Politechnika Lubelska

Wydział Budownictwa i Architektury

Mechanika Budowli I

Projekt 2: Metoda Sił

Wykonał:

……..

Sprawdził:

dr inż. Jakub Gontarz

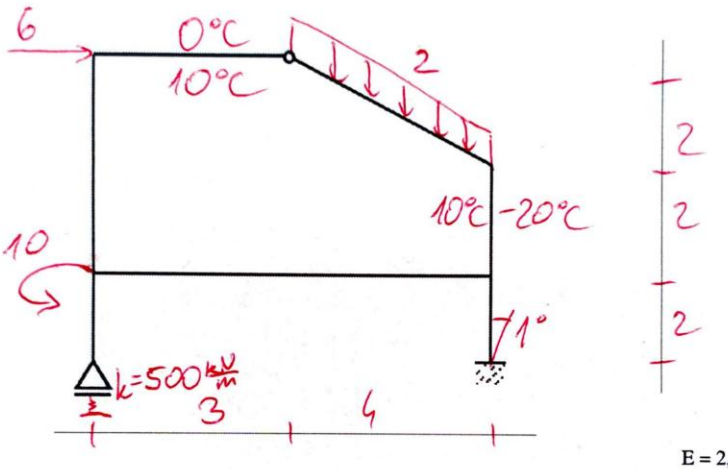
2025

PRZYKŁADOWY TEMAT

Student .....
Grupa .....
Rok akademicki .....

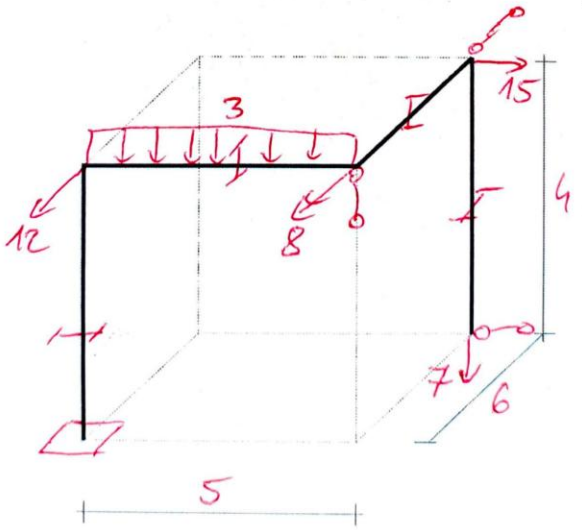
ĆWICZENIE PROJEKTOWE NR 3
Z MECHANIKI BUDOWLI

Dane ustroje statycznie niewyznaczalne rozwiązać metodą sił



$E = 2{,}1 \cdot 10^5$ MPa
$\alpha_t = 1{,}2 \cdot 10^{-5}$ 1/K

Przekrój: IPN 120

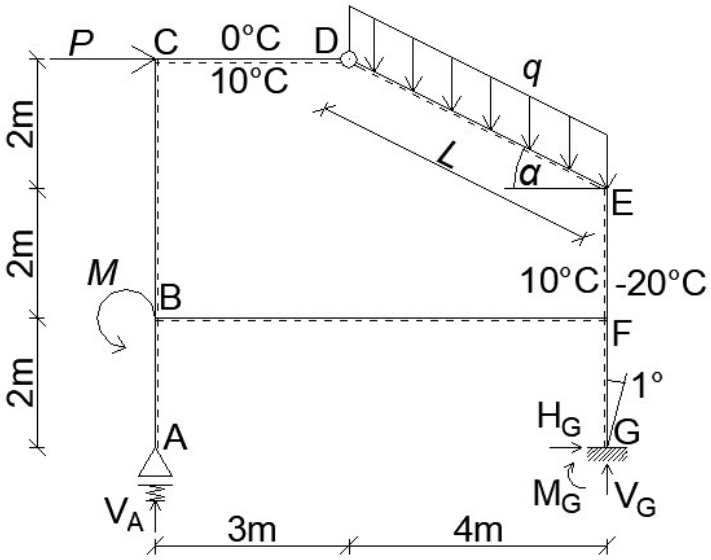


$E = 2{,}1 \cdot 10^5$ MPa
$\alpha_t = 1{,}2 \cdot 10^{-5}$ 1/K

Przekrój: IPE 180

Zadanie 1: Rama 2D

$\mathbf{kNm} := \mathbf{kN} \cdot \mathbf{m}$



$E := 210\ \mathbf{GPa}$

$\alpha_t := 1.2 \cdot 10^{-5}\ \frac{1}{\mathbf{K}}$

Przekrój: IPN 120

$J := 328\ \mathbf{cm}^4$

$EJ := E \cdot J = 688.8\ \mathbf{kN \cdot m^2}$

$h := 120\ \mathbf{mm}$

$P := 6\ \mathbf{kN}$

$M := 10\ \mathbf{kNm}$

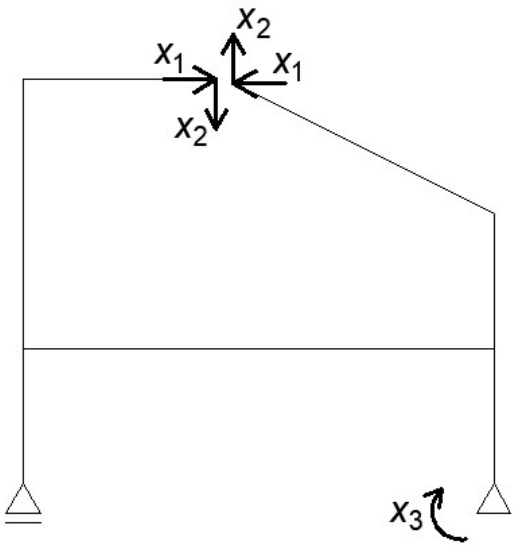
$q := 2\ \frac{\mathbf{kN}}{\mathbf{m}}$

Długość pręta ukośnego:

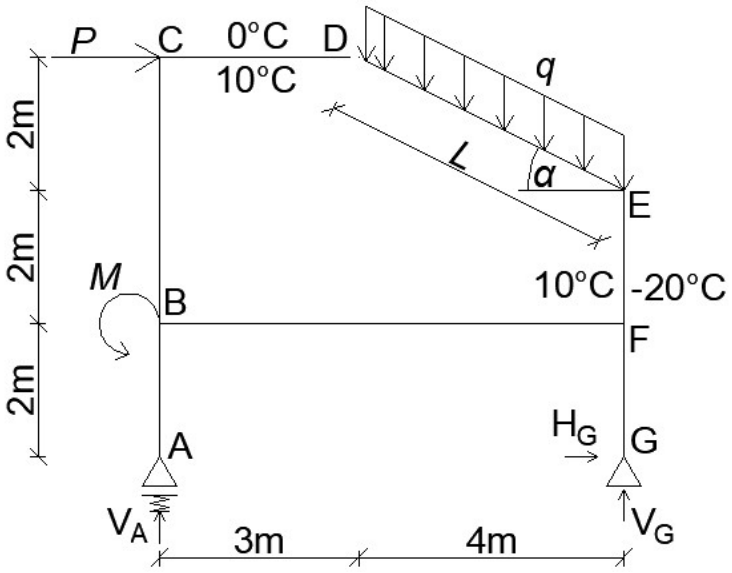
$L := \sqrt{(2\ \mathbf{m})^2 + (4\ \mathbf{m})^2} = 4.472\ \mathbf{m}$

$sin\alpha := \frac{2\ \mathbf{m}}{L} = 0.447 \qquad cos\alpha := \frac{4\ \mathbf{m}}{L} = 0.894$

UPMS



Stan p



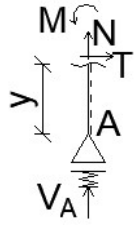
Reakcje:

$\Sigma M_A = P \cdot 6 - M - q \cdot L \cdot 5 - V_G \cdot 7 = 0 \qquad V_G := \frac{P \cdot 6\ \mathbf{m} - M + q \cdot L \cdot 5\ \mathbf{m}}{7\ \mathbf{m}} = 10.103\ \mathbf{kN}$

$\Sigma X = H_G + 6 = 0 \qquad H_G := -6\ \mathbf{kN}$

$\Sigma Y = V_A + V_G - q \cdot L = 0 \qquad V_A := -V_G + q \cdot L = -1.159\ \mathbf{kN}$

Pręt AB y∈(0;2)m



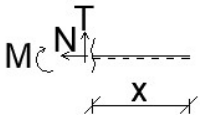
$N_{AB} := -V_A = 1.159\ \mathbf{kN}$

$T_{AB} := 0\ \mathbf{kN}$

$M_{AB}(y) := 0\ \mathbf{kNm}$

$M_{AB}(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_{AB}(2\ \mathbf{m}) = 0\ \mathbf{kNm}$

Pręt CD x∈(0;3)m



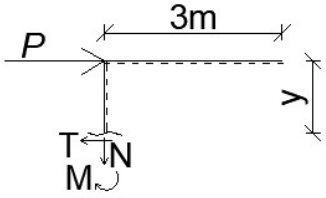
$N_{CD} := 0\ \mathbf{kN}$

$T_{CD} := 0\ \mathbf{kN}$

$M_{CD}(x) := 0\ \mathbf{kNm}$

$M_{CD}(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_{CD}(3\ \mathbf{m}) = 0\ \mathbf{kNm}$

Pręt BC y∈(0;4)m



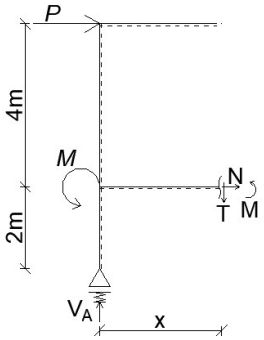
$N_{BC} := 0\ \mathbf{kN}$

$T_{BC} := P = 6\ \mathbf{kN}$

$M_{BC}(y) := -P \cdot y$

$M_{BC}(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_{BC}(4\ \mathbf{m}) = -24\ \mathbf{kNm}$

Pręt BF x∈(0;7)m



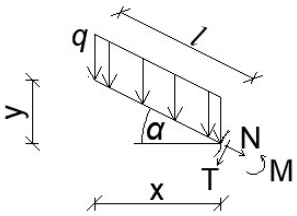
$N_{BF} := -P = -6\ \mathbf{kN}$

$T_{BF} := V_A = -1.159\ \mathbf{kN}$

$M_{BF}(x) := P \cdot 4\ \mathbf{m} - M + V_A \cdot x$

$M_{BF}(0\ \mathbf{m}) = 14\ \mathbf{kNm} \qquad M_{BF}(7\ \mathbf{m}) = 5.889\ \mathbf{kNm}$

Pręt DE x∈(0;4)m $y(x) := 0.5\ x$ $l(x) := \sqrt{y(x)^2 + x^2}$



$N_{DE}(x) := -q \cdot l(x) \cdot \sin\alpha$

$T_{DE}(x) := -q \cdot l(x) \cdot \cos\alpha$

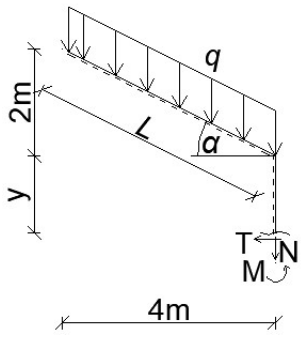
$M_{DE}(x) := -q \cdot l(x) \cdot \frac{x}{2}$

$N_{DE}(0\ \mathbf{m}) = 0\ \mathbf{kN} \qquad N_{DE}(4\ \mathbf{m}) = -4\ \mathbf{kN}$

$T_{DE}(0\ \mathbf{m}) = 0\ \mathbf{kN} \qquad T_{DE}(4\ \mathbf{m}) = -8\ \mathbf{kN}$

$M_{DE}(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_{DE}(4\ \mathbf{m}) = -17.889\ \mathbf{kNm}$

Pręt EF y∈(0;2)m



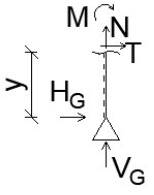
$N_{EF} := -q \cdot L = -8.944\ \mathbf{kN}$

$T_{EF} := 0\ \mathbf{kN}$

$M_{EF}(y) := -q \cdot L \cdot 2\ \mathbf{m}$

$M_{EF}(0\ \mathbf{m}) = -17.889\ \mathbf{kNm} \qquad M_{EF}(7\ \mathbf{m}) = -17.889\ \mathbf{kNm}$

Pręt FG y∈(0;2)m



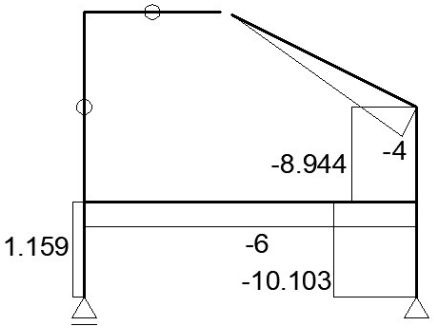
$N_{FG} := -V_G = -10.103\ \mathbf{kN}$

$T_{FG} := -H_G = 6\ \mathbf{kN}$

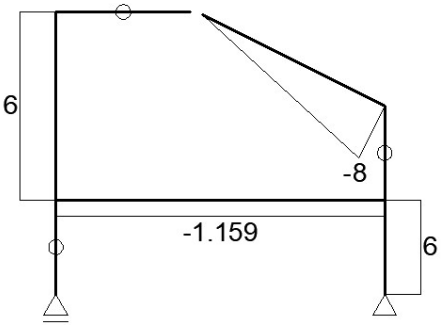
$M_{FG}(y) := H_G \cdot y$

$M_{FG}(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_{FG}(2\ \mathbf{m}) = -12\ \mathbf{kNm}$

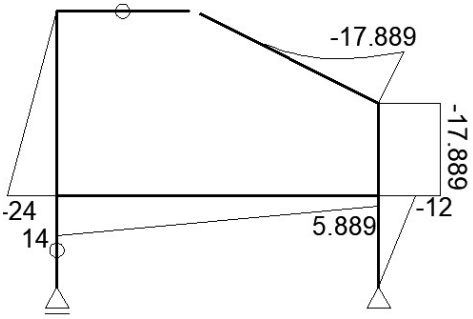
Np [kN]



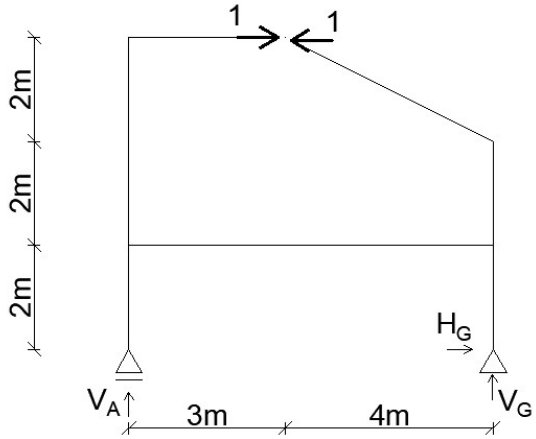
Tp [kN]



Mp [kNm]



Stan x1=1



<u>Reakcje:</u>

$$\Sigma M_A = 1\cdot 6 - 1\cdot 6 - V_G\cdot 7 = 0$$

$$V_G := \frac{1\cdot 6\ \mathbf{m} - 1\cdot 6\ \mathbf{m}}{7\ \mathbf{m}} = 0$$

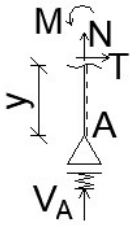
$$\Sigma X = H_G + 1 - 1 = 0$$

$$H_G := -1 + 1 = 0$$

$$\Sigma Y = V_A + V_G = 0$$

$$V_A := -V_G = 0$$

Pręt AB y∈(0;2)m



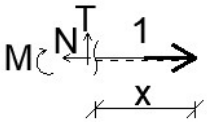
$$N_{AB} := -V_A = 0$$

$$T_{AB} := 0$$

$$M_{AB}(y) := 0\ \mathbf{m}$$

$$M_{AB}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{AB}(2\ \mathbf{m}) = 0\ \mathbf{m}$$

Pręt CD x∈(0;3)m



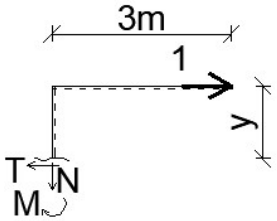
$$N_{CD} := 1$$

$$T_{CD} := 0$$

$$M_{CD}(x) := 0\ \mathbf{m}$$

$$M_{CD}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{CD}(3\ \mathbf{m}) = 0\ \mathbf{m}$$

Pręt BC y∈(0;4)m



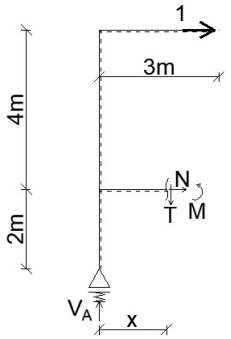
$$N_{BC} := 0$$

$$T_{BC} := 1$$

$$M_{BC}(y) := -1\cdot y$$

$$M_{BC}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{BC}(4\ \mathbf{m}) = -4\ \mathbf{m}$$

Pręt BF x∈(0;7)m



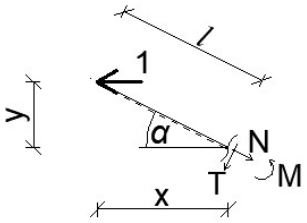
$N_{BF} := -1$

$T_{BF} := V_A = 0$

$M_{BF}(x) := V_A \cdot x + 1 \cdot 4\ \mathbf{m}$

$M_{BF}(0\ \mathbf{m}) = 4\ \mathbf{m}$ $M_{BF}(7\ \mathbf{m}) = 4\ \mathbf{m}$

Pręt DE x∈(0;4)m $y(x) := 0.5\ x$ $l(x) := \sqrt{y(x)^2 + x^2}$



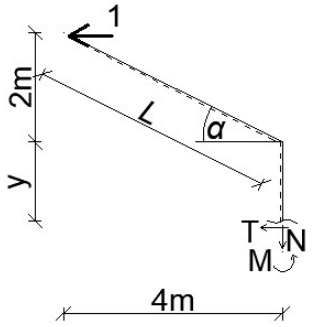
$N_{DE} := 1 \cdot cos\alpha = 0.894$

$T_{DE} := -1 \cdot sin\alpha = -0.447$

$M_{DE}(x) := -1 \cdot y(x)$

$M_{DE}(0\ \mathbf{m}) = 0\ \mathbf{m}$ $M_{DE}(4\ \mathbf{m}) = -2\ \mathbf{m}$

Pręt EF y∈(0;2)m



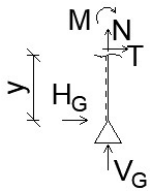
$N_{EF} := 0$

$T_{EF} := -1$

$M_{EF}(y) := -1 \cdot (2\ \mathbf{m} + y)$

$M_{EF}(0\ \mathbf{m}) = -2\ \mathbf{m}$ $M_{EF}(7\ \mathbf{m}) = -9\ \mathbf{m}$

Pręt FG y∈(0;2)m



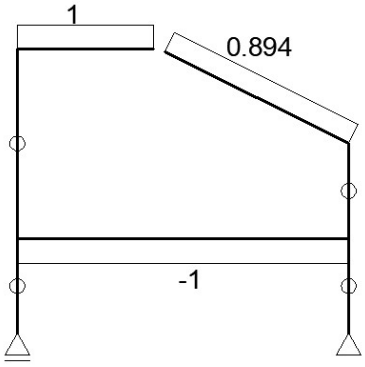
$N_{FG} := -V_G = 0$

$T_{FG} := -H_G = 0$

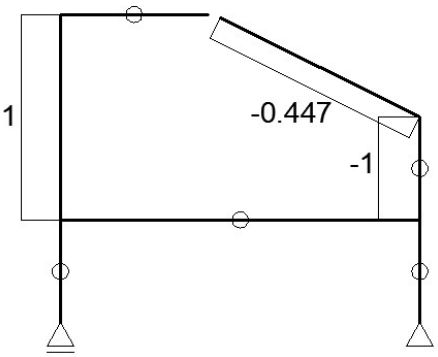
$M_{FG}(y) := H_G \cdot y$

$M_{FG}(0\ \mathbf{m}) = 0\ \mathbf{m}$ $M_{FG}(2\ \mathbf{m}) = 0\ \mathbf{m}$

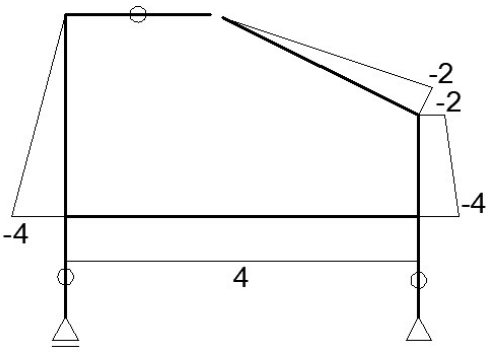
N1 [-]



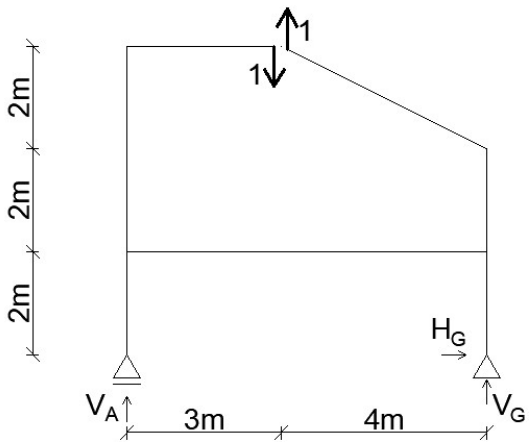
T1 [-]



M1 [m]



Stan x2=1



Reakcje:

$$\Sigma M_A = 1\cdot 3 - 1\cdot 3 - V_G\cdot 7 = 0$$

$$V_G := \frac{1\cdot 3\ \mathbf{m} - 1\cdot 3\ \mathbf{m}}{7\ \mathbf{m}} = 0$$

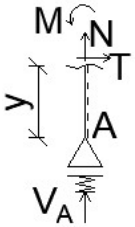
$$\Sigma X = H_G = 0$$

$$H_G := 0$$

$$\Sigma Y = V_A + V_G + 1 - 1 = 0$$

$$V_A := -V_G - 1 + 1 = 0$$

Pręt AB y∈(0;2)m



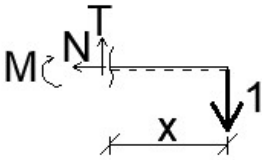
$$N_{AB} := -V_A = 0$$

$$T_{AB} := 0$$

$$M_{AB}(y) := 0\ \mathbf{m}$$

$$M_{AB}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{AB}(2\ \mathbf{m}) = 0\ \mathbf{m}$$

Pręt CD x∈(0;3)m



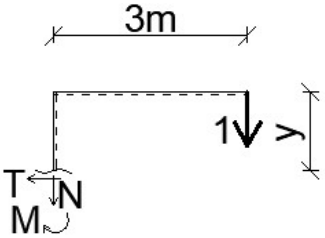
$$N_{CD} := 0$$

$$T_{CD} := 1$$

$$M_{CD}(x) := -1\cdot x$$

$$M_{CD}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{CD}(3\ \mathbf{m}) = -3\ \mathbf{m}$$

Pręt BC y∈(0;4)m



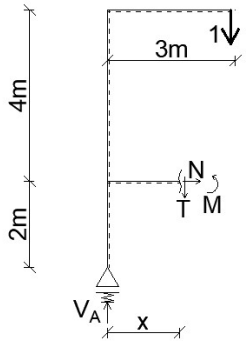
$$N_{BC} := -1$$

$$T_{BC} := 0$$

$$M_{BC}(y) := -1\cdot 3\ \mathbf{m}$$

$$M_{BC}(0\ \mathbf{m}) = -3\ \mathbf{m} \qquad M_{BC}(4\ \mathbf{m}) = -3\ \mathbf{m}$$

Pręt BF x∈(0;7)m



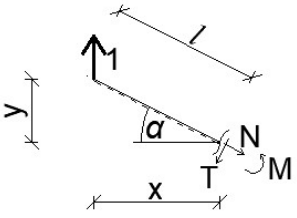
$$N_{BF} := 0$$

$$T_{BF} := V_A - 1 = -1$$

$$M_{BF}(x) := V_A\cdot x - 1\cdot(x - 3\ \mathbf{m})$$

$$M_{BF}(0\ \mathbf{m}) = 3\ \mathbf{m} \qquad M_{BF}(7\ \mathbf{m}) = -4\ \mathbf{m}$$

Pręt DE x∈(0;4)m $y(x) := 0.5\ x$ $l(x) := \sqrt{y(x)^2 + x^2}$



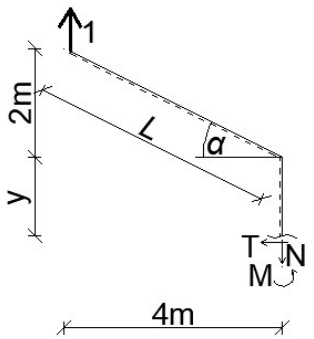
$$N_{DE} := 1\cdot sin\alpha = 0.447$$

$$T_{DE} := 1\cdot cos\alpha = 0.894$$

$$M_{DE}(x) := 1\cdot x$$

$$M_{DE}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{DE}(4\ \mathbf{m}) = 4\ \mathbf{m}$$

Pręt EF y∈(0;2)m



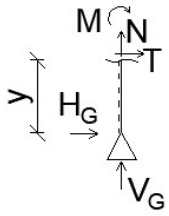
$N_{EF} := 1$

$T_{EF} := 0$

$M_{EF}(y) := 1 \cdot 4\ \mathbf{m}$

$M_{EF}(0\ \mathbf{m}) = 4\ \mathbf{m}$ $\quad M_{EF}(7\ \mathbf{m}) = 4\ \mathbf{m}$

Pręt FG y∈(0;2)m



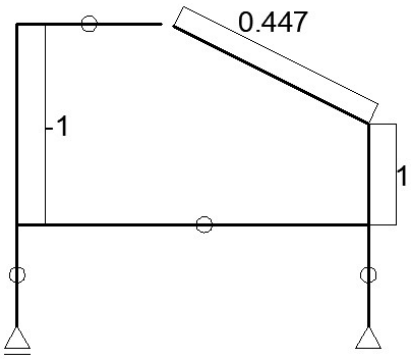
$N_{FG} := -V_G = 0$

$T_{FG} := -H_G = 0$

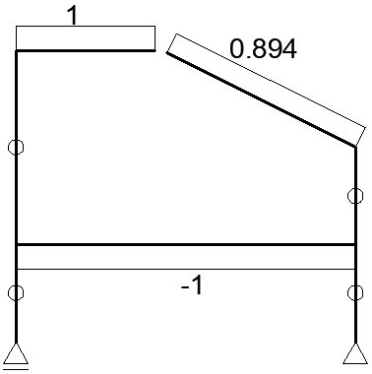
$M_{FG}(y) := H_G \cdot y$

$M_{FG}(0\ \mathbf{m}) = 0\ \mathbf{m}$ $\quad M_{FG}(2\ \mathbf{m}) = 0\ \mathbf{m}$

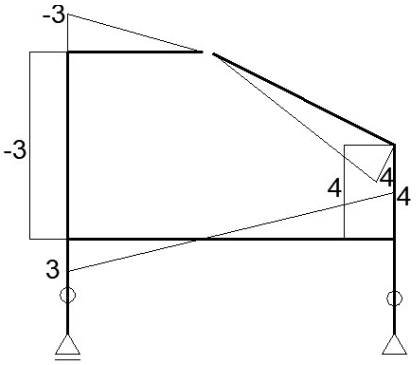
N2 [-]



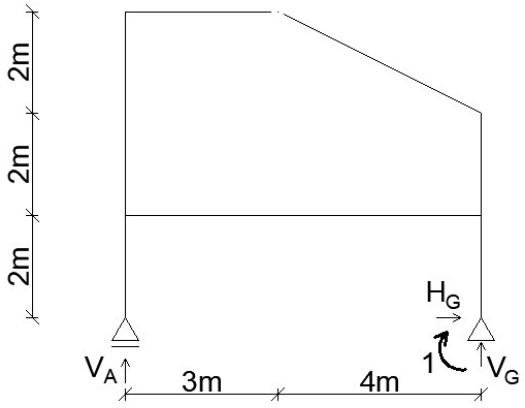
T2 [-]



M2 [m]



Stan x3=1



Reakcje:

$\Sigma M_A = 1 - V_G \cdot 7 = 0$

$V_G := \frac{1}{7\ \mathbf{m}} = 0.143\ \frac{1}{\mathbf{m}}$

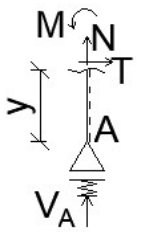
$\Sigma X = H_G = 0$

$H_G := 0\ \frac{1}{\mathbf{m}}$

$\Sigma Y = V_A + V_G = 0$

$V_A := -V_G = -0.143\ \frac{1}{\mathbf{m}}$

Pręt AB y∈(0;2)m



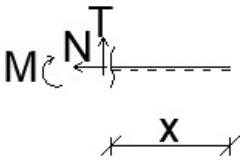
$N_{AB} := -V_A = 0.143\ \frac{1}{\mathbf{m}}$

$T_{AB} := 0\ \frac{1}{\mathbf{m}}$

$M_{AB}(y) := 0$

$M_{AB}(0\ \mathbf{m}) = 0$ $\quad M_{AB}(2\ \mathbf{m}) = 0$

Pręt CD x∈(0;3)m

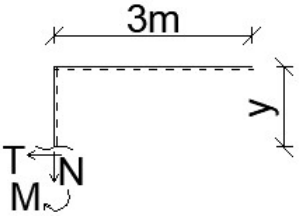


$N_{CD} := 0\ \frac{1}{\mathbf{m}}$ $T_{CD} := 0\ \frac{1}{\mathbf{m}}$

$M_{CD}(x) := 0$

$M_{CD}(0\ \mathbf{m}) = 0$ $M_{CD}(3\ \mathbf{m}) = 0$

Pręt BC y∈(0;4)m

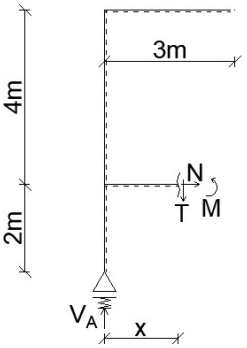


$N_{BC} := 0\ \frac{1}{\mathbf{m}}$ $T_{BC} := 0\ \frac{1}{\mathbf{m}}$

$M_{BC}(y) := 0$

$M_{BC}(0\ \mathbf{m}) = 0$ $M_{BC}(4\ \mathbf{m}) = 0$

Pręt BF x∈(0;7)m



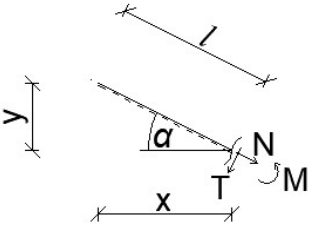
$N_{BF} := 0\ \frac{1}{\mathbf{m}}$

$T_{BF} := V_A = -0.143\ \frac{1}{\mathbf{m}}$

$M_{BF}(x) := V_A \cdot x$

$M_{BF}(0\ \mathbf{m}) = 0$ $M_{BF}(7\ \mathbf{m}) = -1$

Pręt DE x∈(0;4)m $y(x) := 0.5\ x$ $l(x) := \sqrt{y(x)^2 + x^2}$

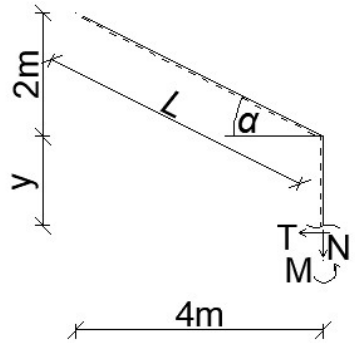


$N_{DE} := 0\ \frac{1}{\mathbf{m}}$ $T_{DE} := 0\ \frac{1}{\mathbf{m}}$

$M_{DE}(x) := 0$

$M_{DE}(0\ \mathbf{m}) = 0$ $M_{DE}(4\ \mathbf{m}) = 0$

Pręt EF y∈(0;2)m

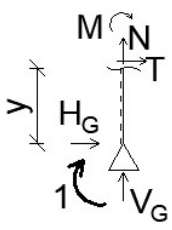


$N_{EF} := 0\ \frac{1}{\mathbf{m}}$ $T_{EF} := 0\ \frac{1}{\mathbf{m}}$

$M_{EF}(y) := 0$

$M_{EF}(0\ \mathbf{m}) = 0$ $M_{EF}(7\ \mathbf{m}) = 0$

Pręt FG y∈(0;2)m



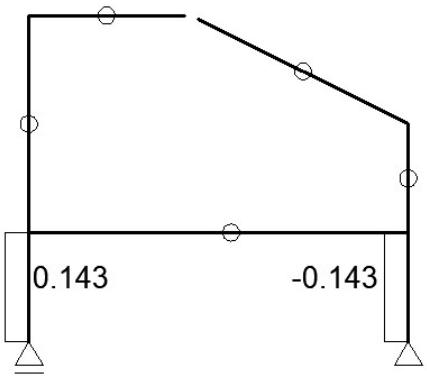
$N_{FG} := -V_G = -0.143\ \frac{1}{\mathbf{m}}$

$T_{FG} := -H_G = 0\ \frac{1}{\mathbf{m}}$

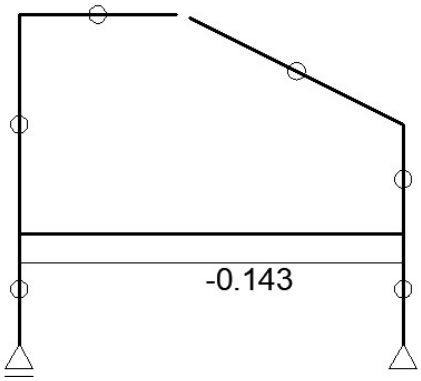
$M_{FG}(y) := H_G \cdot y - 1$

$M_{FG}(0\ \mathbf{m}) = -1$ $M_{FG}(2\ \mathbf{m}) = -1$

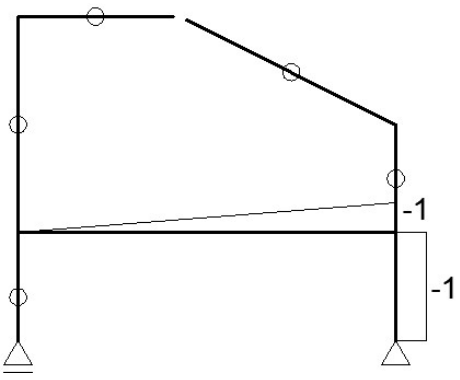
N3 [1/m]



T3 [1/m]



M3 [-]



Obliczenie przemieszczeń

$$\delta_{11} = \frac{1}{EJ}\left[\begin{array}{c} 4 \\ 4 \end{array} \begin{array}{c} 4 \\ 4 \end{array} + \begin{array}{c} 4 \\ 7 \\ 4 \end{array} + \begin{array}{c} 2 \\ 4.472 \\ 2 \end{array} + \begin{array}{c} 2 \\ 4 \end{array} \begin{array}{c} 2 \\ 2 \\ 4 \end{array} \right]$$

$$\delta_{11} := \frac{\mathbf{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 + 4 \cdot 7 \cdot 4 + \frac{1}{2} \cdot 2 \cdot 4.472 \cdot \frac{2}{3} \cdot 2 + \frac{1}{2} \cdot 2 \cdot 2 \cdot \left(\frac{2}{3} \cdot 2 + \frac{1}{3} \cdot 4 \right) + \frac{1}{2} \cdot 2 \cdot 4 \cdot \left(\frac{1}{3} \cdot 2 + \frac{2}{3} \cdot 4 \right) \right)$$

$$\delta_{11} = 0.22933 \ \frac{\mathbf{m}}{\mathbf{kN}}$$

$$\delta_{12} = \frac{1}{EJ}\left[\begin{array}{c} 4 \\ 4 \end{array} \begin{array}{c} \\ 3 \end{array} + \begin{array}{c} 4 \\ 7 \\ 3 \quad 4 \end{array} + \begin{array}{c} 2 \\ 4.472 \\ 4 \end{array} + \begin{array}{c} 2 \\ 4 \end{array} \begin{array}{c} \\ 2 \\ 4 \end{array} \right]$$

$$\delta_{12} := \frac{\mathbf{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 4 \cdot 4 \cdot 3 + 4 \cdot 7 \cdot \frac{3-4}{2} - \frac{1}{2} \cdot 2 \cdot 4.472 \cdot \frac{2}{3} \cdot 4 - \frac{2+4}{2} \cdot 2 \cdot 4 \right) = -0.03764 \ \frac{\mathbf{m}}{\mathbf{kN}} \qquad \delta_{21} := \delta_{12}$$

$$\delta_{13} = \frac{1}{EJ}\left[\begin{array}{c} 4 \\ 7 \\ 1 \end{array} \right]$$

$$\delta_{13} := \frac{\mathbf{m}^2}{EJ} \cdot \left(-4 \cdot 7 \cdot \frac{1}{2} \cdot 1 \right) = -0.02033 \ \frac{1}{\mathbf{kN}} \qquad \delta_{31} := \delta_{13}$$

$$\delta_{22} = \frac{1}{EJ}\left[\begin{array}{c} 3 \\ 3 \end{array} \begin{array}{c} \\ 3 \end{array} + \begin{array}{c} \\ 4 \\ 3 \end{array} \begin{array}{c} \\ \\ 3 \end{array} + \begin{array}{c} 3 \quad 4 \\ 7 \\ 3 \quad 4 \end{array} + \begin{array}{c} 4 \\ 4.472 \\ 4 \end{array} + \begin{array}{c} \\ 2 \\ 4 \end{array} \begin{array}{c} \\ \\ 4 \end{array} \right]$$

$$\delta_{22} := \frac{\mathbf{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 3 \cdot 3 \cdot \frac{2}{3} \cdot 3 + 3 \cdot 4 \cdot 3 + \frac{1}{2} \cdot 3 \cdot 7 \cdot \left(\frac{2}{3} \cdot 3 - \frac{1}{3} \cdot 4 \right) + \frac{1}{2} \cdot 4 \cdot 7 \cdot \left(-\frac{1}{3} \cdot 3 + \frac{2}{3} \cdot 4 \right) + \frac{1}{2} \cdot 4 \cdot 4.472 \cdot \frac{2}{3} \cdot 4 + 4 \cdot 2 \cdot 4 \right)$$

$$\delta_{22} = 0.19045 \ \frac{\mathbf{m}}{\mathbf{kN}}$$

$$\delta_{23} = \frac{1}{EJ}\left[\begin{array}{c} 3 \quad 4 \\ 7 \\ 1 \end{array} \right]$$

$$\delta_{23} := \frac{\mathbf{m}^2}{EJ} \cdot \left(\frac{1}{2} \cdot 1 \cdot 7 \cdot \left(\frac{2}{3} \cdot 4 - \frac{1}{3} \cdot 3 \right) \right) = 0.00847 \ \frac{1}{\mathbf{kN}} \qquad \delta_{32} := \delta_{23}$$

$$\delta_{33} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 1\cdot 7\cdot\frac{2}{3}\cdot 1 + 1\cdot 2\cdot 1\right]$$

$$\delta_{33} := \frac{\mathbf{m}}{EJ}\cdot\left(\frac{1}{2}\cdot 1\cdot 7\cdot\frac{2}{3}\cdot 1 + 1\cdot 2\cdot 1\right) = 0.00629\ \frac{1}{\mathbf{kNm}}$$

$$\delta_{1p} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 24\cdot 4\cdot\frac{2}{3}\cdot 4 + 4\cdot 7\cdot\frac{14+5.889}{2} + \frac{1}{2}\cdot 17.889\cdot 4.472\cdot\frac{2}{3}\cdot 2 + 2\cdot\frac{2+4}{2}\cdot 17.889\right]$$

Pole paraboli: $A_p := \frac{2}{3}\cdot\frac{q\cdot\cos\alpha\cdot L^2}{8}\cdot L\cdot\frac{1}{\mathbf{kN\cdot m^2}} = 13.333$

$$\delta_{1p} := \frac{\mathbf{kN\cdot m^3}}{EJ}\cdot\left(\frac{1}{2}\cdot 24\cdot 4\cdot\frac{2}{3}\cdot 4 + 4\cdot 7\cdot\frac{14+5.889}{2} + \frac{1}{2}\cdot 17.889\cdot 4.472\cdot\frac{2}{3}\cdot 2 - A_p\cdot\frac{1}{2}\cdot 2 + \frac{2+4}{2}\cdot 2\cdot 17.889\right) = 0.80398\ \mathbf{m}$$

$$\delta_{2p} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 24\cdot 4\cdot 3 + \frac{1}{2}\cdot 14\cdot 7\cdot\left(\frac{2}{3}\cdot 3 - \frac{1}{3}\cdot 4\right) + \frac{1}{2}\cdot 5.889\cdot 7\cdot\left(\frac{1}{3}\cdot 3 - \frac{2}{3}\cdot 4\right) - \frac{1}{2}\cdot 17.889\cdot 4.472\cdot\frac{2}{3}\cdot 4 + A_p\cdot\frac{1}{2}\cdot 4 - 17.889\cdot 2\cdot 4\right]$$

$$\delta_{2p1} := \frac{\mathbf{kN\cdot m^3}}{EJ}\cdot\left(\frac{1}{2}\cdot 24\cdot 4\cdot 3 + \frac{1}{2}\cdot 14\cdot 7\cdot\left(\frac{2}{3}\cdot 3 - \frac{1}{3}\cdot 4\right) + \frac{1}{2}\cdot 5.889\cdot 7\cdot\left(\frac{1}{3}\cdot 3 - \frac{2}{3}\cdot 4\right)\right)$$

$$\delta_{2p2} := \frac{\mathbf{kN\cdot m^3}}{EJ}\cdot\left(-\frac{1}{2}\cdot 17.889\cdot 4.472\cdot\frac{2}{3}\cdot 4 + A_p\cdot\frac{1}{2}\cdot 4 - 17.889\cdot 2\cdot 4\right)$$

$$\delta_{2p} := \delta_{2p1} + \delta_{2p2} = -0.1173\ \mathbf{m}$$

$$\delta_{3p} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 14\cdot 7\cdot 1 + 12\cdot 2\cdot 1\right]$$

$$\delta_{3p} := \frac{\mathbf{kN\cdot m^2}}{EJ}\cdot\left(\frac{-1}{2}\cdot 1\cdot 7\cdot\left(\frac{2}{3}\cdot 5.889 + \frac{1}{3}\cdot 14\right) + \frac{1}{2}\cdot 12\cdot 2\cdot 1\right) = -0.02624$$

Temperatura średnia:

$$t_{01} := \frac{0+10}{2}\ \mathbf{K} = 5\ \mathbf{K} \qquad t_{02} := \frac{10-20}{2}\ \mathbf{K} = -5\ \mathbf{K}$$

$$\delta_{1t0} := 1\cdot 3\ \mathbf{m}\cdot t_{01}\cdot\alpha_t + 0\cdot 2\ \mathbf{m}\cdot t_{02}\cdot\alpha_t = 0.00018\ \mathbf{m}$$

$$\delta_{2t0} := 0\cdot 3\ \mathbf{m}\cdot t_{01}\cdot\alpha_t + 1\cdot 2\ \mathbf{m}\cdot t_{02}\cdot\alpha_t = -0.00012\ \mathbf{m}$$

$$\delta_{3t0} := 0\ \frac{1}{\mathbf{m}}\cdot 3\ \mathbf{m}\cdot t_{01}\cdot\alpha_t + 0\ \frac{1}{\mathbf{m}}\cdot 2\ \mathbf{m}\cdot t_{02}\cdot\alpha_t = 0$$

Różnica temperatur:

$$\Delta t_1 := (10-0)\ \mathbf{K} = 10\ \mathbf{K} \qquad \Delta t_2 := (10--20)\ \mathbf{K} = 30\ \mathbf{K}$$

$$\delta_{1\Delta t} := 0\ \mathbf{m}\cdot 3\ \mathbf{m}\cdot\frac{\Delta t_1\cdot\alpha_t}{h} - \frac{2+4}{2}\ \mathbf{m}\cdot 2\ \mathbf{m}\cdot\frac{\Delta t_2\cdot\alpha_t}{h} = -0.018\ \mathbf{m}$$

$$\delta_{2\Delta t} := \frac{-1}{2}\cdot 3\ \mathbf{m}\cdot 3\ \mathbf{m}\cdot\frac{\Delta t_1\cdot\alpha_t}{h} + 4\ \mathbf{m}\cdot 2\ \mathbf{m}\cdot\frac{\Delta t_2\cdot\alpha_t}{h} = 0.0195\ \mathbf{m}$$

$$\delta_{3\Delta t} := 0\cdot 3\ \mathbf{m}\cdot\frac{\Delta t_1\cdot\alpha_t}{h} + 0\cdot 2\ \mathbf{m}\cdot\frac{\Delta t_2\cdot\alpha_t}{h} = 0$$

Wymuszenie w podporze:

$R_1 := 0\ \mathbf{m} \quad R_2 := 0\ \mathbf{m} \quad R_3 := 1 \qquad \Delta := 1\ ° = 0.01745$

$\delta_{1\Delta} := -R_1 \cdot \Delta = 0\ \mathbf{m} \qquad \delta_{2\Delta} := -R_2 \cdot \Delta = 0\ \mathbf{m} \qquad \delta_{3\Delta} := -R_3 \cdot \Delta = -0.01745$

Podpora sprężysta:

$R_p := -1.159\ \mathbf{kN} \qquad R_1 := 0 \qquad R_2 := 0 \qquad R_3 := -0.143\ \frac{1}{\mathbf{m}} \qquad k := 500\ \frac{\mathbf{kN}}{\mathbf{m}}$

$\delta_{11k} := \frac{R_1 \cdot R_1}{k} = 0\ \frac{\mathbf{m}}{\mathbf{kN}} \qquad \delta_{12k} := \frac{R_1 \cdot R_2}{k} = 0\ \frac{\mathbf{m}}{\mathbf{kN}} \qquad \delta_{13k} := \frac{R_1 \cdot R_3}{k} = 0\ \frac{1}{\mathbf{kN}}$

$\delta_{21k} := \frac{R_2 \cdot R_1}{k} = 0\ \frac{\mathbf{m}}{\mathbf{kN}} \qquad \delta_{22k} := \frac{R_2 \cdot R_2}{k} = 0\ \frac{\mathbf{m}}{\mathbf{kN}} \qquad \delta_{23k} := \frac{R_2 \cdot R_3}{k} = 0\ \frac{1}{\mathbf{kN}}$

$\delta_{31k} := \frac{R_3 \cdot R_1}{k} = 0\ \frac{1}{\mathbf{kN}} \qquad \delta_{32k} := \frac{R_3 \cdot R_2}{k} = 0\ \frac{1}{\mathbf{kN}} \qquad \delta_{33k} := \frac{R_3 \cdot R_3}{k} = 0.00004\ \frac{1}{\mathbf{kNm}}$

$\delta_{1k} := \frac{R_1 \cdot R_p}{k} = 0\ \mathbf{m} \qquad \delta_{2k} := \frac{R_2 \cdot R_p}{k} = 0\ \mathbf{m} \qquad \delta_{3k} := \frac{R_3 \cdot R_p}{k} = 0.00033$

Sumaryczne przemieszczenia:

$\delta_{1P} := \delta_{1p} + \delta_{1t0} + \delta_{1\Delta t} + \delta_{1\Delta} + \delta_{1k} = 0.78616\ \mathbf{m}$

$\delta_{2P} := \delta_{2p} + \delta_{2t0} + \delta_{2\Delta t} + \delta_{2\Delta} + \delta_{2k} = -0.09792\ \mathbf{m}$

$\delta_{3P} := \delta_{3p} + \delta_{3t0} + \delta_{3\Delta t} + \delta_{3\Delta} + \delta_{3k} = -0.04336$

Rozwiązanie układu równań

$(\delta_{11} + \delta_{11k})\ x_1 + (\delta_{12} + \delta_{12k})\ x_2 + (\delta_{13} + \delta_{13k})\ x_3 + \delta_{1P} = 0$

$(\delta_{21} + \delta_{21k})\ x_1 + (\delta_{22} + \delta_{22k})\ x_2 + (\delta_{23} + \delta_{23k})\ x_3 + \delta_{2P} = 0$

$(\delta_{31} + \delta_{31k})\ x_1 + (\delta_{32} + \delta_{32k})\ x_2 + (\delta_{33} + \delta_{33k})\ x_3 + \delta_{3P} = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} \delta_{11} + \delta_{11k} & \delta_{12} + \delta_{12k} & \delta_{13} + \delta_{13k} \\ \delta_{21} + \delta_{21k} & \delta_{22} + \delta_{22k} & \delta_{23} + \delta_{23k} \\ \delta_{31} + \delta_{31k} & \delta_{32} + \delta_{32k} & \delta_{33} + \delta_{33k} \end{bmatrix}^{-1} \cdot - \begin{bmatrix} \delta_{1P} \\ \delta_{2P} \\ \delta_{3P} \end{bmatrix}$$

$x_1 = -3.943\ \mathbf{kN} \qquad x_2 = -0.007\ \mathbf{kN} \qquad x_3 = -5.8\ \mathbf{kNm}$

Wykresy ostateczne

$R_{ost} = R_1 \cdot x_1 + R_2 \cdot x_2 + R_3 \cdot x_3 + R_p$

$V_A := 0\ x_1 + 0\ x_2 + \left(-0.143 \cdot \frac{1}{\mathbf{m}}\right) x_3 + \left(-1.159\ \mathbf{kN}\right) = -0.33\ \mathbf{kN}$

$V_G := 0\ x_1 + 0\ x_2 + \left(0.143 \cdot \frac{1}{\mathbf{m}}\right) x_3 + \left(10.103\ \mathbf{kN}\right) = 9.274\ \mathbf{kN}$

$H_G := 0\ x_1 + 0\ x_2 + \left(0 \cdot \frac{1}{\mathbf{m}}\right) x_3 + \left(-6\ \mathbf{kN}\right) = -6\ \mathbf{kN}$

$M_G := 0\ \mathbf{m}\ x_1 + 0\ \mathbf{m}\ x_2 + (1)\ x_3 + (0\ \mathbf{kNm}) = -5.8\ \mathbf{kNm}$

$$N_{ost} = N_1 \cdot x_1 + N_2 \cdot x_2 + N_3 \cdot x_3 + N_p$$

$$N_{ost} := \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0.894 \\ 0.894 \\ 0 \\ 0 \end{bmatrix} \cdot x_1 + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0.447 \\ 0.447 \\ 1 \\ 0 \end{bmatrix} \cdot x_2 + \begin{bmatrix} 0.143 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.143 \end{bmatrix} \frac{1}{\mathbf{m}} x_3 + \begin{bmatrix} 1.159 \\ 0 \\ 0 \\ -6 \\ 0 \\ -4 \\ -8.944 \\ -10.103 \end{bmatrix} \mathbf{kN} = \begin{bmatrix} 0.33 \\ -3.943 \\ 0.007 \\ -2.057 \\ -3.529 \\ -7.529 \\ -8.951 \\ -9.274 \end{bmatrix} \mathbf{kN}$$

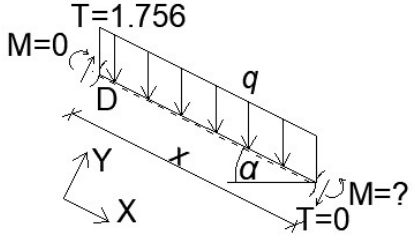
$$T_{ost} = T_1 \cdot x_1 + T_2 \cdot x_2 + T_3 \cdot x_3 + T_p$$

$$T_{ost} := \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -0.447 \\ -0.447 \\ -1 \\ 0 \end{bmatrix} \cdot x_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0.894 \\ 0.894 \\ 0 \\ 0 \end{bmatrix} \cdot x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.143 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \frac{1}{\mathbf{m}} x_3 + \begin{bmatrix} 0 \\ 0 \\ 6 \\ -1.159 \\ 0 \\ -8 \\ 0 \\ 6 \end{bmatrix} \mathbf{kN} = \begin{bmatrix} 0 \\ -0.007 \\ 2.057 \\ -0.322 \\ 1.756 \\ -6.244 \\ 3.943 \\ 6 \end{bmatrix} \mathbf{kN}$$

$$M_{ost} = M_1 \cdot x_1 + M_2 \cdot x_2 + M_3 \cdot x_3 + M_p$$

$$M_{ost} := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -4 \\ 4 \\ 4 \\ 0 \\ -2 \\ -2 \\ -4 \\ 0 \\ 0 \end{bmatrix} \mathbf{m} \cdot x_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3 \\ -3 \\ -3 \\ 3 \\ -4 \\ 0 \\ 4 \\ 4 \\ 4 \\ 0 \\ 0 \end{bmatrix} \mathbf{m} \cdot x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -24 \\ 14 \\ 5.889 \\ 0 \\ -17.889 \\ -17.889 \\ -17.889 \\ -12 \\ 0 \end{bmatrix} \mathbf{kNm} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.022 \\ 0.022 \\ -8.205 \\ -1.795 \\ -4.055 \\ 0 \\ -10.031 \\ -10.031 \\ -2.145 \\ -6.2 \\ 5.8 \end{bmatrix} \mathbf{kNm}$$

Obliczenie ekstremum paraboli



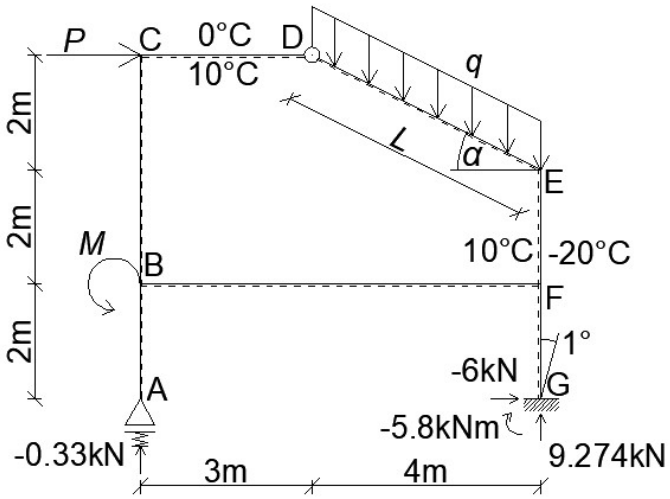
$$\Sigma Y = 1.756 - 0 - q \cdot cos\alpha \cdot x = 0$$

$$x := \frac{1.756\ \mathbf{kN} - 0\ \mathbf{kN}}{q \cdot cos\alpha} = 0.982\ \mathbf{m}$$

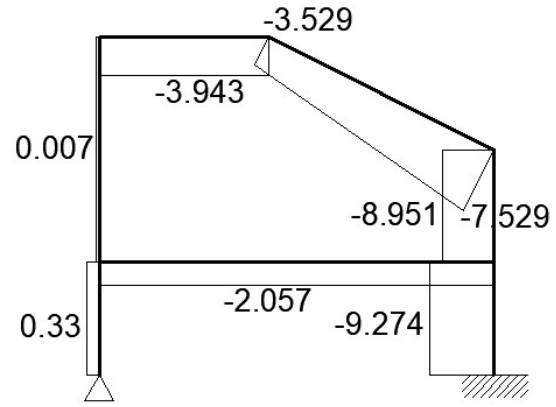
$$\Sigma M_D = 0 - M_{max} + q \cdot x \cdot \frac{x \cdot cos\alpha}{2} = 0$$

$$M_{max} := q \cdot x \cdot \frac{x \cdot cos\alpha}{2} = 0.862\ \mathbf{kNm}$$

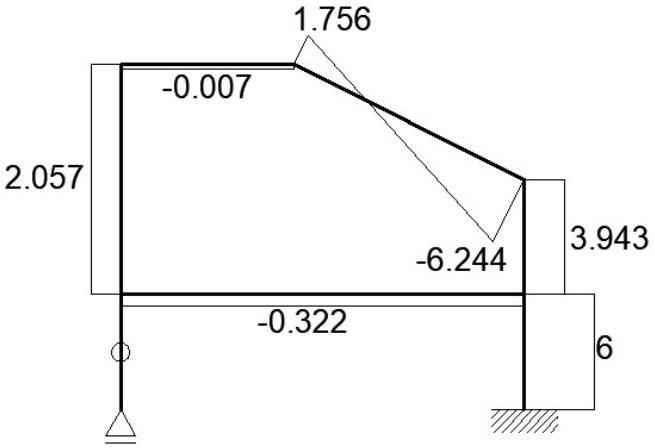
Wykresy ostateczne



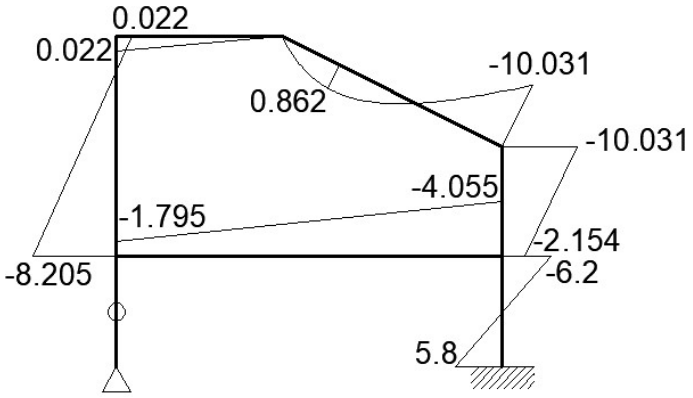
N [kN]



T [kN]

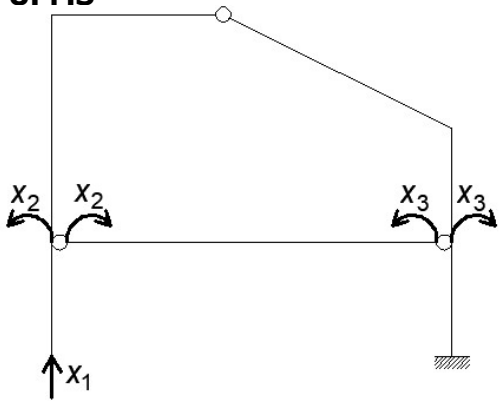


M [kNm]

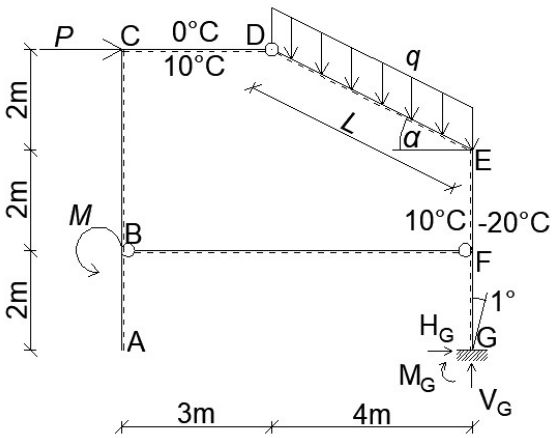


Sprawdzenie kinematyczne

UPMS*



stan p*



Reakcje:

$\Sigma M_G = P \cdot 6 - M - q \cdot L \cdot 2 + M_G = 0$

$M_G := -P \cdot 6\ \mathbf{m} + M + q \cdot L \cdot 2\ \mathbf{m} = -8.111\ \mathbf{kNm}$

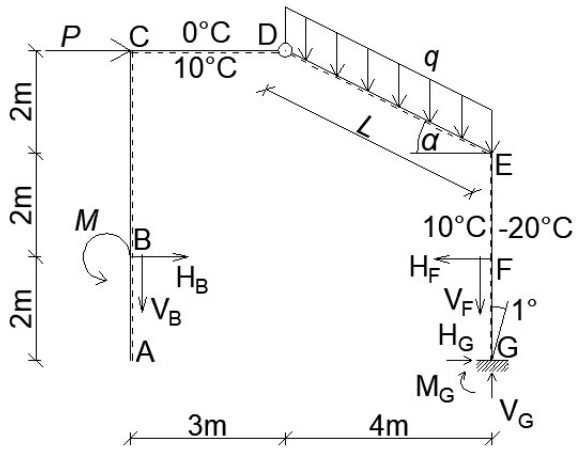
$\Sigma X = H_G + 6 = 0$

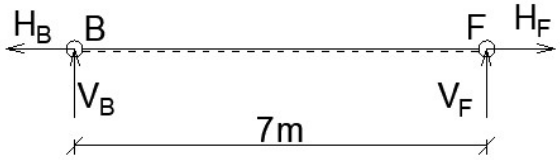
$H_G := -6\ \mathbf{kN}$

$\Sigma Y = V_G - q \cdot L = 0$

$V_G := q \cdot L = 8.944\ \mathbf{kN}$

Siły w ściągu:





Ściąg: $\Sigma M_B = -V_F \cdot 7 = 0 \qquad V_F := 0\ \mathbf{kN}$

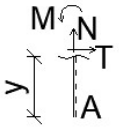
Ściąg: $\Sigma Y = V_B + V_F = 0 \qquad V_B := -V_F = 0\ \mathbf{kN}$

Rama: $\Sigma M_D{}^L = -M - H_B \cdot 4 - V_B \cdot 3 = 0$

$$H_B := \frac{-M - V_B \cdot 3\ \mathbf{m}}{4\ \mathbf{m}} = -2.5\ \mathbf{kN}$$

Ściąg: $\Sigma X = -H_B + H_F = 0 \qquad H_F := H_B = -2.5\ \mathbf{kN}$

Pręt AB y∈(0;2)m



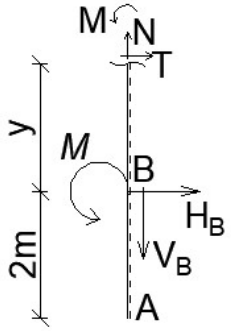
$N_{AB} := 0\ \mathbf{kN}$

$T_{AB} := 0\ \mathbf{kN}$

$M_{AB}(y) := 0\ \mathbf{kNm}$

$M_{AB}(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_{AB}(2\ \mathbf{m}) = 0\ \mathbf{kNm}$

Pręt BC y∈(0;4)m



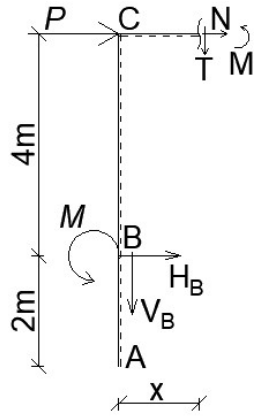
$N_{BC} := V_B = 0\ \mathbf{kN}$

$T_{BC} := -H_B = 2.5\ \mathbf{kN}$

$M_{BC}(y) := -M - H_B \cdot y$

$M_{BC}(0\ \mathbf{m}) = -10\ \mathbf{kNm} \qquad M_{BC}(4\ \mathbf{m}) = 0\ \mathbf{kNm}$

Pręt CD x∈(0;3)m



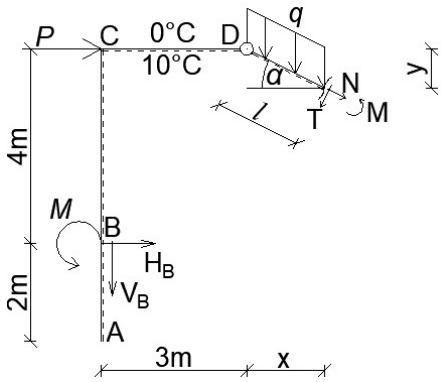
$N_{CD} := -H_B - P = -3.5\ \mathbf{kN}$

$T_{CD} := -V_B = 0\ \mathbf{kN}$

$M_{CD}(x) := -M - H_B \cdot 4\ \mathbf{m} - V_B \cdot x$

$M_{CD}(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_{CD}(3\ \mathbf{m}) = 0\ \mathbf{kNm}$

Pręt DE x∈(0;4)m $y(x) := 0.5\ x$ $l(x) := \sqrt{y(x)^2 + x^2}$



$N_{DE}(x) := -H_B \cdot cos\alpha - V_B \cdot sin\alpha - P \cdot cos\alpha - q \cdot l(x) \cdot sin\alpha$

$T_{DE}(x) := H_B \cdot sin\alpha - V_B \cdot cos\alpha + P \cdot sin\alpha - q \cdot l(x) \cdot cos\alpha$

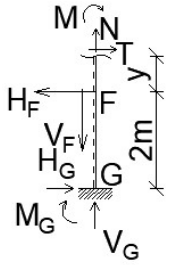
$M_{DE}(x) := -M - H_B \cdot (4\ \mathbf{m} - y(x)) - V_B \cdot (3\ \mathbf{m} + x) + P \cdot y(x) - q \cdot l(x) \cdot \frac{x}{2}$

$N_{DE}(0\ \mathbf{m}) = -3.13\ \mathbf{kN}$ $N_{DE}(4\ \mathbf{m}) = -7.13\ \mathbf{kN}$

$T_{DE}(0\ \mathbf{m}) = 1.565\ \mathbf{kN}$ $T_{DE}(4\ \mathbf{m}) = -6.435\ \mathbf{kN}$

$M_{DE}(0\ \mathbf{m}) = 0\ \mathbf{kNm}$ $M_{DE}(4\ \mathbf{m}) = -10.889\ \mathbf{kNm}$

Pręt EF y∈(0;2)m



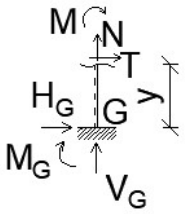
$N_{EF} := V_F - V_G = -8.944\ \mathbf{kN}$

$T_{EF} := H_F - H_G = 3.5\ \mathbf{kN}$

$M_{EF}(y) := -H_F \cdot y + H_G \cdot (2\ \mathbf{m} + y) - M_G$

$M_{EF}(0\ \mathbf{m}) = -3.889\ \mathbf{kNm}$ $M_{EF}(2\ \mathbf{m}) = -10.889\ \mathbf{kNm}$

Pręt FG y∈(0;2)m



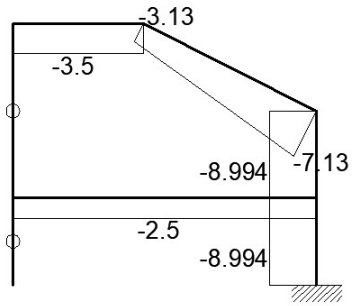
$N_{FG} := -V_G = -8.944\ \mathbf{kN}$

$T_{FG} := -H_G = 6\ \mathbf{kN}$

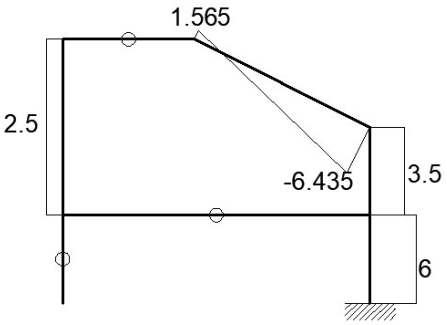
$M_{FG}(y) := H_G \cdot y - M_G$

$M_{FG}(0\ \mathbf{m}) = 8.111\ \mathbf{kNm}$ $M_{FG}(2\ \mathbf{m}) = -3.889\ \mathbf{kNm}$

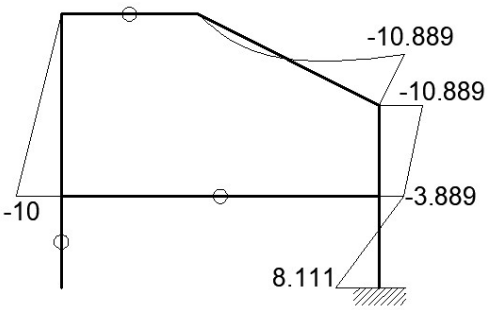
Np* [kN]



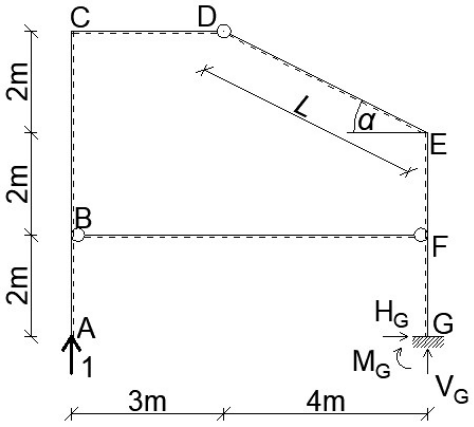
Tp* [kN]



Mp* [kNm]



Stan x1*=1



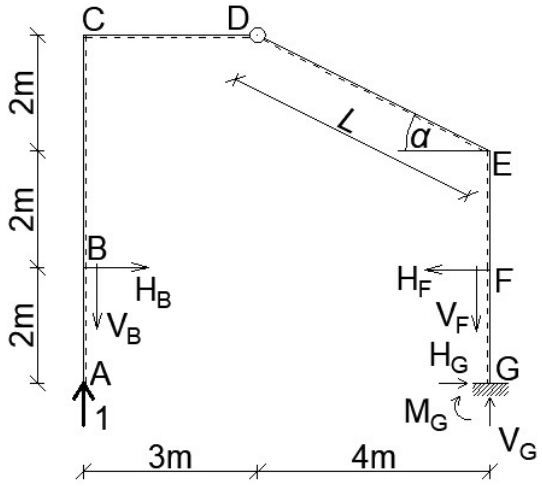
<u>Reakcje:</u>

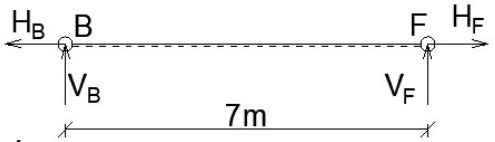
$\Sigma M_G = 1 \cdot 7 + M_G = 0$ $M_G := -1 \cdot 7\ \mathbf{m} = -7\ \mathbf{m}$

$\Sigma X = H_G = 0$ $H_G := 0$

$\Sigma Y = V_G + 1 = 0$ $V_G := -1$

Siły w ściągu:





Ściąg: $\Sigma M_B = -V_F \cdot 7 = 0 \qquad V_F := 0$

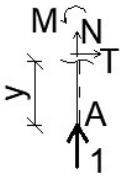
Ściąg: $\Sigma Y = V_B + V_F = 0 \qquad V_B := -V_F = 0$

Rama: $\Sigma M_D{}^L = 1 \cdot 3 - H_B \cdot 4 - V_B \cdot 3 = 0$

$$H_B := \frac{1 \cdot 3\ \mathbf{m} - V_B \cdot 3\ \mathbf{m}}{4\ \mathbf{m}} = 0.75$$

Ściąg: $\Sigma X = -H_B + H_F = 0 \qquad H_F := H_B = 0.75$

Pręt AB y∈(0;2)m



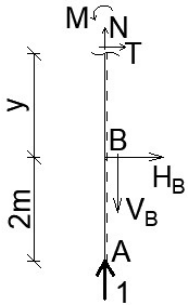
$N_{AB} := -1$

$T_{AB} := 0$

$M_{AB}(y) := 0\ \mathbf{m}$

$M_{AB}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{AB}(2\ \mathbf{m}) = 0\ \mathbf{m}$

Pręt BC y∈(0;4)m



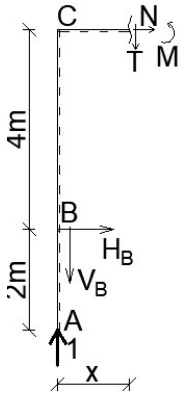
$N_{BC} := -1 + V_B = -1$

$T_{BC} := -H_B = -0.75$

$M_{BC}(y) := -H_B \cdot y$

$M_{BC}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{BC}(4\ \mathbf{m}) = -3\ \mathbf{m}$

Pręt CD x∈(0;3)m



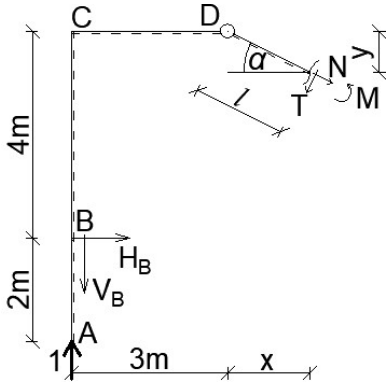
$N_{CD} := -H_B = -0.75$

$T_{CD} := 1 - V_B = 1$

$M_{CD}(x) := 1 \cdot x - H_B \cdot 4\ \mathbf{m} - V_B \cdot x$

$M_{CD}(0\ \mathbf{m}) = -3\ \mathbf{m} \qquad M_{CD}(3\ \mathbf{m}) = 0\ \mathbf{m}$

Pręt DE x∈(0;4)m



$y(x) := 0.5\ x \qquad l(x) := \sqrt{y(x)^2 + x^2}$

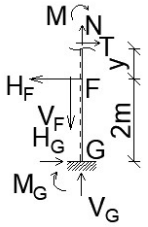
$N_{DE} := 1 \cdot sin\alpha - H_B \cdot cos\alpha - V_B \cdot sin\alpha = -0.224$

$T_{DE} := 1 \cdot cos\alpha + H_B \cdot sin\alpha - V_B \cdot cos\alpha = 1.23$

$M_{DE}(x) := 1 \cdot (3\ \mathbf{m} + x) - H_B \cdot (4\ \mathbf{m} - y(x)) - V_B \cdot (3\ \mathbf{m} + x)$

$M_{DE}(0\ \mathbf{m}) = 0\ \mathbf{m} \qquad M_{DE}(4\ \mathbf{m}) = 5.5\ \mathbf{m}$

Pręt EF y∈(0;2)m



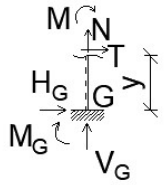
$$N_{EF} := V_F - V_G = 1$$

$$T_{EF} := H_F - H_G = 0.75$$

$$M_{EF}(y) := -H_F \cdot y + H_G \cdot (2\ \mathbf{m} + y) - M_G$$

$$M_{EF}(0\ \mathbf{m}) = 7\ \mathbf{m} \qquad M_{EF}(2\ \mathbf{m}) = 5.5\ \mathbf{m}$$

Pręt FG y∈(0;2)m



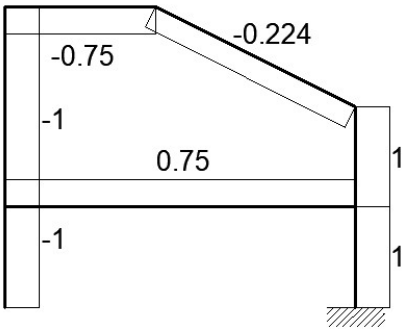
$$N_{FG} := -V_G = 1$$

$$T_{FG} := -H_G = 0$$

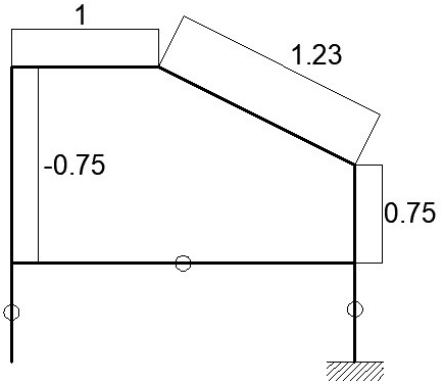
$$M_{FG}(y) := H_G \cdot y - M_G$$

$$M_{FG}(0\ \mathbf{m}) = 7\ \mathbf{m} \qquad M_{FG}(2\ \mathbf{m}) = 7\ \mathbf{m}$$

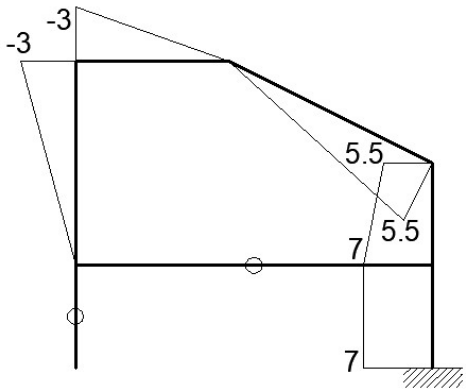
N1* [-]



T1* [-]

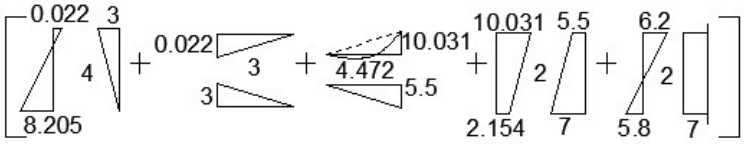


M1* [m]



Obliczenie przemieszczeń

$$\delta_{1ost} = \frac{1}{EJ}$$



$$\delta_{1ost1} := \frac{\mathbf{kN}\cdot\mathbf{m}^3}{EJ} \cdot \left(\frac{1}{2} \cdot 3 \cdot 4 \cdot \left(\frac{1}{3} \cdot 8.205 - \frac{2}{3} \cdot 0.022 \right) - \frac{1}{2} \cdot 0.022 \cdot 3 \cdot \frac{2}{3} \cdot 3 - \frac{1}{2} \cdot 10.031 \cdot 4.472 \cdot \frac{2}{3} \cdot 5.5 \right)$$

$$\delta_{1ost2} := \frac{\mathbf{kN}\cdot\mathbf{m}^3}{EJ} \cdot \left(A_p \cdot \frac{1}{2} \cdot 5.5 - \frac{1}{2} \cdot 10.031 \cdot 2 \cdot \left(\frac{2}{3} \cdot 5.5 + \frac{1}{3} \cdot 7 \right) - \frac{1}{2} \cdot 2.154 \cdot 2 \cdot \left(\frac{1}{3} \cdot 5.5 + \frac{2}{3} \cdot 7 \right) + 7 \cdot 2 \cdot \frac{5.8 - 6.2}{2} \right)$$

$$\delta_{1ost} := \delta_{1ost1} + \delta_{1ost2} = -0.15433\ \mathbf{m}$$

$$\delta_{1t0} := -0.75 \cdot 3\ \mathbf{m} \cdot t_{01} \cdot \alpha_t + 1 \cdot 2\ \mathbf{m} \cdot t_{02} \cdot \alpha_t = -0.00026\ \mathbf{m}$$

$$\delta_{1\Delta t} := -\frac{1}{2} \cdot 3\ \mathbf{m} \cdot 3\ \mathbf{m} \cdot \frac{\Delta t_1 \cdot \alpha_t}{h} + \frac{5.5 + 7}{2}\ \mathbf{m} \cdot 2\ \mathbf{m} \cdot \frac{\Delta t_2 \cdot \alpha_t}{h} = 0.033\ \mathbf{m}$$

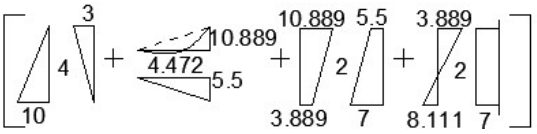
$$\delta_{1\Delta} := -(-7\ \mathbf{m}) \cdot \Delta = 0.12217\ \mathbf{m}$$

$$\delta_{1k} = \frac{R_1 \cdot R_{ost}}{k} \qquad \delta_{1k} := \frac{1 \cdot -0.33\ \mathbf{kN}}{k} = -0.00066\ \mathbf{m}$$

$$\delta_1 := \delta_{1ost} + \delta_{1t0} + \delta_{1\Delta t} + \delta_{1\Delta} + \delta_{1k} = -0.00008\ \mathbf{m}$$

Sprawdzenie względne:

$$\delta_{1p} = \frac{1}{EJ}\left[\; + \; + \; + \; \right]$$



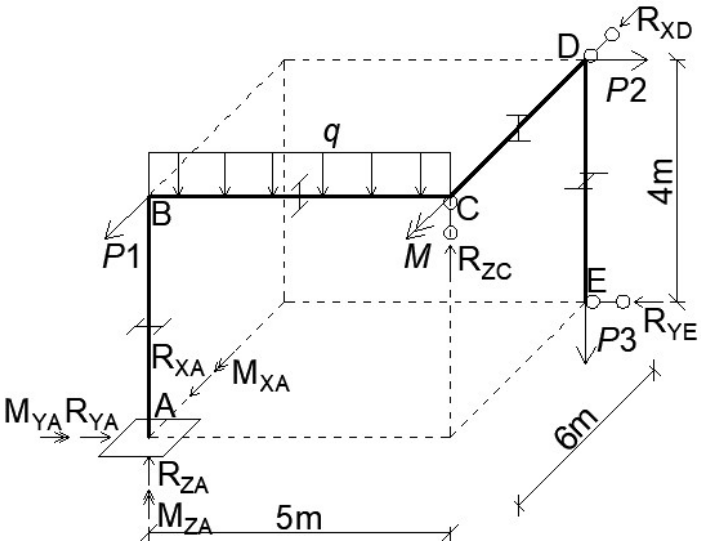
$$\delta_{1p1} := \frac{\mathbf{kN \cdot m^3}}{EJ} \cdot \left(\frac{1}{2} \cdot 10 \cdot 4 \cdot \frac{1}{3 \cdot 3} - \frac{1}{2} \cdot 10.889 \cdot 4.472 \cdot \frac{2}{3} \cdot 5.5 + A_p \cdot \frac{1}{2} \cdot 5.5 \right)$$

$$\delta_{1p2} := \frac{\mathbf{kN \cdot m^3}}{EJ} \cdot \left(-\frac{1}{2} \cdot 10.889 \cdot 2 \cdot \left(\frac{2}{3} \cdot 5.5 + \frac{1}{3} \cdot 7 \right) - \frac{1}{2} \cdot 3.889 \cdot 2 \cdot \left(\frac{1}{3} \cdot 5.5 + \frac{2}{3} \cdot 7 \right) + 7 \cdot 2 \cdot \frac{8.111 - 3.889}{2} \right)$$

$$\delta_{1p} := \delta_{1p1} + \delta_{1p2} = -0.1618 \ \mathbf{m}$$

$$\frac{\delta_1}{\delta_{1p}} = 0.047\% \quad <5\% \quad \text{Warunek spełniony}$$

Zadanie 2: Rama 3D



$E := 210\ \mathbf{GPa}$ $\quad v := 0.3$

$G := \frac{E}{2 \cdot (1+v)} = 80.769\ \mathbf{GPa}$

Przekrój: IPE 180

$J_2 := 1320\ \mathbf{cm}^4$

$EJ_2 := E \cdot J_2 = 2772\ \mathbf{kN \cdot m}^2$

$J_3 := 101\ \mathbf{cm}^4$

$EJ_3 := E \cdot J_3 = 212.1\ \mathbf{kN \cdot m}^2$

$J_0 := 4.79\ \mathbf{cm}^4$

$GJ_0 := G \cdot J_0 = 3.869\ \mathbf{kN \cdot m}^2$

$P1 := 12\ \mathbf{kN}$

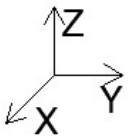
$P2 := 15\ \mathbf{kN}$

$P3 := 7\ \mathbf{kN}$

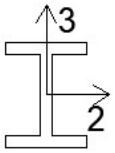
$M := 8\ \mathbf{kNm}$

$q := 3\ \frac{\mathbf{kN}}{\mathbf{m}}$

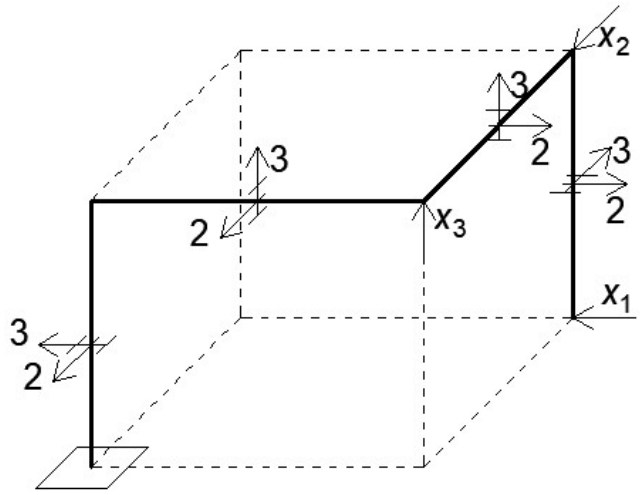
Układ współrzędnych globalnych:



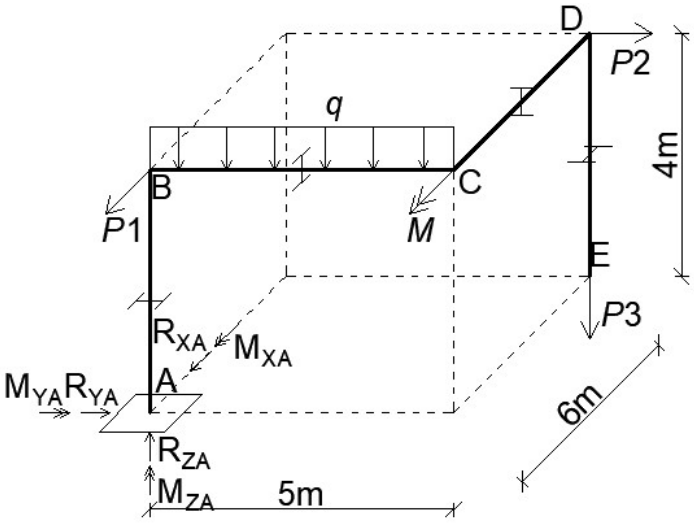
Układ współrzędnych lokalnych:



UPMS i przyjęcie układów lokalnych



Stan p



Reakcje:

$\Sigma X = R_{XA} + P1 = 0$ $\quad R_{XA} := -P1 = -12\ \mathbf{kN}$

$\Sigma Y = R_{YA} + P2 = 0$ $\quad R_{YA} := -P2 = -15\ \mathbf{kN}$

$\Sigma Z = R_{ZA} - P3 - q \cdot 5 = 0$ $\quad R_{ZA} := P3 + q \cdot 5\ \mathbf{m} = 22\ \mathbf{kN}$

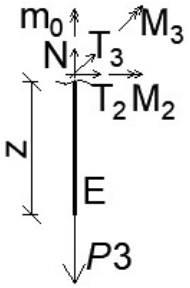
$\Sigma M_{AX} = M_{XA} + M - q \cdot 5 \cdot 2.5 - P2 \cdot 4 - P3 \cdot 5 = 0$

$M_{XA} := -M + q \cdot 5\ \mathbf{m} \cdot 2.5\ \mathbf{m} + P2 \cdot 4\ \mathbf{m} + P3 \cdot 5\ \mathbf{m} = 124.5\ \mathbf{kNm}$

$\Sigma M_{AY} = M_{YA} + P1 \cdot 4 - P3 \cdot 6 = 0$ $\quad M_{YA} := -P1 \cdot 4\ \mathbf{m} + P3 \cdot 6\ \mathbf{m} = -6\ \mathbf{kNm}$

$\Sigma M_{AZ} = M_{ZA} - P2 \cdot 6 = 0$ $\quad M_{ZA} := P2 \cdot 6\ \mathbf{m} = 90\ \mathbf{kNm}$

Pręt ED z∈(0;4)m



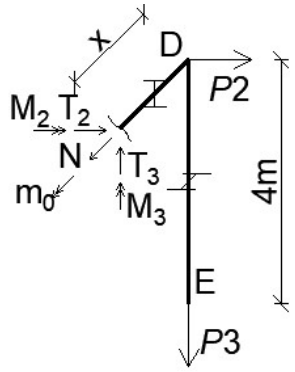
$N := P3 = 7\ \mathbf{kN}$

$T_2 := 0\ \mathbf{kN}$ $T_3 := 0\ \mathbf{kN}$

$m_0 := 0\ \mathbf{kNm}$

$M_2 := 0\ \mathbf{kNm}$ $M_3 := 0\ \mathbf{kNm}$

Pręt DC x∈(0;6)m



$N := 0\ \mathbf{kN}$

$T_2 := -P2 = -15\ \mathbf{kN}$ $T_3 := P3 = 7\ \mathbf{kN}$

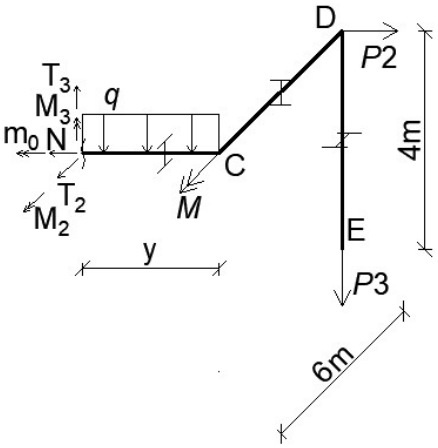
$m_0 := 0\ \mathbf{kNm}$

$M_2(x) := P3 \cdot x$ $M_3(x) := P2 \cdot x$

$M_2(0\ \mathbf{m}) = 0\ \mathbf{kNm}$ $M_3(0\ \mathbf{m}) = 0\ \mathbf{kNm}$

$M_2(6\ \mathbf{m}) = 42\ \mathbf{kNm}$ $M_3(6\ \mathbf{m}) = 90\ \mathbf{kNm}$

Pręt CB y∈(0;5)m



$N := P2 = 15\ \mathbf{kN}$

$T_2 := 0\ \mathbf{kN}$ $T_3(y) := P3 + q \cdot y$

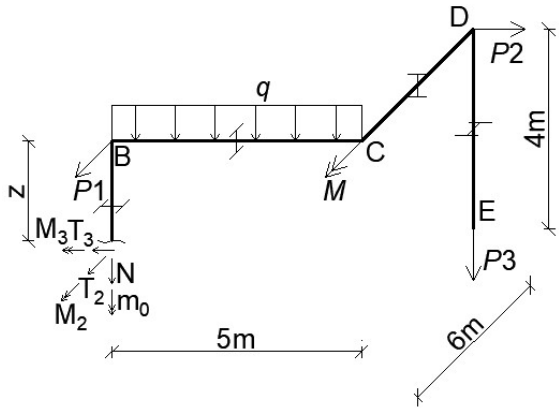
$m_0 := -P3 \cdot 6\ \mathbf{m} = -42\ \mathbf{kNm}$

$M_2(y) := P3 \cdot y - M + q \cdot y \cdot \frac{y}{2}$ $M_3 := P2 \cdot 6\ \mathbf{m} = 90\ \mathbf{kNm}$

$M_2(0\ \mathbf{m}) = -8\ \mathbf{kNm}$ $T_3(0\ \mathbf{m}) = 7\ \mathbf{kN}$

$M_2(5\ \mathbf{m}) = 64.5\ \mathbf{kNm}$ $T_3(5\ \mathbf{m}) = 22\ \mathbf{kN}$

Pręt BA z∈(0;4)m



$N := -P3 - q \cdot 5\ \mathbf{m} = -22\ \mathbf{kN}$

$T_2 := -P1 = -12\ \mathbf{kN}$ $T_3 := P2 = 15\ \mathbf{kN}$

$m_0 := -P2 \cdot 6\ \mathbf{m} = -90\ \mathbf{kNm}$

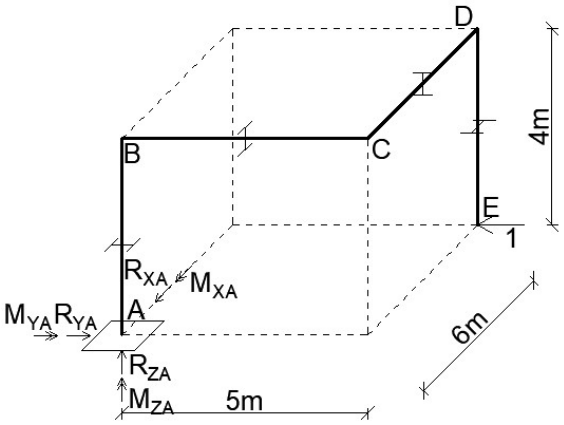
$M_2(z) := -M + q \cdot 5\ \mathbf{m} \cdot 2.5\ \mathbf{m} + P2 \cdot z + P3 \cdot 5\ \mathbf{m}$

$M_3(z) := P1 \cdot z - P3 \cdot 6\ \mathbf{m}$

$M_2(0\ \mathbf{m}) = 64.5\ \mathbf{kNm}$ $M_3(0\ \mathbf{m}) = -42\ \mathbf{kNm}$

$M_2(4\ \mathbf{m}) = 124.5\ \mathbf{kNm}$ $M_3(4\ \mathbf{m}) = 6\ \mathbf{kNm}$

Stan x1=1



Reakcje:

$\Sigma X = R_{XA} = 0$ $\quad R_{XA} := 0$

$\Sigma Y = R_{YA} - 1 = 0$ $\quad R_{YA} := 1$

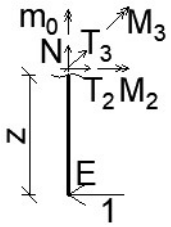
$\Sigma Z = R_{ZA} = 0$ $\quad R_{ZA} := 0$

$\Sigma M_{AX} = M_{XA} = 0$ $\quad M_{XA} := 0\ \mathbf{m}$

$\Sigma M_{AY} = M_{YA} = 0$ $\quad M_{YA} := 0\ \mathbf{m}$

$\Sigma M_{AZ} = M_{ZA} + 1 \cdot 6 = 0$ $\quad M_{ZA} := -1 \cdot 6\ \mathbf{m} = -6\ \mathbf{m}$

Pręt ED $\quad z \in (0;4)\mathrm{m}$



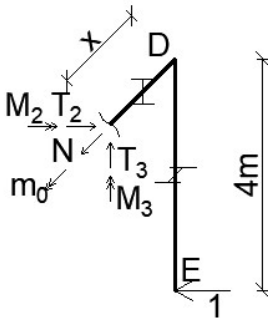
$N := 0$

$T_2 := 1$ $\quad T_3 := 0$

$m_0 := 0\ \mathbf{m}$

$M_2 := 0\ \mathbf{m}$ $\quad M_3(z) := -1 \cdot z$ $\quad M_3(0\ \mathbf{m}) = 0\ \mathbf{m}$ $\quad M_3(4\ \mathbf{m}) = -4\ \mathbf{m}$

Pręt DC $\quad x \in (0;6)\mathrm{m}$



$N := 0$

$T_2 := 1$ $\quad T_3 := 0$

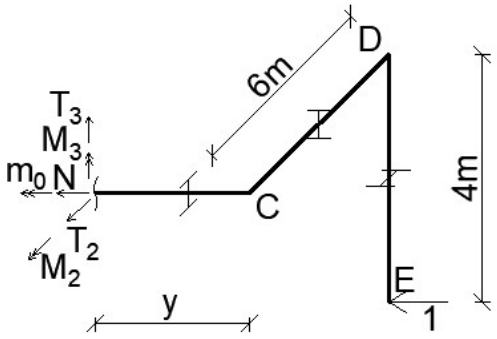
$m_0 := 1 \cdot 4\ \mathbf{m} = 4\ \mathbf{m}$

$M_2 := 0\ \mathbf{m}$ $\quad M_3(x) := -1 \cdot x$

$M_3(0\ \mathbf{m}) = 0\ \mathbf{m}$

$M_3(6\ \mathbf{m}) = -6\ \mathbf{m}$

Pręt CB $\quad y \in (0;5)\mathrm{m}$



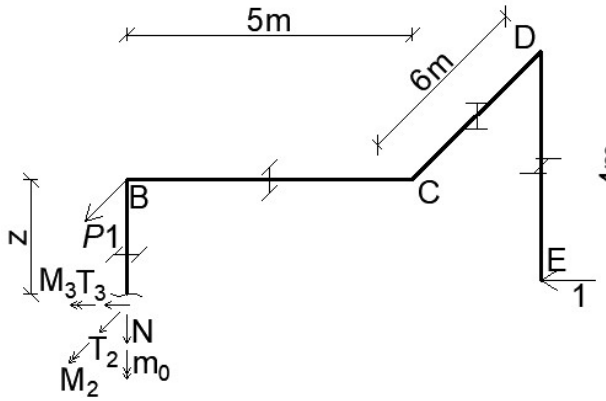
$N := -1$

$T_2 := 0$ $\quad T_3 := 0$

$m_0 := 0\ \mathbf{m}$

$M_2 := 1 \cdot 4\ \mathbf{m} = 4\ \mathbf{m}$ $\quad M_3 := -1 \cdot 6\ \mathbf{m} = -6\ \mathbf{m}$

Pręt BA $\quad z \in (0;4)\mathrm{m}$



$N := 0$

$T_2 := 0$ $\quad T_3 := -1$

$m_0 := 1 \cdot 6\ \mathbf{m} = 6\ \mathbf{m}$

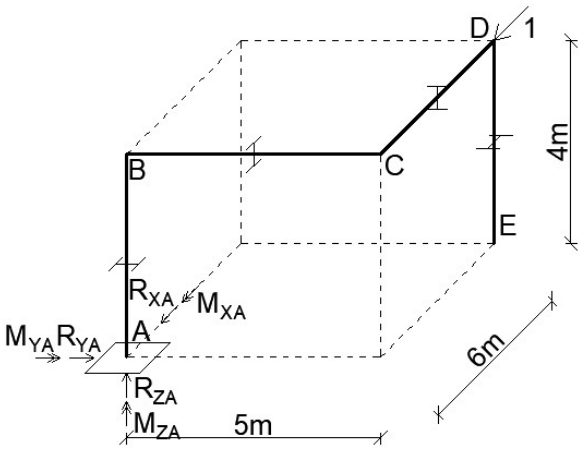
$M_2(z) := 1 \cdot (4\ \mathbf{m} - z)$

$M_3 := 0$

$M_2(0\ \mathbf{m}) = 4\ \mathbf{m}$

$M_2(4\ \mathbf{m}) = 0\ \mathbf{m}$

Stan x2=1



Reakcje:

$\Sigma X = R_{XA} + 1 = 0 \qquad R_{XA} := -1$

$\Sigma Y = R_{YA} = 0 \qquad R_{YA} := 0$

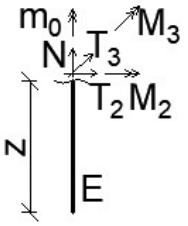
$\Sigma Z = R_{ZA} = 0 \qquad R_{ZA} := 0$

$\Sigma M_{AX} = M_{XA} = 0 \qquad M_{XA} := 0\ \mathbf{m}$

$\Sigma M_{AY} = M_{YA} + 1 \cdot 4 = 0 \qquad M_{YA} := -1 \cdot 4\ \mathbf{m} = -4\ \mathbf{m}$

$\Sigma M_{AZ} = M_{ZA} - 1 \cdot 5 = 0 \qquad M_{ZA} := 1 \cdot 5\ \mathbf{m} = 5\ \mathbf{m}$

Pręt ED z∈(0;4)m



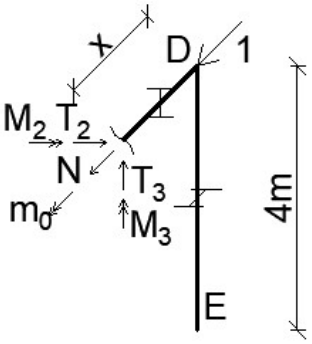
$N := 0$

$T_2 := 0 \qquad T_3 := 0$

$m_0 := 0\ \mathbf{m}$

$M_2 := 0\ \mathbf{m} \qquad M_3 := 0\ \mathbf{m}$

Pręt DC x∈(0;6)m



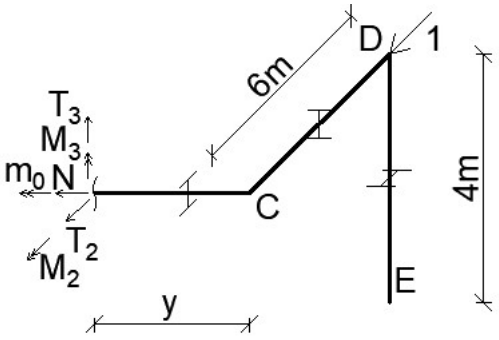
$N := -1$

$T_2 := 0 \qquad T_3 := 0$

$m_0 := 0\ \mathbf{m}$

$M_2 := 0\ \mathbf{m} \qquad M_3 := 0\ \mathbf{m}$

Pręt CB y∈(0;5)m



$N := 0$

$T_2 := -1 \qquad T_3 := 0$

$m_0 := 0\ \mathbf{m}$

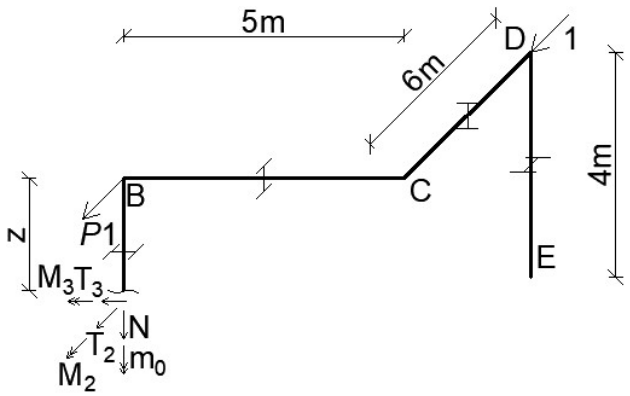
$M_2 := 0\ \mathbf{m}$

$M_3(y) := 1 \cdot y$

$M_3(0\ \mathbf{m}) = 0\ \mathbf{m}$

$M_3(5\ \mathbf{m}) = 5\ \mathbf{m}$

Pręt BA z∈(0;4)m



$N := 0$

$T_2 := -1 \qquad T_3 := 0$

$m_0 := -1 \cdot 5\ \mathbf{m} = -5\ \mathbf{m}$

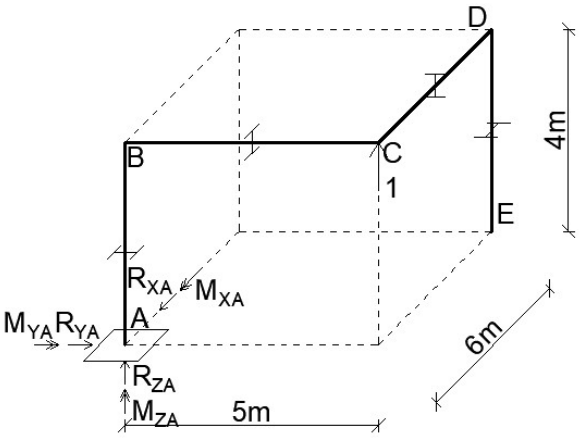
$M_2 := 0\ \mathbf{m}$

$M_3(z) := 1 \cdot z$

$M_3(0\ \mathbf{m}) = 0\ \mathbf{m}$

$M_3(4\ \mathbf{m}) = 4\ \mathbf{m}$

Stan x3=1



Reakcje:

$\Sigma X = R_{XA} = 0$ $\quad R_{XA} := 0$

$\Sigma Y = R_{YA} = 0$ $\quad R_{YA} := 0$

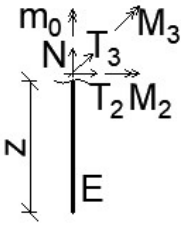
$\Sigma Z = R_{ZA} + 1 = 0$ $\quad R_{ZA} := -1$

$\Sigma M_{AX} = M_{XA} + 1 \cdot 5 = 0$ $\quad M_{XA} := -1 \cdot 5\ \mathbf{m}$

$\Sigma M_{AY} = M_{YA} = 0$ $\quad M_{YA} := 0\ \mathbf{m}$

$\Sigma M_{AZ} = M_{ZA} = 0$ $\quad M_{ZA} := 0\ \mathbf{m}$

Pręt ED z∈(0;4)m



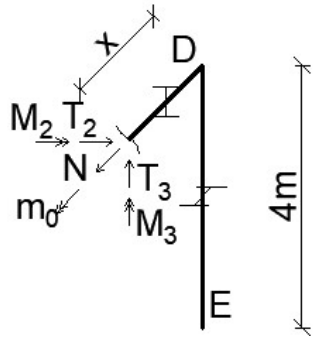
$N := 0$

$T_2 := 0$ $\quad T_3 := 0$

$m_0 := 0\ \mathbf{m}$

$M_2 := 0\ \mathbf{m}$ $\quad M_3 := 0\ \mathbf{m}$

Pręt DC x∈(0;6)m



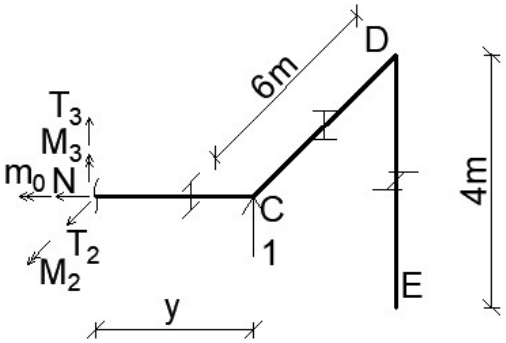
$N := 0$

$T_2 := 0$ $\quad T_3 := 0$

$m_0 := 0\ \mathbf{m}$

$M_2 := 0\ \mathbf{m}$ $\quad M_3 := 0\ \mathbf{m}$

Pręt CB y∈(0;5)m



$N := 0$

$T_2 := 0$ $\quad T_3 := -1$

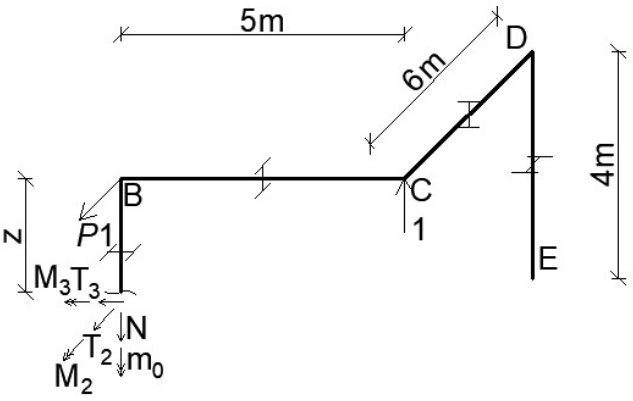
$m_0 := 0\ \mathbf{m}$

$M_2(y) := -1 \cdot y$ $\quad M_3 := 0$

$M_2(0\ \mathbf{m}) = 0\ \mathbf{m}$

$M_2(5\ \mathbf{m}) = -5\ \mathbf{m}$

Pręt BA z∈(0;4)m



$N := 1$

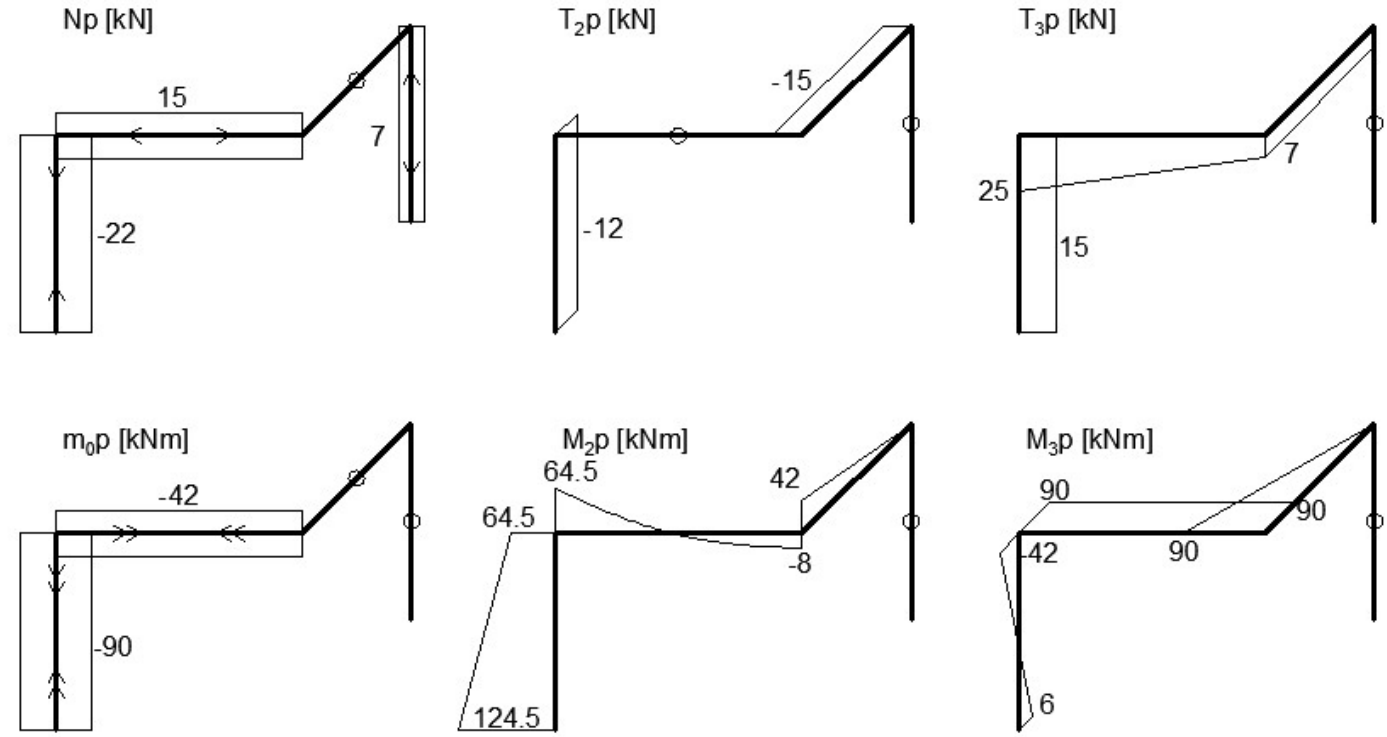
$T_2 := 0$ $\quad T_3 := 0$

$m_0 := 0\ \mathbf{m}$

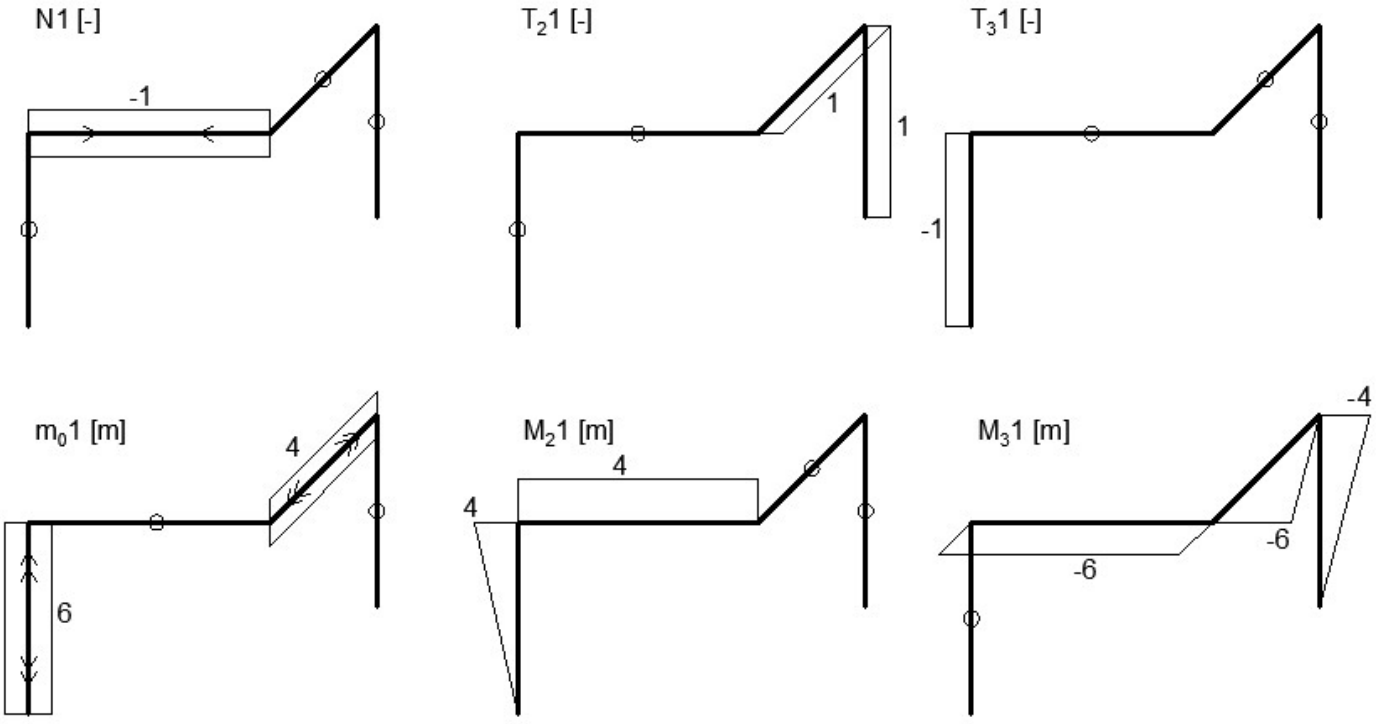
$M_2 := -1 \cdot 5\ \mathbf{m} = -5\ \mathbf{m}$ $\quad M_3 := 0\ \mathbf{m}$

Wykresy

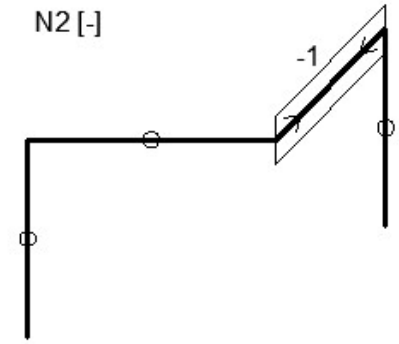
Stan p

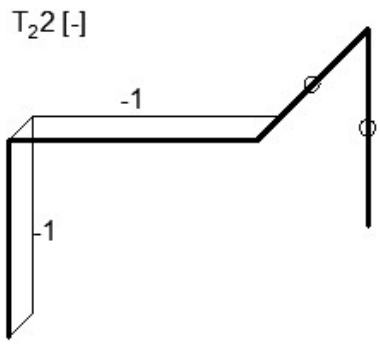


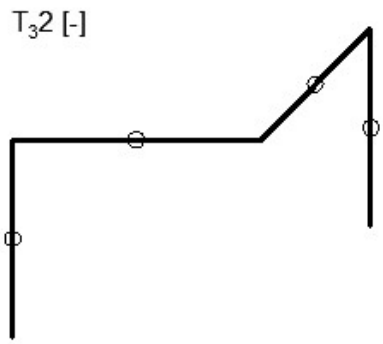
Stan 1

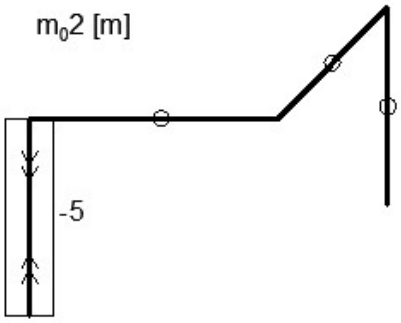


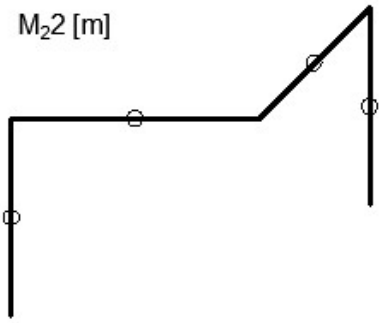
Stan 2

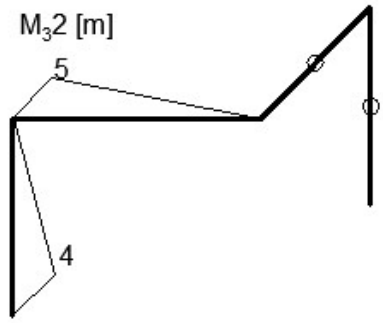




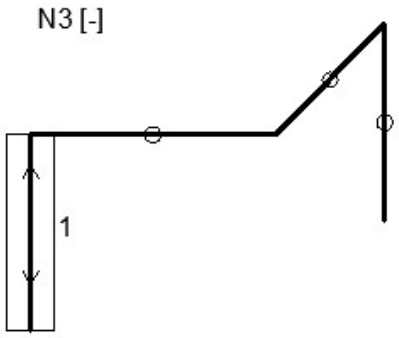


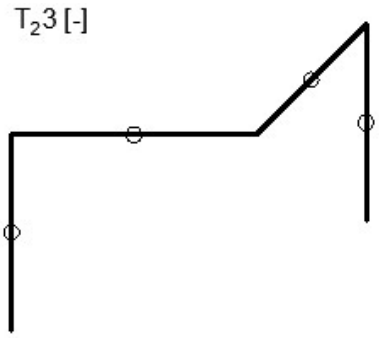


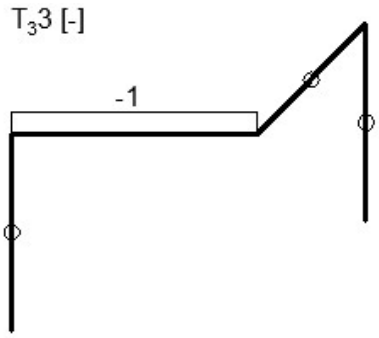


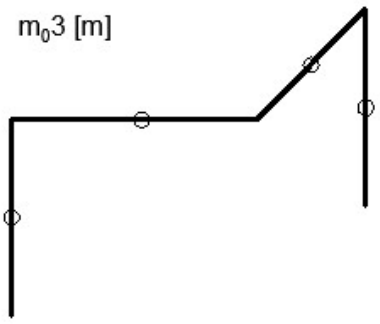


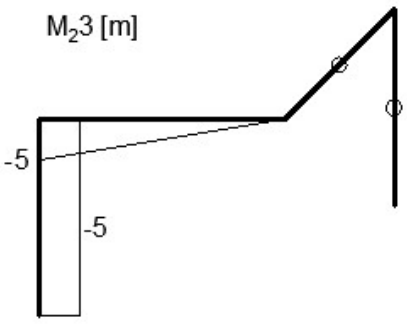
Stan 3

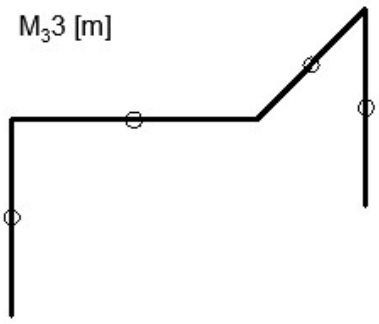




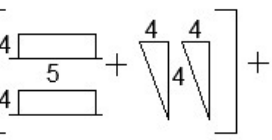
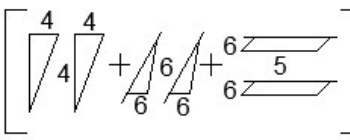
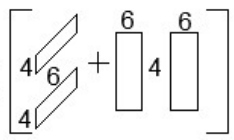






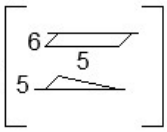
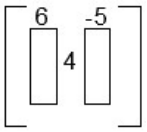


Przemieszczenia

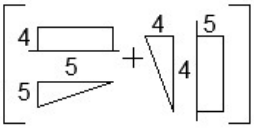
$\delta_{11} = \frac{1}{EJ_2}$  $+ \frac{1}{EJ_3}$  $+ \frac{1}{GJ_0}$ 

$$\delta_{11} := \frac{\mathbf{m}^3}{EJ_2}\left(4\cdot 5\cdot 4+\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 4\right)+\frac{\mathbf{m}^3}{EJ_3}\left(\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 4+\frac{1}{2}\cdot 6\cdot 6\cdot\frac{2}{3}\cdot 6+6\cdot 5\cdot 6\right)+\frac{\mathbf{m}^3}{GJ_0}\left(4\cdot 6\cdot 4+6\cdot 4\cdot 6\right)$$

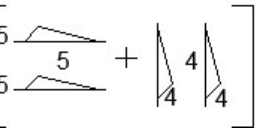
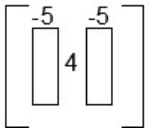
$$\delta_{11} = 63.359\ \frac{\mathbf{m}}{\mathbf{kN}}$$

$\delta_{12} = \frac{1}{EJ_3}$  $+ \frac{1}{GJ_0}$ 

$$\delta_{12} := \frac{\mathbf{m}^3}{EJ_2}\cdot 0+\frac{\mathbf{m}^3}{EJ_3}\left(-6\cdot 5\cdot\frac{1}{2}\cdot 5\right)+\frac{\mathbf{m}^3}{GJ_0}\left(6\cdot 4\cdot -5\right) = -31.371\ \frac{\mathbf{m}}{\mathbf{kN}} \qquad \delta_{21} := \delta_{12}$$

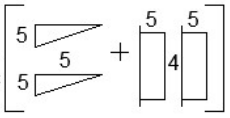
$\delta_{13} = \frac{1}{EJ_2}$ 

$$\delta_{13} := \frac{\mathbf{m}^3}{EJ_2}\left(-4\cdot 5\cdot\frac{1}{2}\cdot 5-\frac{1}{2}\cdot 4\cdot 4\cdot 5\right)+\frac{\mathbf{m}^3}{EJ_3}\cdot 0+\frac{\mathbf{m}^3}{GJ_0}\cdot 0 \qquad \delta_{31} := \delta_{13}$$

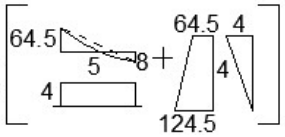
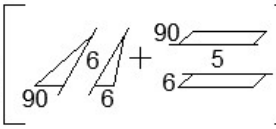
$\delta_{22} = \frac{1}{EJ_3}$  $+ \frac{1}{GJ_0}$ 

$$\delta_{22} := \frac{\mathbf{m}^3}{EJ_2}\cdot 0+\frac{\mathbf{m}^3}{EJ_3}\left(\frac{1}{2}\cdot 5\cdot 5\cdot\frac{2}{3}\cdot 5+\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 4\right)+\frac{\mathbf{m}^3}{GJ_0}\left(-5\cdot 4\cdot -5\right) = 26.145\ \frac{\mathbf{m}}{\mathbf{kN}}$$

$$\delta_{23} := \frac{\mathbf{m}^3}{EJ_2}\cdot 0+\frac{\mathbf{m}^3}{EJ_3}\cdot 0+\frac{\mathbf{m}^3}{GJ_0}\cdot 0 \qquad \delta_{32} := \delta_{23}$$

$\delta_{33} = \frac{1}{EJ_2}$ 

$$\delta_{33} := \frac{\mathbf{m}^3}{EJ_2}\left(\frac{1}{2}\cdot 5\cdot 5\cdot\frac{2}{3}\cdot 5+5\cdot 4\cdot 5\right) = 0.051\ \frac{\mathbf{m}}{\mathbf{kN}}$$

$\delta_{1p} = \frac{1}{EJ_2}$  $+ \frac{1}{EJ_3}$  $+ \frac{1}{GJ_0}$

Pole paraboli: $A_p := \frac{2}{3}\cdot\frac{q\cdot(5\ \mathbf{m})^2}{8}\cdot 5\ \mathbf{m}\cdot\frac{1}{\mathbf{kN}\cdot\mathbf{m}^2} = 31.25$

$$\delta_{1p2} := 4\cdot 5\cdot\frac{64.5-8}{2}-A_p\cdot 4+\frac{1}{2}\cdot 4\cdot 4\cdot\left(\frac{2}{3}\cdot 64.5+\frac{1}{3}\cdot 124.5\right) = 1116$$

$$\delta_{1p3} := -\frac{1}{2}\cdot 90\cdot 6\cdot\frac{2}{3}\cdot 6-90\cdot 5\cdot 6 = -3780$$

$$\delta_{1p0} := -90\cdot 4\cdot 6 = -2160$$

$$\delta_{1p} := \frac{\mathbf{kN \cdot m^3}}{EJ_2} \cdot \delta_{1p2} + \frac{\mathbf{kN \cdot m^3}}{EJ_3} \cdot \delta_{1p3} + \frac{\mathbf{kN \cdot m^3}}{GJ_0} \cdot \delta_{1p0} = -575.725\ \mathbf{m}$$

$$\delta_{2p} = \frac{1}{EJ_3}\left[\begin{array}{c} 90 \\ 5 \quad 5 \end{array} + \begin{array}{c} 42 \\ 4 \\ 6 \quad 4 \end{array}\right] + \frac{1}{GJ_0}\left[\begin{array}{c} -90 \quad -5 \\ \square \quad 4 \quad \square \end{array}\right]$$

$$\delta_{2p2} := 0$$

$$\delta_{2p3} := 90 \cdot 5 \cdot \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \left(\frac{2}{3} \cdot 6 - \frac{1}{3} \cdot 42\right) = 1045$$

$$\delta_{2p0} := -90 \cdot 4 \cdot -5 = 1800$$

$$\delta_{2p} := \frac{\mathbf{kN \cdot m^3}}{EJ_2} \cdot \delta_{2p2} + \frac{\mathbf{kN \cdot m^3}}{EJ_3} \cdot \delta_{2p3} + \frac{\mathbf{kN \cdot m^3}}{GJ_0} \cdot \delta_{2p0} = 470.182\ \mathbf{m}$$

$$\delta_{3p} = \frac{1}{EJ_2}\left[\begin{array}{c} 64.5 \\ 5 \quad 8 \\ 5 \end{array} + \begin{array}{c} 64.5 \quad 5 \\ 4 \quad \square \\ 124.5 \end{array}\right]$$

$$\delta_{3p2} := \frac{1}{2} \cdot 5 \cdot 5 \cdot \left(\frac{1}{3} \cdot 8 - \frac{2}{3} \cdot 64.5\right) + A_p \cdot \frac{1}{2} \cdot 5 - 5 \cdot 4 \cdot \frac{64.5 + 124.5}{2} = -2316.042$$

$$\delta_{3p3} := 0$$

$$\delta_{3p0} := 0$$

$$\delta_{3p} := \frac{\mathbf{kN \cdot m^3}}{EJ_2} \cdot \delta_{3p2} + \frac{\mathbf{kN \cdot m^3}}{EJ_3} \cdot \delta_{3p3} + \frac{\mathbf{kN \cdot m^3}}{GJ_0} \cdot \delta_{3p0} = -0.836\ \mathbf{m}$$

Rozwiązanie układu równań

$$\delta_{11}\ x_1 + \delta_{12}\ x_2 + \delta_{13}\ x_3 + \delta_{1p} = 0$$

$$\delta_{21}\ x_1 + \delta_{22}\ x_2 + \delta_{23}\ x_3 + \delta_{2p} = 0$$

$$\delta_{31}\ x_1 + \delta_{32}\ x_2 + \delta_{33}\ x_3 + \delta_{3p} = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix}^{-1} \cdot - \begin{bmatrix} \delta_{1p} \\ \delta_{2p} \\ \delta_{3p} \end{bmatrix}$$

$x_1 = 0.47\ \mathbf{kN}$ $x_2 = -17.419\ \mathbf{kN}$ $x_3 = 16.647\ \mathbf{kN}$

Reakcje ostateczne

$$R_{ost} = R_1 \cdot x_1 + R_2 \cdot x_2 + R_3 \cdot x_3 + R_p$$

$$R_{XA} := 0\ x_1 + -1\ x_2 + 0\ x_3 + -12\ \mathbf{kN} = 5.419\ \mathbf{kN}$$

$$R_{YA} := 1\ x_1 + 0\ x_2 + 0\ x_3 + -15\ \mathbf{kN} = -14.53\ \mathbf{kN}$$

$$R_{ZA} := 0\ x_1 + 0\ x_2 + -1\ x_3 + 22\ \mathbf{kN} = 5.353\ \mathbf{kN}$$

$$M_{XA} := 0\ \mathbf{m}\ x_1 + 0\ \mathbf{m}\ x_2 + -5\ \mathbf{m}\ x_3 + 124.5\ \mathbf{kNm} = 41.263\ \mathbf{kNm}$$

$$M_{YA} := 0\ \mathbf{m}\ x_1 + -4\ \mathbf{m}\ x_2 + 0\ \mathbf{m}\ x_3 + -6\ \mathbf{kNm} = 63.678\ \mathbf{kNm}$$

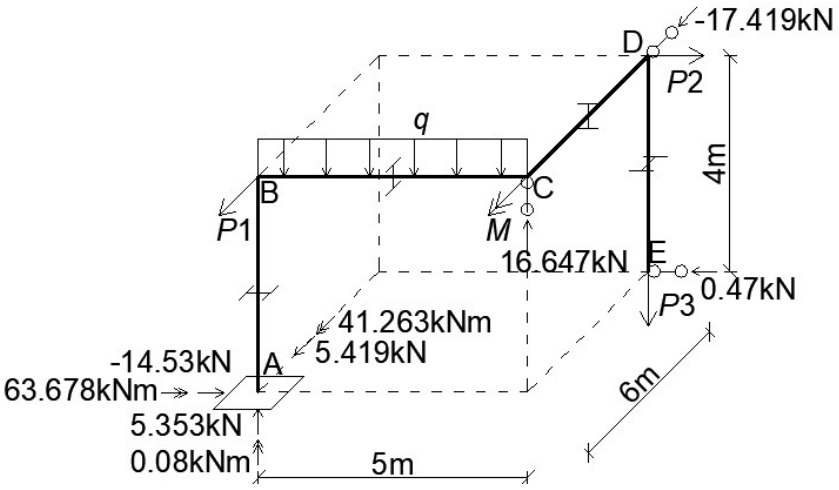
$$M_{ZA} := -6\ \mathbf{m}\ x_1 + 5\ \mathbf{m}\ x_2 + 0\ \mathbf{m}\ x_3 + 90\ \mathbf{kNm} = 0.08\ \mathbf{kNm}$$

$R_{YE} := x_1 = 0.47\ \mathbf{kN}$

$R_{XD} := x_2 = -17.419\ \mathbf{kN}$

$R_{ZC} := x_3 = 16.647\ \mathbf{kN}$

Reakcje ostateczne



Sprawdzenie statyczne

$\Sigma X := R_{XA} + R_{XD} + P1 = 0\ \mathbf{kN}$

$\Sigma Y := R_{YA} - R_{YE} + P2 = 0\ \mathbf{kN}$

$\Sigma Z := R_{ZA} + R_{ZC} - P3 - q \cdot 5\ \mathbf{m} = 0\ \mathbf{kN}$

$\Sigma M_{AX} := M_{XA} + R_{ZC} \cdot 5\ \mathbf{m} + M - q \cdot 5\ \mathbf{m} \cdot 2.5\ \mathbf{m} - P2 \cdot 4\ \mathbf{m} - P3 \cdot 5\ \mathbf{m} = 0\ \mathbf{kNm}$

$\Sigma M_{AY} := M_{YA} + R_{XD} \cdot 4\ \mathbf{m} + P1 \cdot 4\ \mathbf{m} - P3 \cdot 6\ \mathbf{m} = 0\ \mathbf{kNm}$

$\Sigma M_{AZ} := M_{ZA} - R_{XD} \cdot 5\ \mathbf{m} + R_{YE} \cdot 6\ \mathbf{m} - P2 \cdot 6\ \mathbf{m} = 0\ \mathbf{kNm}$

Wykresy ostateczne

$N_{ost} = N_1 \cdot x_1 + N_2 \cdot x_2 + N_3 \cdot x_3 + N_p$

$$N_{ost} := \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_3 + \begin{bmatrix} 7 \\ 0 \\ 15 \\ -22 \end{bmatrix} \mathbf{kN} = \begin{bmatrix} 7 \\ 17.419 \\ 14.53 \\ -5.353 \end{bmatrix} \mathbf{kN}$$

$T_{ost} = T_1 \cdot x_1 + T_2 \cdot x_2 + T_3 \cdot x_3 + T_p$

$$T_{2ost} := \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot x_3 + \begin{bmatrix} 0 \\ -15 \\ 0 \\ -12 \end{bmatrix} \mathbf{kN} = \begin{bmatrix} 0.47 \\ -14.53 \\ 17.419 \\ 5.419 \end{bmatrix} \mathbf{kN}$$

$$T_{3ost} := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 7 \\ 7 \\ 25 \\ 15 \end{bmatrix} \mathbf{kN} = \begin{bmatrix} 0 \\ 7 \\ -9.647 \\ 8.353 \\ 14.53 \end{bmatrix} \mathbf{kN}$$

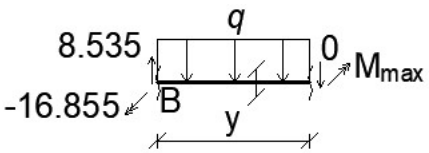
$M_{ost} = M_1 \cdot x_1 + M_2 \cdot x_2 + M_3 \cdot x_3 + M_p$

$$m_{ost} := \begin{bmatrix} 0 \\ 4 \\ 0 \\ 6 \end{bmatrix} \mathbf{m} \cdot x_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -5 \end{bmatrix} \mathbf{m} \cdot x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mathbf{m} \cdot x_3 + \begin{bmatrix} 0 \\ 0 \\ -42 \\ -90 \end{bmatrix} \mathbf{kNm} = \begin{bmatrix} 0 \\ 1.882 \\ -42 \\ -0.08 \end{bmatrix} \mathbf{kNm}$$

$$M_{2ost} := \begin{bmatrix}0\\0\\0\\0\\4\\4\\4\\0\end{bmatrix} \mathbf{m}\cdot x_1 + \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix} \mathbf{m}\cdot x_2 + \begin{bmatrix}0\\0\\0\\0\\0\\-5\\-5\\-5\end{bmatrix} \mathbf{m}\cdot x_3 + \begin{bmatrix}0\\0\\0\\42\\-8\\64.5\\64.5\\124.5\end{bmatrix} \mathbf{kNm} = \begin{bmatrix}0\\0\\0\\42\\-6.118\\-16.855\\-16.855\\41.263\end{bmatrix} \mathbf{kNm}$$

$$M_{3ost} := \begin{bmatrix}0\\-4\\0\\-6\\-6\\-6\\0\\0\end{bmatrix} \mathbf{m}\cdot x_1 + \begin{bmatrix}0\\0\\0\\0\\0\\5\\0\\4\end{bmatrix} \mathbf{m}\cdot x_2 + \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix} \mathbf{m}\cdot x_3 + \begin{bmatrix}0\\0\\0\\90\\90\\90\\-42\\6\end{bmatrix} \mathbf{kNm} = \begin{bmatrix}0\\-1.882\\0\\87.177\\87.177\\0.08\\-42\\-63.678\end{bmatrix} \mathbf{kNm}$$

Obliczenie ekstremum paraboli



$$\Sigma Z = 8.535 - q\cdot y = 0 \qquad y := \frac{8.535\ \mathbf{kN}}{q} = 2.845\ \mathbf{m}$$

$$\Sigma M_{XB} = -16.855 - q\cdot y\cdot\frac{y}{2} - M_{max} = 0$$

$$M_{max} := -16.855\ \mathbf{kNm} - q\cdot y\cdot\frac{y}{2} = -28.996\ \mathbf{kNm}$$

Wykresy ostateczne