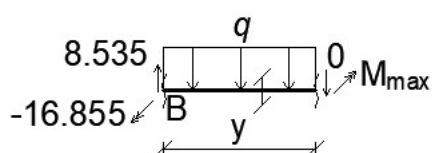
$$M_{ost} = M_1 \cdot x_1 + M_2 \cdot x_2 + M_3 \cdot x_3 + M_p$$

$$m_{ost} := \begin{bmatrix} 0 \\ 4 \\ 0 \\ 6 \end{bmatrix} \text{ m} \cdot x_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -5 \end{bmatrix} \text{ m} \cdot x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ m} \cdot x_3 + \begin{bmatrix} 0 \\ 0 \\ -42 \\ -90 \end{bmatrix} \text{ kNm} = \begin{bmatrix} 0 \\ 1.882 \\ -42 \\ -0.08 \end{bmatrix} \text{ kNm}$$

$$M_{2ost} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 4 & 4 \\ 4 & 0 \end{bmatrix} \mathbf{m} \cdot x_1 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{m} \cdot x_2 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -5 \\ -5 & -5 \end{bmatrix} \mathbf{m} \cdot x_3 + \begin{bmatrix} 0 & 0 \\ 0 & 42 \\ -8 & 64.5 \\ 64.5 & 124.5 \end{bmatrix} \text{ kNm} = \begin{bmatrix} 0 & 0 \\ 0 & 42 \\ -6.118 & -16.855 \\ -16.855 & 41.263 \end{bmatrix} \text{ kNm}$$

$$M_{3ost} := \begin{bmatrix} 0 & -4 \\ 0 & -6 \\ -6 & -6 \\ 0 & 0 \end{bmatrix} \mathbf{m} \cdot x_1 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 5 \\ 0 & 4 \end{bmatrix} \mathbf{m} \cdot x_2 + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{m} \cdot x_3 + \begin{bmatrix} 0 & 0 \\ 0 & 90 \\ 90 & 90 \\ -42 & 6 \end{bmatrix} \text{ kNm} = \begin{bmatrix} 0 & -1.882 \\ 0 & 87.177 \\ 87.177 & 0.08 \\ -42 & -63.678 \end{bmatrix} \text{ kNm}$$

Obliczenie ekstremum paraboli



$$\Sigma Z = 8.535 - q \cdot y = 0 \quad y := \frac{8.535 \text{ kN}}{q} = 2.845 \text{ m}$$

$$\Sigma M_{XB} = -16.855 - q \cdot y \cdot \frac{y}{2} - M_{max} = 0$$

$$M_{max} := -16.855 \text{ kNm} - q \cdot y \cdot \frac{y}{2} = -28.996 \text{ kNm}$$

Wykresy ostateczne

