# Politechnika Lubelska

## Wydział Budownictwa i Architektury

Mechanika Budowli I

Projekt 1: Przemieszczenia


Wykonał:

……..

Sprawdził:

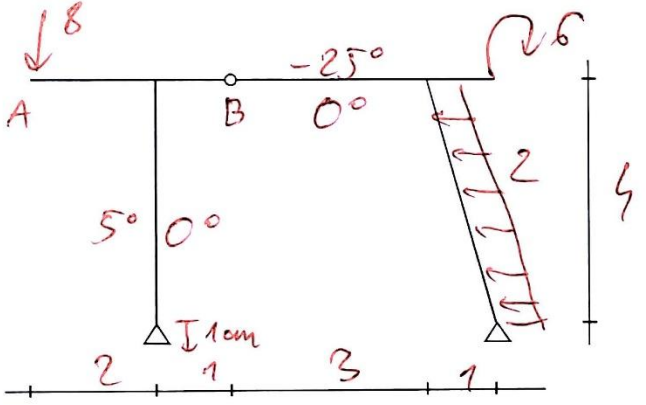
dr inż. Jakub Gontarz


2025

Student.....Grupa .....

# ĆWICZENIE PROJEKTOWE Z MECHANIKI BUDOWLI nr 2

Zadanie: Obliczyć wielkości zaznaczonych przemieszczeń.

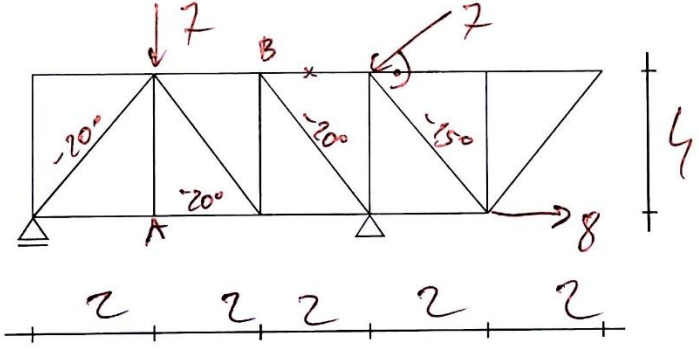



$X_A$, $\Delta\varphi_B$

Przekrój: IPN 220

$E = 2.1 * 10^5$ MPa

$\alpha_t = 1.2 * 10^{-5}$ 1/K




$\varphi_x$, $\Delta L_{A-B}$

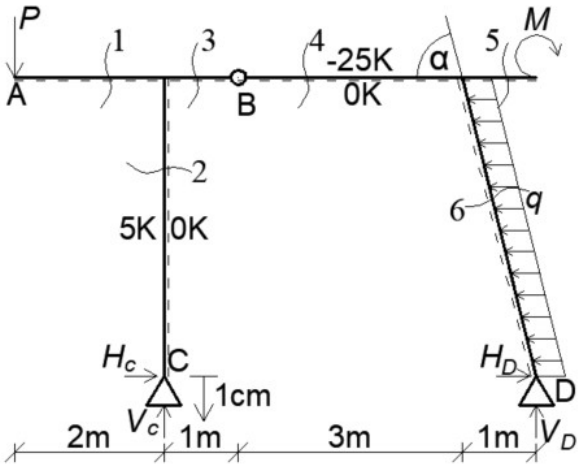
Przekrój: C200

$E = 2.1 * 10^5$ MPa

$\alpha_t = 1.2 * 10^{-5}$ 1/K

## Zadanie 1: Rama

### Stan p




$E := 210\ \mathbf{GPa}$

$\alpha_t := 1.2 \cdot 10^{-5}\ \frac{1}{\mathbf{K}}$

Przekrój: IPN 220

$J := 3060\ \mathbf{cm}^4$

$EJ := E \cdot J = 6426\ \mathbf{kN \cdot m^2}$

$h := 220\ \mathbf{mm}$

$\mathbf{kNm} := \mathbf{kN \cdot m}$

$P := 8\ \mathbf{kN}$

$M := 6\ \mathbf{kNm}$

$q := 2\ \frac{\mathbf{kN}}{\mathbf{m}}$

Długość pręta ukośnego:

$L := \sqrt{(1\ \mathbf{m})^2 + (4\ \mathbf{m})^2} = 4.123\ \mathbf{m}$

$sin\alpha := \frac{4\ \mathbf{m}}{L} = 0.97 \qquad cos\alpha := \frac{1\ \mathbf{m}}{L} = 0.243$

Reakcje:

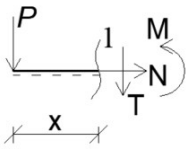
$\Sigma M_c = -P \cdot 2 + M - q \cdot L \cdot 2 - V_D \cdot 5 = 0$ $\qquad V_D := \frac{-P \cdot 2\ \mathbf{m} + M - q \cdot L \cdot 2\ \mathbf{m}}{5\ \mathbf{m}} = -5.298\ \mathbf{kN}$

$\Sigma M_B{}^P = M + q \cdot L \cdot 2 - H_D \cdot 4 - V_D \cdot 4 = 0$ $\qquad H_D := \frac{M + q \cdot L \cdot 2\ \mathbf{m} - V_D \cdot 4\ \mathbf{m}}{4\ \mathbf{m}} = 10.922\ \mathbf{kN}$

$\Sigma X = H_c + H_D - q \cdot L = 0$ $\qquad H_C := -H_D + q \cdot L = -2.675\ \mathbf{kN}$

$\Sigma Y = V_C + V_D - P = 0$ $\qquad V_C := -V_D + P = 13.298\ \mathbf{kN}$

Pręt 1 $\qquad x \in (0;2)\mathbf{m}$



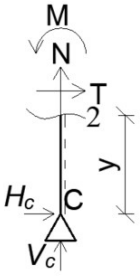

$N_1 := 0$

$T_1 := -8\ \mathbf{kN}$

$M_1(x) := -8\ \mathbf{kN} \cdot x$

$M_1(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_1(2\ \mathbf{m}) = -16\ \mathbf{kNm}$

Pręt 2 $\qquad y \in (0;4)\mathbf{m}$



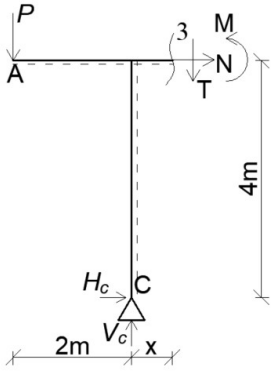

$N_2 := -V_C = -13.298\ \mathbf{kN}$

$T_2 := -H_C = 2.675\ \mathbf{kN}$

$M_2(y) := -H_C \cdot y$

$M_2(0\ \mathbf{m}) = 0\ \mathbf{kNm} \qquad M_2(4\ \mathbf{m}) = 10.702\ \mathbf{kNm}$

Pręt 3 $\qquad x \in (0;1)\mathbf{m}$




$N_3 := -H_C = 2.675\ \mathbf{kN}$

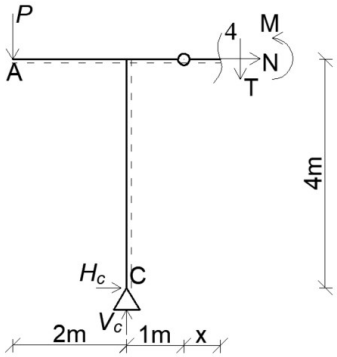
$T_3 := V_C - P = 5.298\ \mathbf{kN}$

$M_3(x) := -P \cdot (x + 2\ \mathbf{m}) - H_C \cdot 4\ \mathbf{m} + V_C \cdot x$

$M_3(0\ \mathbf{m}) = -5.298\ \mathbf{kNm} \qquad M_3(1\ \mathbf{m}) = 0\ \mathbf{kNm}$

Pręt 4

x∈(0;3)m




$N_4 := -H_C = 2.675\ \mathbf{kN}$

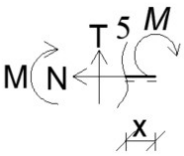
$T_4 := V_C - P = 5.298\ \mathbf{kN}$

$M_4(x) := -P \cdot (x + 3\ \mathbf{m}) - H_C \cdot 4\ \mathbf{m} + V_C \cdot (x + 1\ \mathbf{m})$

$M_4(0\ \mathbf{m}) = 0\ \mathbf{kNm}$ $\quad M_4(3\ \mathbf{m}) = 15.895\ \mathbf{kNm}$

Pręt 5

x∈(0;1)m



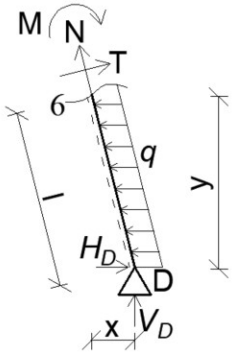

$N_5 := 0\ \mathbf{kN}$

$T_5 := 0\ \mathbf{kN}$

$M_5 := -M = -6\ \mathbf{kNm}$

Pręt 6

x∈(0;1)m $\quad y(x) := \frac{4\ \mathbf{m}}{1\ \mathbf{m}}\, x$ $\quad l(x) := \frac{L}{1\ \mathbf{m}} \cdot x$




$N_6(x) := -V_D \cdot sin\alpha + H_D \cdot cos\alpha - q \cdot l(x) \cdot cos\alpha$

$T_6(x) := -V_D \cdot cos\alpha - H_D \cdot sin\alpha + q \cdot l(x) \cdot sin\alpha$

$M_6(x) := V_D \cdot x + H_D \cdot y(x) - q \cdot l(x) \cdot \frac{y(x)}{2}$

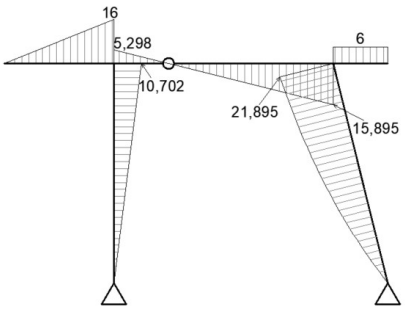
$N_6(0\ \mathbf{m}) = 7.789\ \mathbf{kN}$ $\quad N_6(1\ \mathbf{m}) = 5.789\ \mathbf{kN}$

$T_6(0\ \mathbf{m}) = -9.31\ \mathbf{kN}$ $\quad T_6(1\ \mathbf{m}) = -1.31\ \mathbf{kN}$

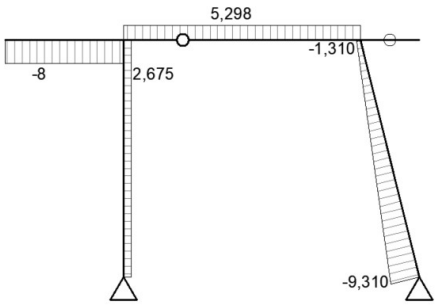
$M_6(0\ \mathbf{m}) = 0\ \mathbf{kNm}$ $\quad M_6(1\ \mathbf{m}) = 21.895\ \mathbf{kNm}$

<u>Wykresy:</u>

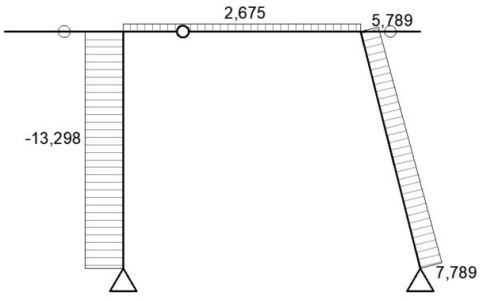
MP [kNm]



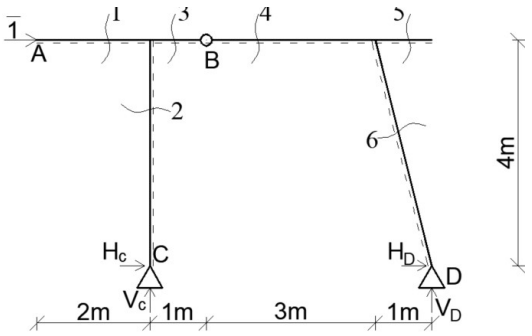

TP [kN]




NP [kN]



**Stan 1** Przemieszczenie poziome w punkcie A




Reakcje:

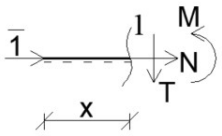
$\Sigma M_C = 1 \cdot 4 - V_D \cdot 5 = 0$ $\quad V_D := \frac{1 \cdot 4\ \mathbf{m}}{5\ \mathbf{m}} = 0.8$

$\Sigma M_B{}^P = -V_D \cdot 4 - H_D \cdot 4 = 0$ $\quad H_D := \frac{-V_D \cdot 4\ \mathbf{m}}{4\ \mathbf{m}} = -0.8$

$\Sigma X = H_C + H_D + 1 = 0$ $\quad H_C := -H_D - 1 = -0.2$

$\Sigma Y = V_C + V_D = 0$ $\quad V_C := -V_D = -0.8$

Pręt 1 x∈(0;2)m




$N_1 := -1$

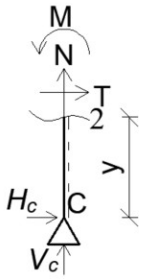
$T_1 := 0$

$M_1 := 0$

Reakcja na wymuszeniu:

$R_1 := V_C$

Pręt 2 y∈(0;4)m



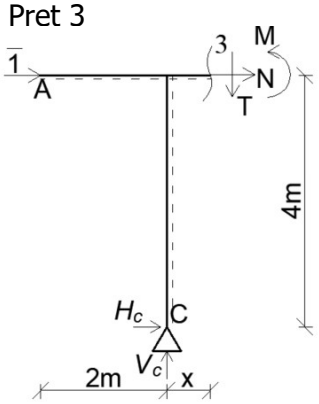

$N_2 := -V_C = 0.8$

$T_2 := -H_C = 0.2$

$M_2(y) := -H_C \cdot y$

$M_2(0\ \mathbf{m}) = 0\ \mathbf{m}$ $\quad M_2(4\ \mathbf{m}) = 0.8\ \mathbf{m}$

Pret 3 x∈(0;1)m



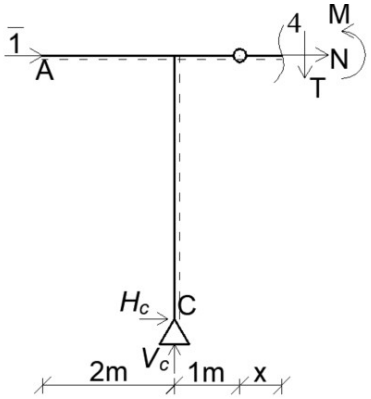

$N_3 := -1 - H_C = -0.8$

$T_3 := V_C = -0.8$

$M_3(x) := -H_C \cdot 4\ \mathbf{m} + V_C \cdot x$

$M_3(0\ \mathbf{m}) = 0.8\ \mathbf{m}$ $\quad M_3(1\ \mathbf{m}) = 0\ \mathbf{m}$

Pręt 4 x∈(0;3)m



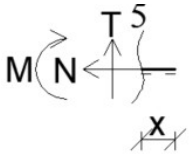

$N_4 := -1 - H_C = -0.8$

$T_4 := V_C = -0.8$

$M_4(x) := -H_C \cdot 4\ \mathbf{m} + V_C \cdot (x + 1\ \mathbf{m})$

$M_4(0\ \mathbf{m}) = 0\ \mathbf{m}$ $\quad M_4(3\ \mathbf{m}) = -2.4\ \mathbf{m}$

Pręt 5 x∈(0;1)m

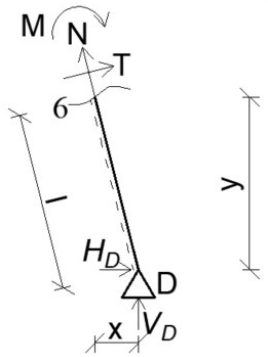



$N_5 := 0$

$T_5 := 0$

$M_5 := 0\ \mathbf{m}$

Pręt 6 x∈(0;1)m $y(x) := \frac{4\ \mathbf{m}}{1\ \mathbf{m}}\, x$ $l(x) := \frac{L}{1\ \mathbf{m}} \cdot x$




$N_6(x) := -V_D \cdot sin\alpha + H_D \cdot cos\alpha$

$T_6(x) := -V_D \cdot cos\alpha - H_D \cdot sin\alpha$

$M_6(x) := V_D \cdot x + H_D \cdot y(x)$

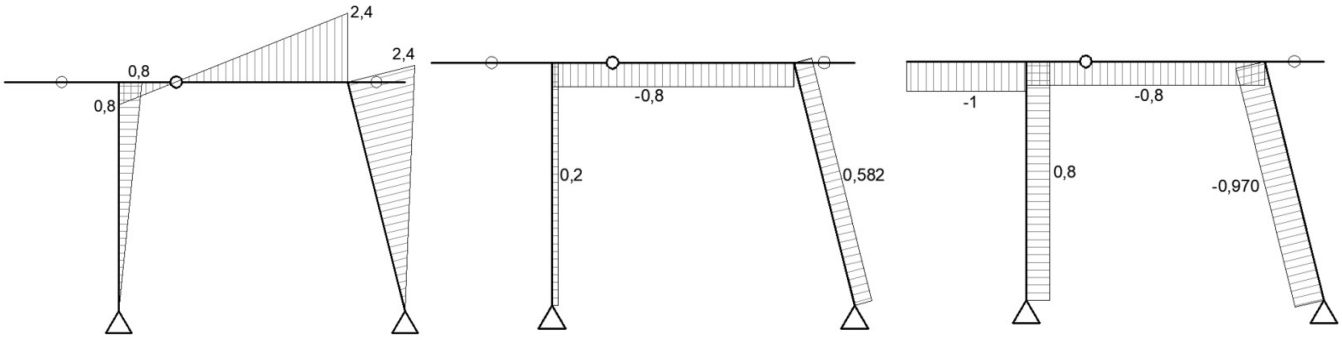
$N_6(0\ \mathbf{m}) = -0.97$ $N_6(1\ \mathbf{m}) = -0.97$

$T_6(0\ \mathbf{m}) = 0.582$ $T_6(1\ \mathbf{m}) = 0.582$

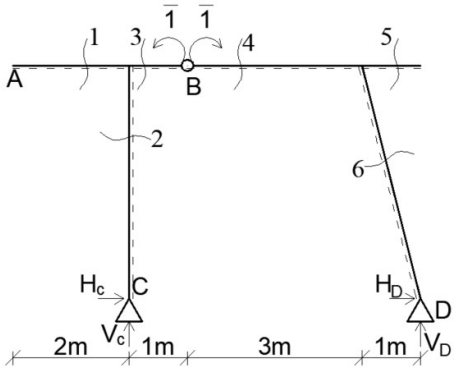
$M_6(0\ \mathbf{m}) = 0\ \mathbf{m}$ $M_6(1\ \mathbf{m}) = -2.4\ \mathbf{m}$

Wykresy:

M1 [m] T1 [-] N1 [-]




**Stan 2** Kąt załamania przegubu w punkcie B




4m

Reakcje:

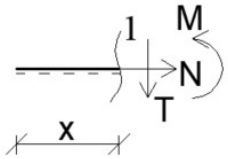
$\Sigma M_C = -V_D \cdot 5 - 1 + 1 = 0$ $V_D := 0 \cdot \frac{1}{\mathbf{m}}$

$\Sigma M_B{}^P = -V_D \cdot 4 - H_D \cdot 4 + 1 = 0$ $H_D := \frac{-V_D \cdot 4\ \mathbf{m} + 1}{4\ \mathbf{m}} = 0.25\ \frac{1}{\mathbf{m}}$

$\Sigma X = H_C + H_D = 0$ $H_C := -H_D = -0.25\ \frac{1}{\mathbf{m}}$

$\Sigma Y = V_C + V_D = 0$ $V_C := -V_D = 0\ \frac{1}{\mathbf{m}}$

Pręt 1 x∈(0;2)m




$N_1 := 0 \cdot \frac{1}{\mathbf{m}}$

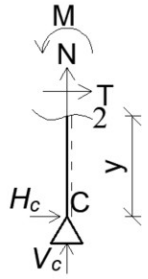
$T_1 := 0 \cdot \frac{1}{\mathbf{m}}$

$M_1 := 0$

Reakcja na wymuszeniu:

$R_2 := V_C$

Pręt 2 y∈(0;4)m



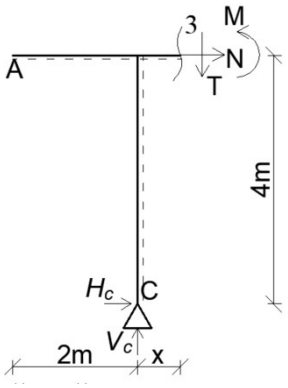

$N_2 := -V_C = 0\ \frac{1}{\mathbf{m}}$

$T_2 := -H_C = 0.25\ \frac{1}{\mathbf{m}}$

$M_2(y) := -H_C \cdot y$

$M_2(0\ \mathbf{m}) = 0 \qquad M_2(4\ \mathbf{m}) = 1$

Pręt 3 x∈(0;1)m



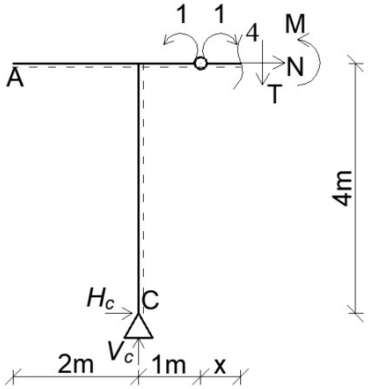

$N_3 := -H_C = 0.25\ \frac{1}{\mathbf{m}}$

$T_3 := V_C = 0\ \frac{1}{\mathbf{m}}$

$M_3(x) := -H_C \cdot 4\ \mathbf{m} + V_C \cdot x$

$M_3(0\ \mathbf{m}) = 1 \qquad M_3(1\ \mathbf{m}) = 1$

Pręt 4 x∈(0;3)m



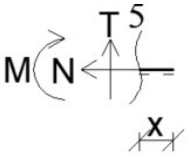

$N_4 := -H_C = 0.25\ \frac{1}{\mathbf{m}}$

$T_4 := V_C = 0\ \frac{1}{\mathbf{m}}$

$M_4(x) := -H_C \cdot 4\ \mathbf{m} + V_C \cdot (x + 1\ \mathbf{m}) - 1 + 1$

$M_4(0\ \mathbf{m}) = 1 \qquad M_4(3\ \mathbf{m}) = 1$

Pręt 5 x∈(0;1)m

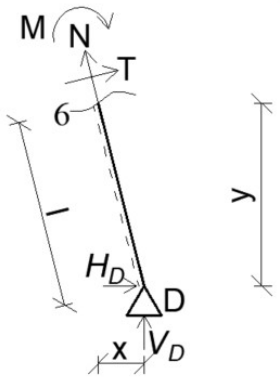



$N_5 := 0 \cdot \frac{1}{\mathbf{m}}$

$T_5 := 0 \cdot \frac{1}{\mathbf{m}}$

$M_5 := 0\ \mathbf{m}$

Pręt 6 x∈(0;1)m $y(x) := \frac{4\ \mathbf{m}}{1\ \mathbf{m}}\ x$ $l(x) := \frac{L}{1\ \mathbf{m}} \cdot x$




$N_6(x) := -V_D \cdot \sin\alpha + H_D \cdot \cos\alpha$

$T_6(x) := -V_D \cdot \cos\alpha - H_D \cdot \sin\alpha$

$M_6(x) := V_D \cdot x + H_D \cdot y(x)$

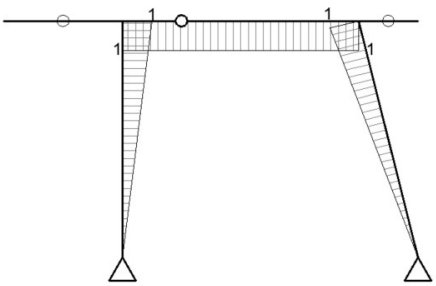
$N_6(0\ \mathbf{m}) = 0.061\ \frac{1}{\mathbf{m}} \qquad N_6(1\ \mathbf{m}) = 0.061\ \frac{1}{\mathbf{m}}$

$T_6(0\ \mathbf{m}) = -0.243\ \frac{1}{\mathbf{m}} \qquad T_6(1\ \mathbf{m}) = -0.243\ \frac{1}{\mathbf{m}}$

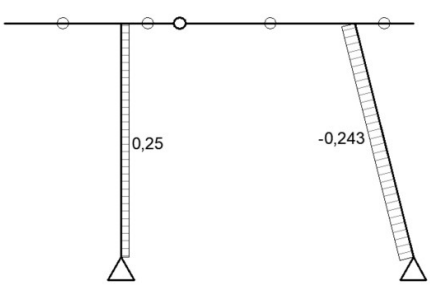
$M_6(0\ \mathbf{m}) = 0 \qquad M_6(1\ \mathbf{m}) = 1$

Wykresy:

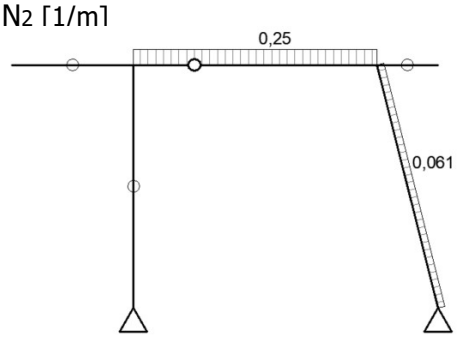
M2 [-]




T2 [1/m]




N2 [1/m]




**Obliczenia przemieszczeń**

Pole paraboli:

$$A_p := \frac{2}{3}\cdot\frac{q\cdot sin\alpha\cdot L^2}{8}\cdot L\cdot\frac{1}{\mathbf{kN}\cdot\mathbf{m}^2} = 11.333$$

Temperatury (podano w kelwinach):

$$\Delta t_1 := |5\ \mathbf{K} - 0\ \mathbf{K}| = 5\ \mathbf{K} \qquad \Delta t_2 := |-25\ \mathbf{K} - 0\ \mathbf{K}| = 25\ \mathbf{K}$$

$$t_{01} := \frac{5\ \mathbf{K} + 0\ \mathbf{K}}{2} = 2.5\ \mathbf{K} \qquad t_{02} := \frac{-25\ \mathbf{K} + 0\ \mathbf{K}}{2} = -12.5\ \mathbf{K}$$

Wymuszenie: $\Delta := -1\ \mathbf{cm}$

Przemieszczenie poziome w punkcie A:

$$\delta_{1p} = \frac{1}{EJ}\left[\begin{array}{c}10{,}702\ \ 0{,}8\\ 4\end{array} + \begin{array}{c}5{,}298\\ 1\\ 0{,}8\end{array} + \begin{array}{c}15{,}895\\ 3\\ 2{,}4\end{array} + \begin{array}{c}21{,}895\ \ 2{,}4\\ 4{,}123\end{array}\right]$$

$$\delta_{1p} := \frac{\mathbf{kN}\cdot\mathbf{m}^3}{EJ}\cdot\left(\frac{1}{2}\cdot 10.702\cdot 4\cdot\frac{2}{3}\cdot 0.8 - \frac{1}{2}\cdot 5.298\cdot 1\cdot\frac{2}{3}\cdot 0.8 - \frac{1}{2}\cdot 15.895\cdot 3\cdot\frac{2}{3}\cdot 2.4 - \frac{1}{2}\cdot 21.895\cdot 4.123\cdot\frac{2}{3}\cdot 2.4 - A_p\cdot\frac{1}{2}\cdot 2.4\right)$$

$$\delta_{1p} = -0.017735\ \mathbf{m}$$

$$\delta_{1\Delta t} = \left[\begin{array}{c}\frac{\alpha_t\Delta t_1}{h}\ \ 0{,}8\\ 4\end{array} + \begin{array}{c}\frac{\alpha_t\Delta t_2}{h}\\ 3\\ 2{,}4\end{array}\right] \qquad \delta_{1\Delta t} := -\frac{1}{2}\cdot 0.8\ \mathbf{m}\cdot 4\ \mathbf{m}\cdot\frac{\alpha_t\cdot\Delta t_1}{h} - \frac{1}{2}\cdot 2.4\ \mathbf{m}\cdot 3\ \mathbf{m}\cdot\frac{\alpha_t\cdot\Delta t_2}{h} = -0.005345\ \mathbf{m}$$

$$\delta_{1t0} = \left[\begin{array}{c}\alpha_t t_{01}\ \ 0{,}8\\ 4\end{array} + \begin{array}{c}\alpha_t t_{02}\\ 3\\ -0{,}8\end{array}\right] \qquad \delta_{1t0} := 0.8\cdot 4\ \mathbf{m}\cdot\alpha_t\cdot t_{01} + (-0.8)\cdot 3\ \mathbf{m}\cdot\alpha_t\cdot t_{02} = 0.000456\ \mathbf{m}$$

$$\delta_{1\Delta} := -R_1\cdot\Delta = -0.008\ \mathbf{m} \qquad \delta_1 := \delta_{1p} + \delta_{1\Delta t} + \delta_{1t0} + \delta_{1\Delta} = -30.624\ \mathbf{mm}$$

Odp.: Punkt A przesunął się o 30.624mm w lewo.

Kąt załamania przegubu B:

$$\delta_{2p} = \frac{1}{EJ}\left[\begin{array}{c}10{,}702\ \ 1\\ 4\end{array} + \begin{array}{c}5{,}298\\ 1\\ 1\end{array} + \begin{array}{c}15{,}895\\ 3\\ 1\end{array} + \begin{array}{c}21{,}895\ \ 1\\ 4{,}123\end{array}\right]$$

$$\delta_{2p} := \frac{\mathbf{kN}\cdot\mathbf{m}^2}{EJ}\cdot\left(\frac{1}{2}\cdot 10.702\cdot 4\cdot\frac{2}{3}\cdot 1 - \frac{1}{2}\cdot 5.298\cdot 1\cdot 1 + \frac{1}{2}\cdot 15.895\cdot 3\cdot 1\cdot 1 + \frac{1}{2}\cdot 21.895\cdot 4.123\cdot\frac{2}{3}\cdot 1 + A_p\cdot\frac{1}{2}\cdot 1\right)$$

$$\delta_{2p} = 0.011083$$

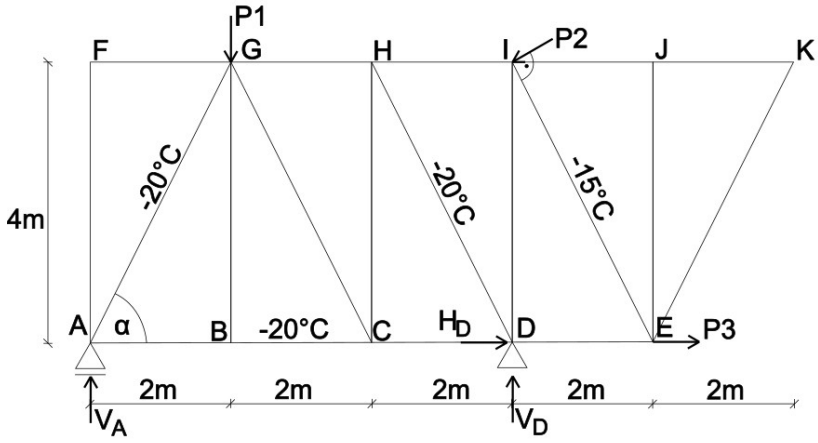
$$\delta_{2\Delta t} = \left[\begin{array}{c}\frac{\alpha_t\Delta t_1}{h}\ \ 1\\ 4\end{array} + \begin{array}{c}\frac{\alpha_t\Delta t_2}{h}\\ 3\\ 1\end{array}\right] \qquad \delta_{2\Delta t} := -\frac{1}{2}\cdot 1\cdot 4\ \mathbf{m}\cdot\frac{\alpha_t\cdot\Delta t_1}{h} + 1\cdot 3\ \mathbf{m}\cdot\frac{\alpha_t\cdot\Delta t_2}{h} = 0.003545$$

$$\delta_{2t0} = \left[\begin{array}{c}\alpha_t t_{02}\\ 3\\ 0{,}25\end{array}\right] \qquad \delta_{2t0} := 0.25\ \frac{1}{\mathbf{m}}\cdot 3\ \mathbf{m}\cdot\alpha_t\cdot t_{02} = -0.000113$$

$$\delta_{2\Delta} := -R_2\cdot\Delta = 0 \qquad \delta_2 := \delta_{2p} + \delta_{2\Delta t} + \delta_{2t0} + \delta_{2\Delta} = 0.832\ °$$

Odp.: Przegub B załamał się o 0,832°. Kąt rozwarty powstał na górze.

**Zadanie 2: Kratownica**




$E := 210\ \mathbf{GPa}$

$\alpha_t := 1.2 \cdot 10^{-5}\ \frac{1}{\mathbf{K}}$

$P1 := 7\ \mathbf{kN}$

$P2 := 7\ \mathbf{kN}$

$P3 := 8\ \mathbf{kN}$

Przekrój: C 200

$A := 32.2\ \mathbf{cm}^2$

$EA := E \cdot A = 676200\ \mathbf{kN}$

Długość pręta ukośnego:

$L := \sqrt{(2\ \mathbf{m})^2 + (4\ \mathbf{m})^2} = 4.472\ \mathbf{m}$

$sin\alpha := \frac{4\ \mathbf{m}}{L} = 0.894 \qquad cos\alpha := \frac{2\ \mathbf{m}}{L} = 0.447$

**Stan p**

Reakcje:

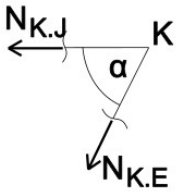
$\Sigma M_A = P1 \cdot 2\ \mathbf{m} + P2 \cdot cos\alpha \cdot 6\ \mathbf{m} - P2 \cdot sin\alpha \cdot 4\ \mathbf{m} - V_D \cdot 6\ \mathbf{m} = 0$

$$V_D := \frac{P1 \cdot 2\ \mathbf{m} + P2 \cdot cos\alpha \cdot 6\ \mathbf{m} - P2 \cdot sin\alpha \cdot 4\ \mathbf{m}}{6\ \mathbf{m}} = 1.29\ \mathbf{kN}$$

$\Sigma X = H_D + P3 - P2 \cdot sin\alpha = 0 \qquad H_D := -P3 + P2 \cdot sin\alpha = -1.739\ \mathbf{kN}$

$\Sigma Y = V_A + V_D - P1 - P2 \cdot cos\alpha = 0 \qquad V_A := -V_D + P1 + P2 \cdot cos\alpha = 8.841\ \mathbf{kN}$

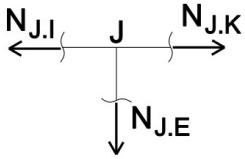
Węzeł K




$\Sigma Y = -N_{K.E} \cdot sin\alpha = 0 \qquad N_{K.E} := \frac{0\ \mathbf{kN}}{sin\alpha} = 0\ \mathbf{kN} \qquad N_{E.K} := N_{K.E}$

$\Sigma X = -N_{K.J} - N_{K.E} \cdot cos\alpha = 0 \qquad N_{K.J} := -N_{K.E} \cdot cos\alpha = 0\ \mathbf{kN} \qquad N_{J.K} := N_{K.J}$

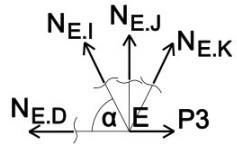
Węzeł J




$\Sigma Y = -N_{J.E} = 0 \qquad N_{J.E} := 0\ \mathbf{kN} \qquad N_{E.J} := N_{J.E}$

$\Sigma X = -N_{J.I} + N_{J.K} = 0 \qquad N_{J.I} := N_{J.K} = 0\ \mathbf{kN} \qquad N_{I.J} := N_{J.I}$

Węzeł E



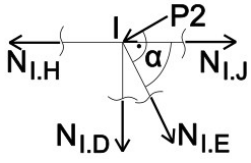

$\Sigma Y = N_{E.I} \cdot sin\alpha + N_{E.J} + N_{E.K} \cdot sin\alpha = 0$

$$N_{E.I} := \frac{-N_{E.J} - N_{E.K} \cdot sin\alpha}{sin\alpha} = 0\ \mathbf{kN} \qquad N_{I.E} := N_{E.I}$$

$\Sigma X = -N_{E.D} - N_{E.I} \cdot cos\alpha + N_{E.K} \cdot cos\alpha + P3 = 0$

$N_{E.D} := -N_{E.I} \cdot cos\alpha + N_{E.K} \cdot cos\alpha + P3 = 8\ \mathbf{kN} \qquad N_{D.E} := N_{E.D}$

Węzeł I



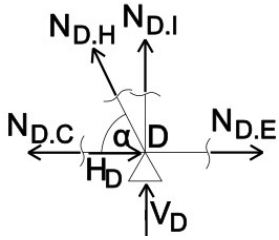

$\Sigma X = -N_{I.H} + N_{I.J} + N_{I.E} \cdot cos\alpha - P2 \cdot sin\alpha = 0$

$N_{I.H} := N_{I.J} + N_{I.E} \cdot cos\alpha - P2 \cdot sin\alpha = -6.261\ \mathbf{kN} \qquad N_{H.I} := N_{I.H}$

$\Sigma Y = -N_{I.D} - N_{I.E} \cdot sin\alpha - P2 \cdot sin\alpha = 0$

$N_{I.D} := -N_{I.E} \cdot sin\alpha - P2 \cdot cos\alpha = -3.13\ \mathbf{kN} \qquad N_{D.I} := N_{I.D}$

Węzeł D



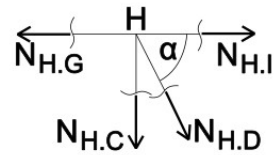

$$\Sigma Y = V_D + N_{D.I} + N_{D.H} \cdot \sin\alpha = 0$$

$$N_{D.H} := \frac{-V_D - N_{D.I}}{\sin\alpha} = 2.058 \text{ kN} \qquad N_{H.D} := N_{D.H}$$

$$\Sigma X = -N_{D.C} + H_D - N_{D.H} \cdot \cos\alpha + N_{D.E} = 0$$

$$N_{D.C} := H_D - N_{D.H} \cdot \cos\alpha + N_{D.E} = 5.341 \text{ kN} \qquad N_{C.D} := N_{D.C}$$

Węzeł H



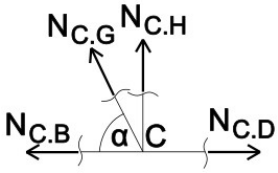

$$\Sigma X = -N_{H.G} + H_{H.I} + N_{H.D} \cdot \cos\alpha = 0$$

$$N_{H.G} := N_{H.I} + N_{H.D} \cdot \cos\alpha = -5.341 \text{ kN} \qquad N_{G.H} := N_{H.G}$$

$$\Sigma Y = -N_{H.C} - N_{H.D} \cdot \sin\alpha = 0$$

$$N_{H.C} := -N_{H.D} \cdot \sin\alpha = -1.841 \text{ kN} \qquad N_{C.H} := N_{H.C}$$

Węzeł C

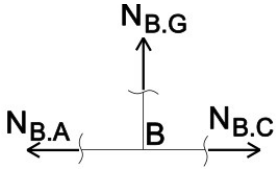



$$\Sigma Y = N_{C.H} + N_{C.G} \cdot \sin\alpha = 0 \qquad N_{C.G} := \frac{-N_{C.H}}{\sin\alpha} = 2.058 \text{ kN} \qquad N_{G.C} := N_{C.G}$$

$$\Sigma X = -N_{C.B} - N_{C.G} \cdot \cos\alpha + N_{C.D} = 0$$

$$N_{C.B} := -N_{C.G} \cdot \cos\alpha + N_{C.D} = 4.42 \text{ kN} \qquad N_{B.C} := N_{C.B}$$

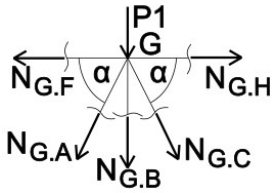
Węzeł B




$$\Sigma Y = N_{B.G} = 0 \qquad N_{B.G} := 0 \text{ kN} \qquad N_{G.B} := N_{B.G}$$

$$\Sigma X = -N_{B.A} + N_{B.C} = 0 \qquad N_{B.A} := N_{B.C} = 4.42 \text{ kN} \qquad N_{A.B} := N_{B.A}$$

Węzeł G



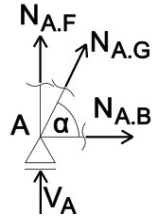

$$\Sigma Y = -N_{G.A} \cdot \sin\alpha - N_{G.B} - N_{G.C} \cdot \sin\alpha - P1 = 0$$

$$N_{G.A} := \frac{-N_{G.B} - N_{G.C} \cdot \sin\alpha - P1}{\sin\alpha} = -9.884 \text{ kN} \qquad N_{A.G} := N_{G.A}$$

$$\Sigma X = -N_{G.F} - N_{G.A} \cdot \cos\alpha + N_{G.C} \cdot \cos\alpha + N_{G.H} = 0$$

$$N_{G.F} := -N_{G.A} \cdot \cos\alpha + N_{G.C} \cdot \cos\alpha + N_{G.H} = 0 \text{ kN} \qquad N_{F.G} := N_{G.F}$$

Węzeł A

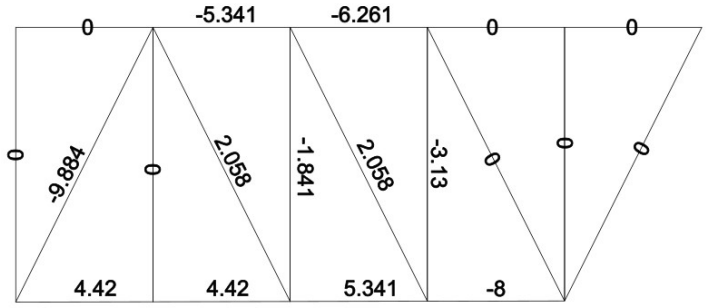



$$\Sigma Y = N_{A.F} + V_A + N_{A.G} \cdot \sin\alpha = 0 \qquad N_{A.F} := -V_A - N_{A.G} \cdot \sin\alpha = 0 \text{ kN} \qquad N_{F.A} := N_{A.F}$$

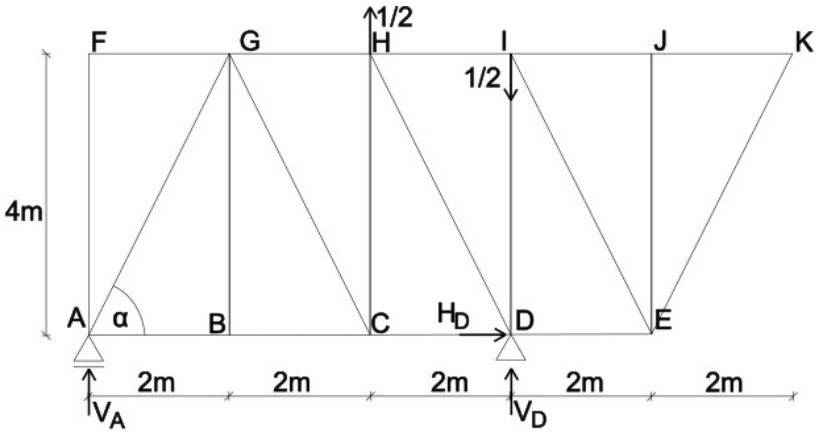
sprawdzenie:

$$\Sigma X = 0 \qquad N_{A.G} \cdot \cos\alpha + N_{A.B} = 0 \text{ kN}$$

$N_P$ [kN]



**Stan 1** Obrót pręta H.I




Reakcje:

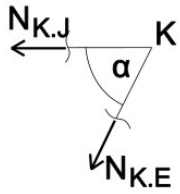
$$\Sigma M_A = \frac{-1}{2\ \mathbf{m}} \cdot 4\ \mathbf{m} + \frac{1}{2\ \mathbf{m}} \cdot 6\ \mathbf{m} - V_D \cdot 6\ \mathbf{m} = 0$$

$$V_D := \frac{\frac{-1}{2\ \mathbf{m}} \cdot 4\ \mathbf{m} + \frac{1}{2\ \mathbf{m}} \cdot 6\ \mathbf{m}}{6\ \mathbf{m}} = 0.167\ \frac{1}{\mathbf{m}}$$

$$\Sigma X = H_D = 0 \qquad H_D := 0\ \frac{1}{\mathbf{m}}$$

$$\Sigma Y = V_A + V_D - \frac{1}{2\ \mathbf{m}} + \frac{1}{2\ \mathbf{m}} = 0 \qquad V_A := -V_D + \frac{1}{2\ \mathbf{m}} - \frac{1}{2\ \mathbf{m}} = -0.167\ \frac{1}{\mathbf{m}}$$

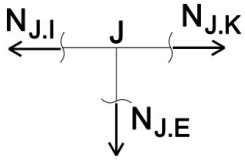
Węzeł K




$$\Sigma Y = -N_{K.E} \cdot sin\alpha = 0 \qquad N_{K.E} := \frac{0}{sin\alpha}\ \frac{1}{\mathbf{m}} = 0\ \frac{1}{\mathbf{m}} \qquad N_{E.K} := N_{K.E}$$

$$\Sigma X = -N_{K.J} - N_{K.E} \cdot cos\alpha = 0 \qquad N_{K.J} := -N_{K.E} \cdot cos\alpha = 0\ \frac{1}{\mathbf{m}} \qquad N_{J.K} := N_{K.J}$$

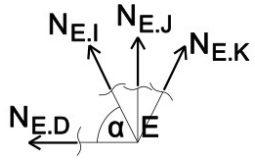
Węzeł J




$$\Sigma Y = -N_{J.E} = 0 \qquad N_{J.E} := 0\ \frac{1}{\mathbf{m}} \qquad N_{E.J} := N_{J.E}$$

$$\Sigma X = -N_{J.I} + N_{J.K} = 0 \qquad N_{J.I} := N_{J.K} = 0\ \frac{1}{\mathbf{m}} \qquad N_{I.J} := N_{J.I}$$

Węzeł E



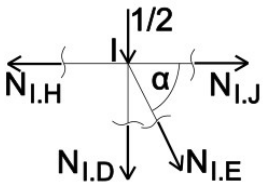

$$\Sigma Y = N_{E.I} \cdot sin\alpha + N_{E.J} + N_{E.K} \cdot sin\alpha = 0$$

$$N_{E.I} := \frac{-N_{E.J} - N_{E.K} \cdot sin\alpha}{sin\alpha} = 0\ \frac{1}{\mathbf{m}} \qquad N_{I.E} := N_{E.I}$$

$$\Sigma X = -N_{E.D} - N_{E.I} \cdot cos\alpha + N_{E.K} \cdot cos\alpha = 0$$

$$N_{E.D} := -N_{E.I} \cdot cos\alpha + N_{E.K} \cdot cos\alpha = 0\ \frac{1}{\mathbf{m}} \qquad N_{D.E} := N_{E.D}$$

Węzeł I



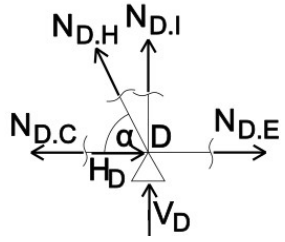

$$\Sigma X = -N_{I.H} + N_{I.J} + N_{I.E} \cdot cos\alpha = 0$$

$$N_{I.H} := N_{I.J} + N_{I.E} \cdot cos\alpha = 0\ \frac{1}{\mathbf{m}} \qquad N_{H.I} := N_{I.H}$$

$$\Sigma Y = -N_{I.D} - N_{I.E} \cdot sin\alpha - \frac{1}{2\ \mathbf{m}} = 0$$

$$N_{I.D} := -N_{I.E} \cdot sin\alpha - \frac{1}{2\ \mathbf{m}} = -0.5\ \frac{1}{\mathbf{m}} \qquad N_{D.I} := N_{I.D}$$

Węzeł D



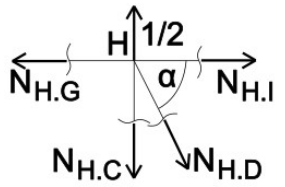

$$\Sigma Y = V_D + N_{D.I} + N_{D.H} \cdot \sin\alpha = 0$$

$$N_{D.H} := \frac{-V_D - N_{D.I}}{\sin\alpha} = 0.373 \ \frac{1}{\mathbf{m}} \qquad N_{H.D} := N_{D.H}$$

$$\Sigma X = -N_{D.C} + H_D - N_{D.H} \cdot \cos\alpha + N_{D.E} = 0$$

$$N_{D.C} := H_D - N_{D.H} \cdot \cos\alpha + N_{D.E} = -0.167 \ \frac{1}{\mathbf{m}} \qquad N_{C.D} := N_{D.C}$$

Węzeł H



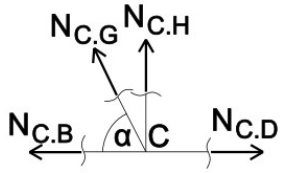

$$\Sigma X = -N_{H.G} + H_{H.I} + N_{H.D} \cdot \cos\alpha = 0$$

$$N_{H.G} := N_{H.I} + N_{H.D} \cdot \cos\alpha = 0.167 \ \frac{1}{\mathbf{m}} \qquad N_{G.H} := N_{H.G}$$

$$\Sigma Y = -N_{H.C} - N_{H.D} \cdot \sin\alpha + \frac{1}{2\ \mathbf{m}} = 0$$

$$N_{H.C} := -N_{H.D} \cdot \sin\alpha + \frac{1}{2\ \mathbf{m}} = 0.167 \ \frac{1}{\mathbf{m}} \qquad N_{C.H} := N_{H.C}$$

Węzeł C

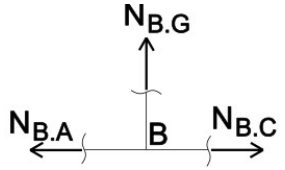



$$\Sigma Y = N_{C.H} + N_{C.G} \cdot \sin\alpha = 0 \qquad N_{C.G} := \frac{-N_{C.H}}{\sin\alpha} = -0.186 \ \frac{1}{\mathbf{m}} \qquad N_{G.C} := N_{C.G}$$

$$\Sigma X = -N_{C.B} - N_{C.G} \cdot \cos\alpha + N_{C.D} = 0$$

$$N_{C.B} := -N_{C.G} \cdot \cos\alpha + N_{C.D} = -0.083 \ \frac{1}{\mathbf{m}} \qquad N_{B.C} := N_{C.B}$$

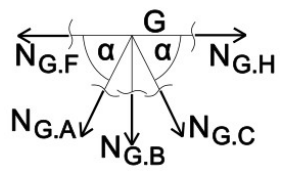
Węzeł B




$$\Sigma Y = N_{B.G} = 0 \qquad N_{B.G} := 0 \ \frac{1}{\mathbf{m}} \qquad N_{G.B} := N_{B.G}$$

$$\Sigma X = -N_{B.A} + N_{B.C} = 0 \qquad N_{B.A} := N_{B.C} = -0.083 \ \frac{1}{\mathbf{m}} \qquad N_{A.B} := N_{B.A}$$

Węzeł G



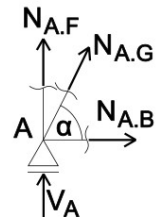

$$\Sigma Y = -N_{G.A} \cdot \sin\alpha - N_{G.B} - N_{G.C} \cdot \sin\alpha = 0$$

$$N_{G.A} := \frac{-N_{G.B} - N_{G.C} \cdot \sin\alpha}{\sin\alpha} = 0.186 \ \frac{1}{\mathbf{m}} \qquad N_{A.G} := N_{G.A}$$

$$\Sigma X = -N_{G.F} - N_{G.A} \cdot \cos\alpha + N_{G.C} \cdot \cos\alpha + N_{G.H} = 0$$

$$N_{G.F} := -N_{G.A} \cdot \cos\alpha + N_{G.C} \cdot \cos\alpha + N_{G.H} = 0 \ \frac{1}{\mathbf{m}} \qquad N_{F.G} := N_{G.F}$$

Węzeł A

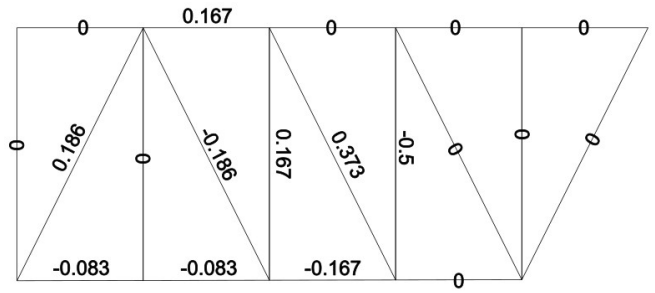



$$\Sigma Y = N_{A.F} + V_A + N_{A.G} \cdot \sin\alpha = 0 \qquad N_{A.F} := -V_A - N_{A.G} \cdot \sin\alpha = 0 \ \frac{1}{\mathbf{m}} \qquad N_{F.A} := N_{A.F}$$

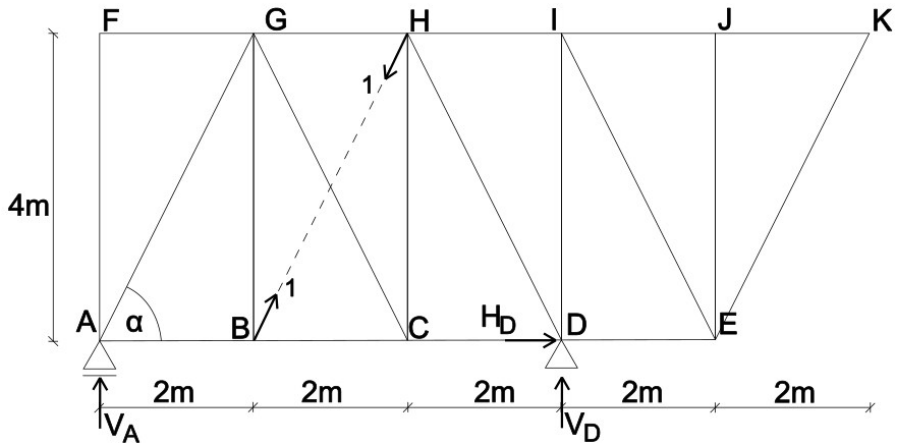
sprawdzenie:

$$\Sigma X = 0 \qquad N_{A.G} \cdot \cos\alpha + N_{A.B} = 0 \ \frac{1}{\mathbf{m}}$$

$N_1$ [1/m]



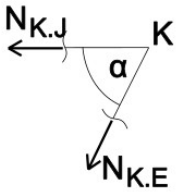
**Stan 2** Zmiana odległości między punktami B i H




Reakcje:

$V_D := 0 \qquad H_D := 0 \qquad V_A := 0$

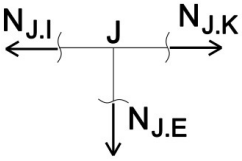
Węzeł K




$\Sigma Y = -N_{K.E} \cdot \sin\alpha = 0 \qquad N_{K.E} := \frac{0}{\sin\alpha} = 0 \qquad N_{E.K} := N_{K.E}$

$\Sigma X = -N_{K.J} - N_{K.E} \cdot \cos\alpha = 0 \qquad N_{K.J} := -N_{K.E} \cdot \cos\alpha = 0 \qquad N_{J.K} := N_{K.J}$

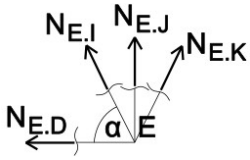
Węzeł J




$\Sigma Y = -N_{J.E} = 0 \qquad N_{J.E} := 0 \qquad N_{E.J} := N_{J.E}$

$\Sigma X = -N_{J.I} + N_{J.K} = 0 \qquad N_{J.I} := N_{J.K} = 0 \qquad N_{I.J} := N_{J.I}$

Węzeł E



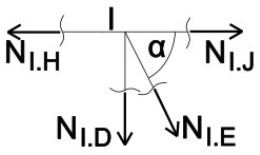

$\Sigma Y = N_{E.I} \cdot \sin\alpha + N_{E.J} + N_{E.K} \cdot \sin\alpha = 0$

$N_{E.I} := \frac{-N_{E.J} - N_{E.K} \cdot \sin\alpha}{\sin\alpha} = 0 \qquad N_{I.E} := N_{E.I}$

$\Sigma X = -N_{E.D} - N_{E.I} \cdot \cos\alpha + N_{E.K} \cdot \cos\alpha = 0$

$N_{E.D} := -N_{E.I} \cdot \cos\alpha + N_{E.K} \cdot \cos\alpha = 0 \qquad N_{D.E} := N_{E.D}$

Węzeł I



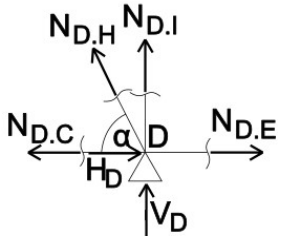

$\Sigma X = -N_{I.H} + N_{I.J} + N_{I.E} \cdot \cos\alpha = 0$

$N_{I.H} := N_{I.J} + N_{I.E} \cdot \cos\alpha = 0 \qquad N_{H.I} := N_{I.H}$

$\Sigma Y = -N_{I.D} - N_{I.E} \cdot \sin\alpha - \frac{1}{2\ \mathrm{m}} = 0$

$N_{I.D} := -N_{I.E} \cdot \sin\alpha = 0 \qquad N_{D.I} := N_{I.D}$

Węzeł D



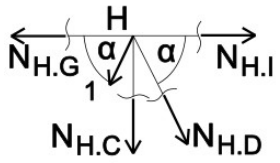

$\Sigma Y = V_D + N_{D.I} + N_{D.H} \cdot \sin\alpha = 0$

$N_{D.H} := \frac{-V_D - N_{D.I}}{\sin\alpha} = 0 \qquad N_{H.D} := N_{D.H}$

$\Sigma X = -N_{D.C} + H_D - N_{D.H} \cdot \cos\alpha + N_{D.E} = 0$

$N_{D.C} := H_D - N_{D.H} \cdot \cos\alpha + N_{D.E} = 0 \qquad N_{C.D} := N_{D.C}$

Węzeł H



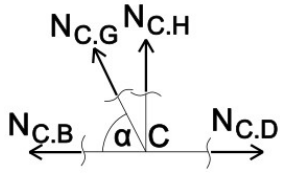

$$\Sigma X = -N_{H.G} + H_{H.I} + N_{H.D} \cdot cos\alpha - 1 \cdot cos\alpha = 0$$

$$N_{H.G} := N_{H.I} + N_{H.D} \cdot cos\alpha - 1 \cdot cos\alpha = -0.447 \qquad N_{G.H} := N_{H.G}$$

$$\Sigma Y = -N_{H.C} - N_{H.D} \cdot sin\alpha - 1 \cdot sin\alpha = 0$$

$$N_{H.C} := -N_{H.D} \cdot sin\alpha - 1 \cdot sin\alpha = -0.894 \qquad N_{C.H} := N_{H.C}$$

Węzeł C

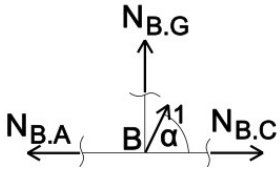



$$\Sigma Y = N_{C.H} + N_{C.G} \cdot sin\alpha = 0 \qquad N_{C.G} := \frac{-N_{C.H}}{sin\alpha} = 1 \qquad N_{G.C} := N_{C.G}$$

$$\Sigma X = -N_{C.B} - N_{C.G} \cdot cos\alpha + N_{C.D} = 0$$

$$N_{C.B} := -N_{C.G} \cdot cos\alpha + N_{C.D} = -0.447 \qquad N_{B.C} := N_{C.B}$$

Węzeł B

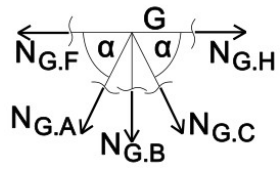



$$\Sigma Y = N_{B.G} + 1 \cdot sin\alpha = 0 \qquad N_{B.G} := -1 \; sin\alpha = -0.894 \qquad N_{G.B} := N_{B.G}$$

$$\Sigma X = -N_{B.A} + N_{B.C} + 1 \cdot cos\alpha = 0$$

$$N_{B.A} := N_{B.C} + 1 \cdot cos\alpha = 0 \qquad N_{A.B} := N_{B.A}$$

Węzeł G



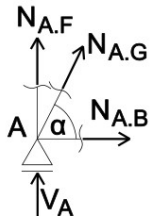

$$\Sigma Y = -N_{G.A} \cdot sin\alpha - N_{G.B} - N_{G.C} \cdot sin\alpha = 0$$

$$N_{G.A} := \frac{-N_{G.B} - N_{G.C} \cdot sin\alpha}{sin\alpha} = 0 \qquad N_{A.G} := N_{G.A}$$

$$\Sigma X = -N_{G.F} - N_{G.A} \cdot cos\alpha + N_{G.C} \cdot cos\alpha + N_{G.H} = 0$$

$$N_{G.F} := -N_{G.A} \cdot cos\alpha + N_{G.C} \cdot cos\alpha + N_{G.H} = 0 \qquad N_{F.G} := N_{G.F}$$

Węzeł A

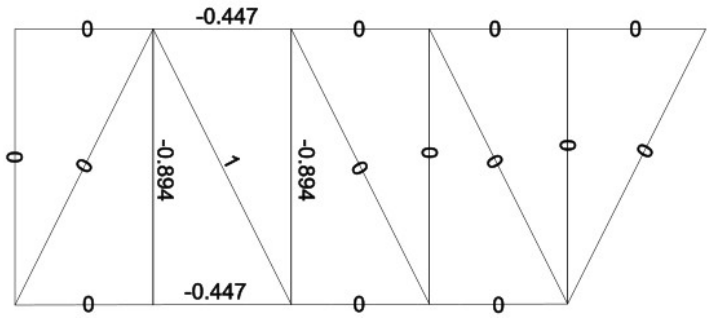



$$\Sigma Y = N_{A.F} + V_A + N_{A.G} \cdot sin\alpha = 0 \qquad N_{A.F} := -V_A - N_{A.G} \cdot sin\alpha = 0$$

sprawdzenie:

$$\Sigma X = 0 \qquad N_{A.G} \cdot cos\alpha + N_{A.B} = 0$$

$N_2$ [-]



**Obliczenia przemieszczeń**

| Pręt | L [m] | Np [kN] | N1 [1/m] | N2 [-] | t0 [°C] | Np·N1·L | Np·N2·L | t0·N1·L | t0·N2·L |
|---|---|---|---|---|---|---|---|---|---|
| A.B | 2 | 4.42 | -0.083 | 0 | 0 | -0.73372 | 0 | 0 | 0 |
| B.C | 2 | 4.42 | -0.083 | -0.447 | -20 | -0.73372 | -3.95148 | 3.32 | 17.88 |
| C.D | 2 | 5.341 | -0.167 | 0 | 0 | -1.78389 | 0 | 0 | 0 |
| D.E | 2 | -8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| F.G | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G.H | 2 | -5.341 | 0.167 | -0.447 | 0 | -1.78389 | 4.774854 | 0 | 0 |
| H.I | 2 | -6.261 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| I.J | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| J.K | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A.F | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B.G | 4 | 0 | 0 | -0.894 | 0 | 0 | 0 | 0 | 0 |
| C.H | 4 | -1.841 | 0.167 | -0.894 | 0 | -1.22979 | 6.583416 | 0 | 0 |
| D.I | 4 | -3.13 | -0.5 | 0 | 0 | 6.26 | 0 | 0 | 0 |
| E.J | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A.G | 4.472 | -9.884 | 0.186 | 0 | -20 | -8.22143 | 0 | -16.6358 | 0 |
| G.C | 4.472 | 2.058 | -0.186 | 1 | 0 | -1.71183 | 9.203376 | 0 | 0 |
| H.D | 4.472 | 2.058 | 0.373 | 0 | -20 | 3.432859 | 0 | -33.3611 | 0 |
| I.E | 4.472 | 0 | 0 | 0 | -15 | 0 | 0 | 0 | 0 |
| E.K | 4.472 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | Σ | **-6.50542** | **16.61017** | **-46.677** | **17.88** |

$$\delta_{1p} := \frac{-6.50542\ \mathbf{kN}}{EA} = -9.621 \cdot 10^{-6}$$

$$\delta_{1t0} := \alpha_t \cdot -46.677\ \mathbf{K} = -5.601 \cdot 10^{-4}$$

$$\delta_{1P} := \delta_{1p} + \delta_{1t0} = -5.697 \cdot 10^{-4} \qquad \delta_{1P} = -0.033\ °$$

Odp.: Pręt H.I obrócił się o 0.033° przeciwnie do ruchu wskazówek zegara.

$$\delta_{1p} := \frac{16.61017\ \mathbf{kNm}}{EA} = \left(2.456 \cdot 10^{-5}\right)\ \mathbf{m}$$

$$\delta_{1t0} := \alpha_t \cdot 17.88\ \mathbf{K \cdot m} = \left(2.146 \cdot 10^{-4}\right)\ \mathbf{m}$$

$$\delta_{1P} := \delta_{1p} + \delta_{1t0} = \left(2.391 \cdot 10^{-4}\right)\ \mathbf{m} \qquad \delta_{1P} = 0.239\ \mathbf{mm}$$

Odp.: Punkty B i H zbliżyły się do siebie o 0.239mm.