Politechnika Lubelska

Wydział Budownictwa i Architektury

Mechanika Budowli II

Projekt 2: Drgania

Wykonał:

……………………..

Sprawdził:

dr inż. Jakub Gontarz

2025

PRZYKŁADOWY TEMAT

Student ....

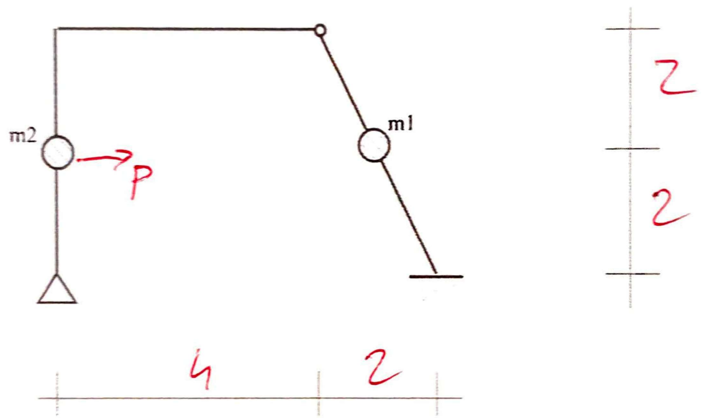
Grupa ....

Rok akademicki ....

ĆWICZENIE PROJEKTOWE NR 2 Z MECHANIKI BUDOWLI

1. Wyznaczyć częstości drgań własnych oraz naszkicować postacie tych drgań.

2. Wyznaczyć amplitudy sił wewnętrznych dla konstrukcji obciążonej siłą wymuszającą $P=P_o \sin\theta t$.



m1=...500 kg...

m2=...300 kg...

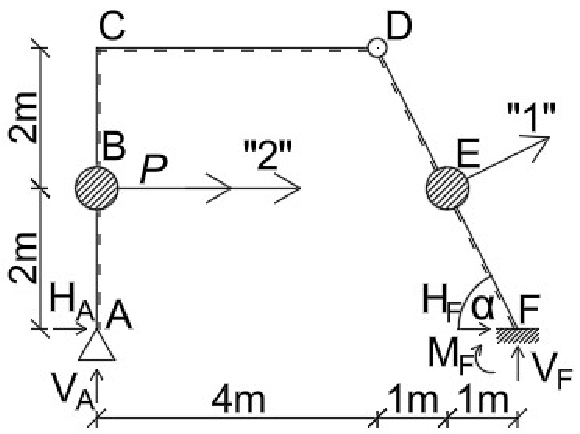
m3=....................

E=...stal...

I=...IPN 160...

θ=...5 Hz...

Po=...11 kN...



$$kNm := kN \cdot m$$

$$E := 210\ GPa$$

$$m_1 := 500\ kg$$

$$\theta := 5\ Hz \cdot 2\ \pi = 31.416\ \frac{rad}{s}$$

$$m_2 := 300\ kg$$

$$P_0 := 11\ kN$$

Przekrój: IPN 160

$$J := 935\ cm^4$$

$$EJ := E \cdot J = 1963.5\ kN \cdot m^2$$

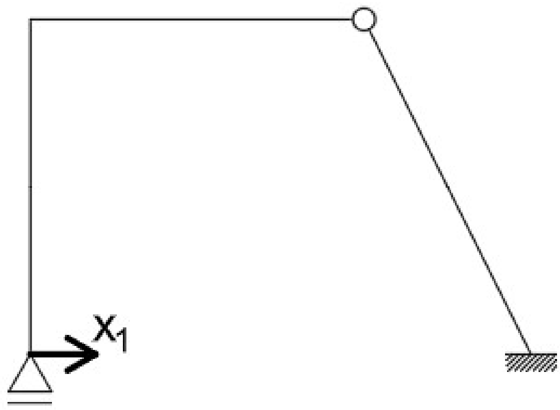
Długość pręta ukośnego:

$$L := \sqrt{(2\ m)^2 + (4\ m)^2} = 4.472\ m$$

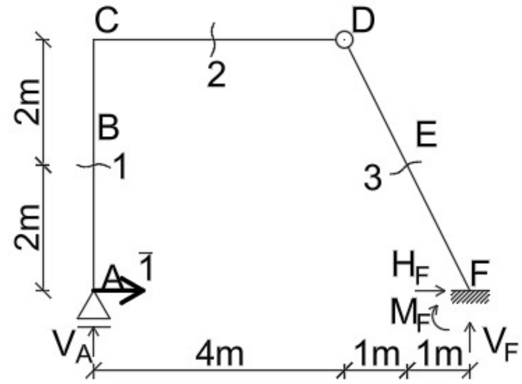
$$sin\alpha := \frac{4\ m}{L} = 0.894 \qquad cos\alpha := \frac{2\ m}{L} = 0.447$$

Drgania własne

UPMS



Stan 1



Reakcje:

$$\Sigma M_D{}^L = V_A \cdot 4\ m - 1 \cdot 4\ m = 0 \qquad V_A := \frac{1 \cdot 4\ m}{4\ m} = 1$$

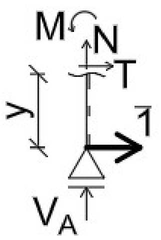
$$\Sigma Y = V_A + V_F = 0 \qquad V_F := -V_A = -1$$

$$\Sigma X = 1 + H_F = 0 \qquad H_F := -1$$

$$\Sigma M_D{}^P = -V_F \cdot 2\ m - H_F \cdot 4\ m + M_F = 0$$

$$M_F := V_F \cdot 2\ m + H_F \cdot 4\ m = -6\ m$$

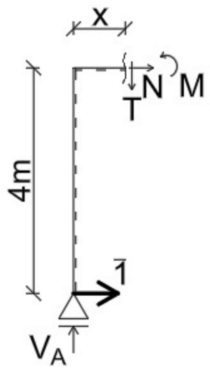
Pręt 1 $\qquad$ y∈(0;4)m



$$M_1(y) := -1 \cdot y$$

$$M_1(0\ m) = 0\ m \qquad M_1(4\ m) = -4\ m$$

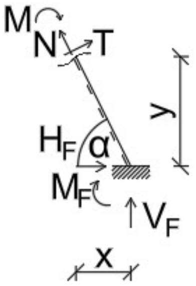
Pręt 2 x∈(0;4)m



$$M_2(x) := -1 \cdot 4\ \mathrm{m} + V_A \cdot x$$

$$M_2(0\ \mathrm{m}) = -4\ \mathrm{m} \qquad M_2(4\ \mathrm{m}) = 0\ \mathrm{m}$$

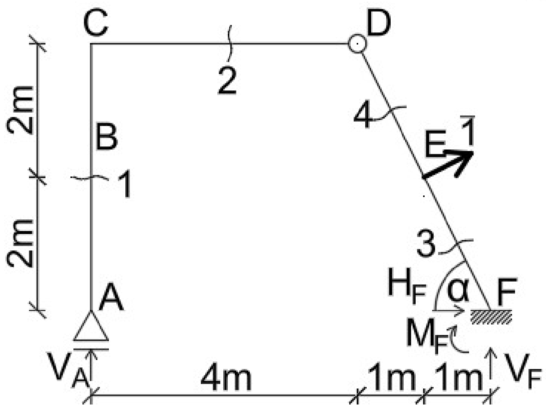
Pręt 3 x∈(0;2)m $y(x) := 2\ x$



$$M_3(x) := V_F \cdot x + H_F \cdot y(x) - M_F$$

$$M_3(0\ \mathrm{m}) = 6\ \mathrm{m} \qquad M_3(2\ \mathrm{m}) = 0\ \mathrm{m}$$

Stan P1



Reakcje:

$$\Sigma M_D{}^L = V_A \cdot 4\ m = 0 \qquad V_A := 0$$

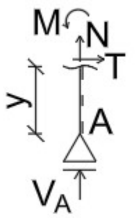
$$\Sigma Y = V_A + V_F + 1 \cdot cos\alpha = 0 \qquad V_F := -V_A - 1 \cdot cos\alpha = -0.447$$

$$\Sigma X = H_F + 1 \cdot sin\alpha = 0 \qquad H_F := -1 \cdot sin\alpha = -0.894$$

$$\Sigma M_D{}^P = -1 \cdot \frac{L}{2} - V_F \cdot 2\ m - H_F \cdot 4\ m + M_F = 0$$

$$M_F := 1 \cdot \frac{L}{2} + V_F \cdot 2\ \mathrm{m} + H_F \cdot 4\ \mathrm{m} = -2.236\ \mathrm{m}$$

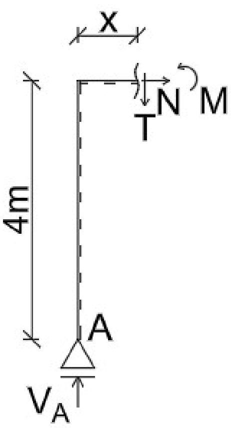
Pręt 1 y∈(0;4)m



$$M_1(y) := 0$$

$$M_1(0\ \mathrm{m}) = 0\ \mathrm{m} \qquad M_1(4\ \mathrm{m}) = 0$$

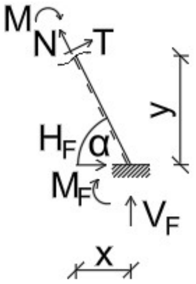
Pręt 2 x∈(0;4)m



$$M_2(x) := -1 \cdot 4\ \mathrm{m} + V_A \cdot x$$

$$M_2(0\ \mathrm{m}) = -4\ \mathrm{m} \qquad M_2(4\ \mathrm{m}) = -4\ \mathrm{m}$$

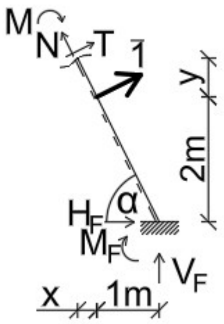
Pręt 3 $x \in (0;2)m$ $y(x) := 2\ x$



$$M_3(x) := V_F \cdot x + H_F \cdot y(x) - M_F$$

$$M_3(0\ \mathrm{m}) = 2.236\ \mathrm{m} \quad M_3(2\ \mathrm{m}) = -2.236\ \mathrm{m}$$

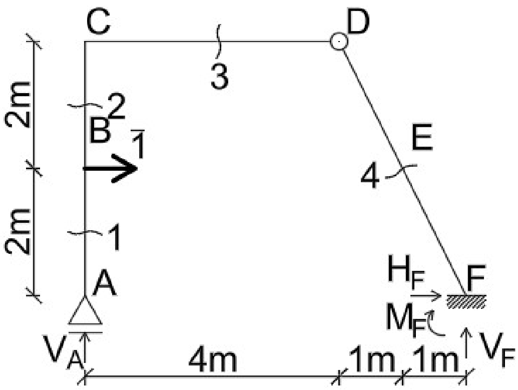
Pręt 4 $x \in (0;2)m$ $y(x) := 2\ x$



$$M_3(x) := V_F \cdot (x + 1\ \mathrm{m}) + H_F \cdot (y(x) + 2\ \mathrm{m}) + 1 \cdot \sqrt{x^2 + y(x)^2} - M_F$$

$$M_3(0\ \mathrm{m}) = 0\ \mathrm{m} \qquad M_3(2\ \mathrm{m}) = 0\ \mathrm{m}$$

Stan P2



Reakcje:

$$\Sigma M_D^{\,L} = V_A \cdot 4\ m - 1 \cdot 2\ m = 0 \qquad V_A := \frac{1 \cdot 2\ \mathrm{m}}{4\ \mathrm{m}} = 0.5$$

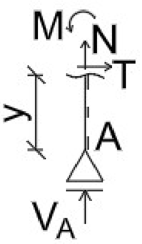
$$\Sigma Y = V_A + V_F = 0 \qquad V_F := -V_A = -0.5$$

$$\Sigma X = 1 + H_F = 0 \qquad H_F := -1$$

$$\Sigma M_D^{\,P} = -V_F \cdot 2\ m - H_F \cdot 4\ m + M_F = 0$$

$$M_F := V_F \cdot 2\ \mathrm{m} + H_F \cdot 4\ \mathrm{m} = -5\ \mathrm{m}$$

Pręt 1 $y \in (0;2)m$

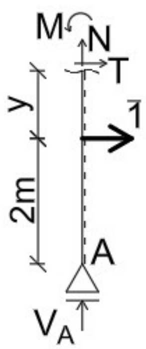


$$M_1(y) := 0$$

$$M_1(0\ \mathrm{m}) = 0\ \mathrm{m} \qquad M_1(2\ \mathrm{m}) = 0$$

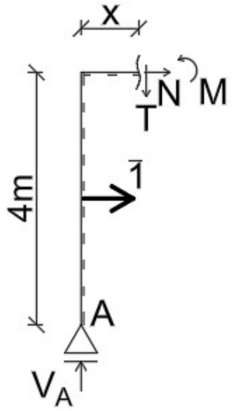
Pręt 2 $y \in (0;2)m$

$$M_2(y) := -1 \cdot y$$



$$M_2(0\ \mathrm{m}) = 0\ \mathrm{m} \qquad M_2(2\ \mathrm{m}) = -2\ \mathrm{m}$$

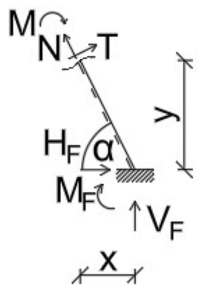
Pręt 3 $\quad x \in (0;4)\mathrm{m}$



$$M_3(x) := -1 \cdot 2\ \mathrm{m} + V_A \cdot x$$

$$M_3(0\ \mathrm{m}) = -2\ \mathrm{m} \qquad M_3(4\ \mathrm{m}) = 0\ \mathrm{m}$$

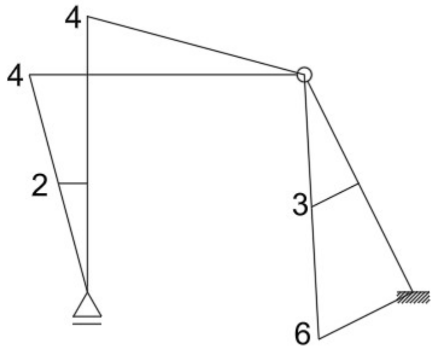
Pręt 4 $\quad x \in (0;2)\mathrm{m} \qquad y(x) := 2\ x$



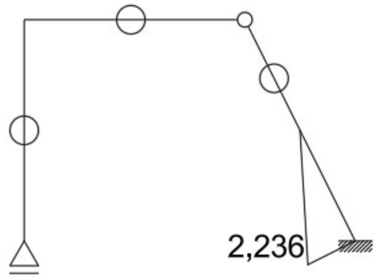
$$M_4(x) := V_F \cdot x + H_F \cdot y(x) - M_F$$

$$M_4(0\ \mathrm{m}) = 5\ \mathrm{m} \qquad M_4(2\ \mathrm{m}) = 0\ \mathrm{m}$$

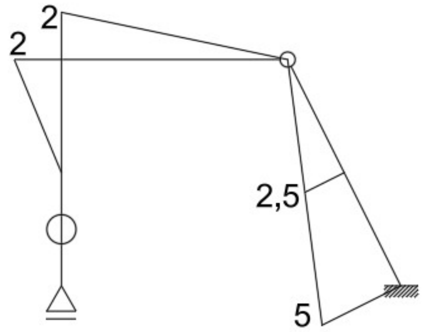
M1 [m]



MP1 [m]

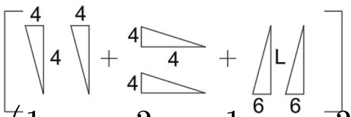


MP2 [m]



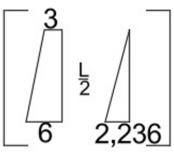
$$M_1 := \begin{bmatrix} 0 & -2 \\ -2 & -4 \\ -4 & 0 \\ 0 & 3 \\ 3 & 6 \end{bmatrix} \mathrm{m} \qquad M_{P1} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 2.236 \end{bmatrix} \mathrm{m} \qquad M_{P2} := \begin{bmatrix} 0 & 0 \\ 0 & -2 \\ -2 & 0 \\ 0 & 2.5 \\ 2.5 & 5 \end{bmatrix} \mathrm{m}$$

$$\delta_{11} = \frac{1}{EJ}$$

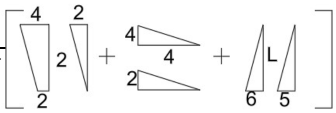


$$\delta_{11} := \frac{\mathrm{m}^3}{EJ} \left(\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 + \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 + \frac{1}{2} \cdot 6 \cdot 4.472 \cdot \frac{2}{3} \cdot 6 \right) = 96.331\ \frac{\mathrm{m}^3}{EJ} \qquad \delta_{11} = 0.04906\ \frac{\mathrm{m}}{\mathrm{kN}}$$

$$\delta_{1P1} = \frac{1}{EJ}$$



$$\delta_{1P1} := \frac{\mathrm{m}^3}{EJ} \left(\frac{1}{2} \cdot 2.236 \cdot 2.236 \cdot \left(\frac{2}{3} \cdot 6 + \frac{1}{3} \cdot 3 \right) \right) = 12.499\ \frac{\mathrm{m}^3}{EJ} \qquad \delta_{1P1} = 0.00637\ \frac{\mathrm{m}}{\mathrm{kN}}$$

$\delta_{1P2} = \frac{1}{EJ}$ 

$$\delta_{1P2} := \frac{m^3}{EJ}\left(\frac{1}{2}\cdot 2\cdot 2\cdot\left(\frac{2}{3}\cdot 4+\frac{1}{3}\cdot 2\right)+\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 2+\frac{1}{2}\cdot 6\cdot 4.472\cdot\frac{2}{3}\cdot 5\right)=62.053\ \frac{m^3}{EJ} \qquad \delta_{1P2}=0.0316\ \frac{m}{kN}$$

Obliczenia niewiadomych metody sił

$\delta_{11}\cdot x_{11}+\delta_{1P1}=0$

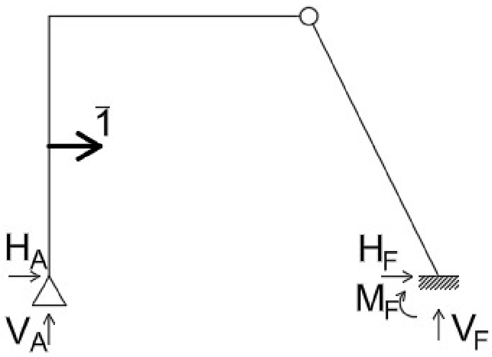
$x_{11} := \frac{-\delta_{1P1}}{\delta_{11}} = -0.13$

$\delta_{11}\cdot x_{11}+\delta_{1P2}=0$

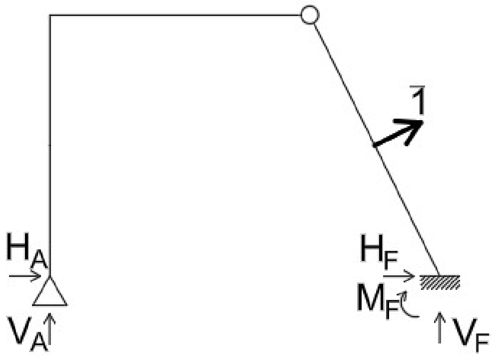
$x_{12} := \frac{-\delta_{1P2}}{\delta_{11}} = -0.644$

Stany ostateczne

Stan ostateczny 1



Stan ostateczny 2

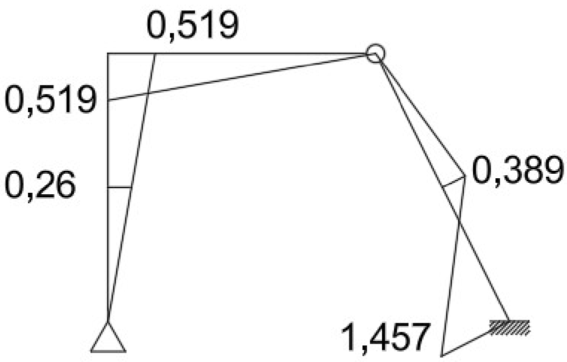


Wykresy ostateczne

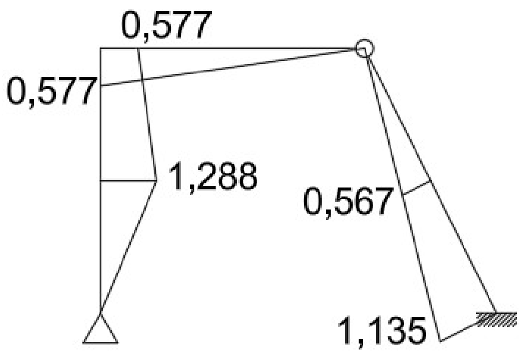
$$M_{ost1} := M_1\cdot x_{11}+M_{P1}=\begin{bmatrix} 0 & 0.26 \\ 0.26 & 0.519 \\ 0.519 & 0 \\ 0 & -0.389 \\ -0.389 & 1.457 \end{bmatrix} m$$

$$M_{ost2} := M_1\cdot x_{12}+M_{P2}=\begin{bmatrix} 0 & 1.288 \\ 1.288 & 0.577 \\ 0.577 & 0 \\ 0 & 0.567 \\ 0.567 & 1.135 \end{bmatrix} m$$

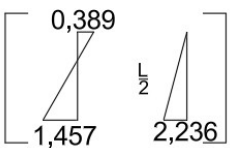
Most1 [m]



Most2 [m]

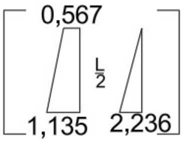


Obliczenie przemieszczeń na kierunkach swobody dynamicznej:

$\delta_{11} = \int_S \frac{M_{ost1}\cdot M_{P1}}{EJ}\,dS = \frac{1}{EJ}$ 

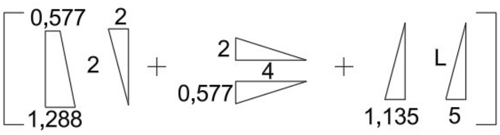
$$\delta_{11} := \frac{m^3}{EJ}\left(\frac{1}{2}\cdot 2.236\cdot 2.236\cdot\left(\frac{2}{3}\cdot 1.457-\frac{1}{3}\cdot 0.389\right)\right)=2.104\ \frac{m^3}{EJ} \qquad \delta_{11}=0.00107\ \frac{m}{kN}$$

$$\delta_{12} = \int_S \frac{M_{ost2} \cdot M_{P1}}{EJ} \, \mathrm{d}S = \frac{1}{EJ}$$



$$\delta_{12} := \frac{\mathbf{m}^3}{EJ} \left(\frac{1}{2} \cdot 2.236 \cdot 2.236 \cdot \left(\frac{2}{3} \cdot 1.135 + \frac{1}{3} \cdot 0.567 \right) \right) = 2.364 \; \frac{\mathbf{m}^3}{EJ} \qquad \delta_{12} = 0.0012 \; \frac{\mathbf{m}}{\mathbf{kN}} \qquad \delta_{21} := \delta_{12}$$

$$\delta_{22} = \int_S \frac{M_{ost2} \cdot M_{P2}}{EJ} \, \mathrm{d}S = \frac{1}{EJ}$$



$$\delta_{22} := \frac{\mathbf{m}^3}{EJ} \left(\frac{-1}{2} \cdot 2 \cdot 2 \cdot \left(\frac{2}{3} \cdot 0.577 + \frac{1}{3} \cdot 1.288 \right) - \frac{1}{2} \cdot 2 \cdot 4 \cdot \frac{2}{3} \cdot 0.577 + \frac{1}{2} \cdot 1.135 \cdot 4.472 \cdot \frac{2}{3} \cdot 5 \right) = 5.293 \; \frac{\mathbf{m}^3}{EJ}$$

$$\delta_{22} = 0.0027 \; \frac{\mathbf{m}}{\mathbf{kN}}$$

Układ równań opisujący ruch:

$$\left(m1 \cdot \delta_{11} - \frac{1}{\omega^2} \right) \cdot A_1 + m2 \cdot \delta_{12} \cdot A_2 = 0$$

$$m1 \cdot \delta_{21} \cdot A_1 + \left(m2 \cdot \delta_{22} - \frac{1}{\omega^2} \right) \cdot A_2 = 0$$

Podstawienie: $X = \frac{m_0 \cdot \omega^2}{EJ} \; \mathbf{m}^3$

$$\delta'_{11} := \frac{\delta_{11} \cdot EJ}{\mathbf{m}^3} = 2.104 \qquad \delta'_{12} := \frac{\delta_{12} \cdot EJ}{\mathbf{m}^3} = 2.364 \qquad \delta'_{22} := \frac{\delta_{22} \cdot EJ}{\mathbf{m}^3} = 5.293 \qquad \delta'_{21} := \delta'_{12}$$

$$m_0 := 100 \; \mathbf{kg} \qquad m_1 = 5 \; m_0 \qquad m_2 = 3 \; m_0 \qquad m'_1 := 5 \qquad m'_2 := 3$$

$$\left(m'_1 \cdot \delta'_{11} \cdot X - 1 \right) \cdot A_1 + m'_2 \cdot \delta'_{12} \cdot X \cdot A_2 = 0$$

$$m'_1 \cdot \delta'_{21} \cdot X \cdot A_1 + \left(m'_2 \cdot \delta'_{22} \cdot X - 1 \right) \cdot A_2 = 0$$

Stąd:

$$\begin{bmatrix} m'_1 \cdot \delta'_{11} \cdot X - 1 & m'_2 \cdot \delta'_{12} \cdot X \\ m'_1 \cdot \delta'_{21} \cdot X & m'_2 \cdot \delta'_{22} \cdot X - 1 \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0$$

Odrzucamy rozwiązanie trywialne:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \neq 0$$

Więc:

$$\left\| \begin{bmatrix} m'_1 \cdot \delta'_{11} \cdot X - 1 & m'_2 \cdot \delta'_{12} \cdot X \\ m'_1 \cdot \delta'_{21} \cdot X & m'_2 \cdot \delta'_{22} \cdot X - 1 \end{bmatrix} \right\| = 0$$

$$\left(m'_1 \cdot \delta'_{11} \cdot X - 1 \right) \cdot \left(m'_2 \cdot \delta'_{22} \cdot X - 1 \right) - m'_1 \cdot \delta'_{21} \cdot X \cdot m'_2 \cdot \delta'_{12} \cdot X = 0$$

$$m'_1 \cdot m'_2 \cdot \left(\delta'_{11} \cdot \delta'_{22} - {\delta'_{12}}^2 \right) \cdot X^2 + \left(-m'_1 \cdot \delta'_{11} - m'_2 \cdot \delta'_{22} \right) \cdot X + 1 = 0$$

$$a := m'_1 \cdot m'_2 \cdot \left(\delta'_{11} \cdot \delta'_{22} - {\delta'_{12}}^2 \right) \qquad b := -m'_1 \cdot \delta'_{11} - m'_2 \cdot \delta'_{22} \qquad c := 1$$

$$\Delta := b^2 - 4 \cdot a \cdot c$$

$$X_1 := \frac{-b - \sqrt{\Delta}}{2\ a} = 0.044 \qquad X_2 := \frac{-b + \sqrt{\Delta}}{2\ a} = 0.273$$

$$\omega_1 := \sqrt{\frac{X_1 \cdot EJ}{m_0 \cdot \mathbf{m}^3}} = 29.385\ \frac{\mathbf{rad}}{\mathbf{s}} \qquad \omega_2 := \sqrt{\frac{X_2 \cdot EJ}{m_0 \cdot \mathbf{m}^3}} = 73.248\ \frac{\mathbf{rad}}{\mathbf{s}}$$

Obliczenie amplitud drgań własnych:

$$\left(m'_1 \cdot \delta'_{11} \cdot X_1 - 1\right) \cdot A_{11} + m'_2 \cdot \delta'_{12} \cdot X_1 \cdot A_{21} = 0$$

$$m'_1 \cdot \delta'_{21} \cdot X_1 \cdot A_{11} + \left(m'_2 \cdot \delta'_{22} \cdot X_1 - 1\right) \cdot A_{21} = 0$$

$$A_{11} := 1 \qquad A_{21} := \frac{-\left(m'_1 \cdot \delta'_{11} \cdot X_1 - 1\right)}{m'_2 \cdot \delta'_{12} \cdot X_1} = 1.723$$

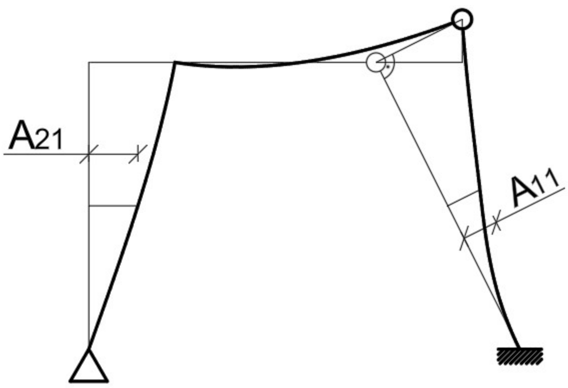
$$\left(m'_1 \cdot \delta'_{11} \cdot X_2 - 1\right) \cdot A_{12} + m'_2 \cdot \delta'_{12} \cdot X_2 \cdot A_{22} = 0$$

$$m'_1 \cdot \delta'_{21} \cdot X_2 \cdot A_{12} + \left(m'_2 \cdot \delta'_{22} \cdot X_2 - 1\right) \cdot A_{22} = 0$$

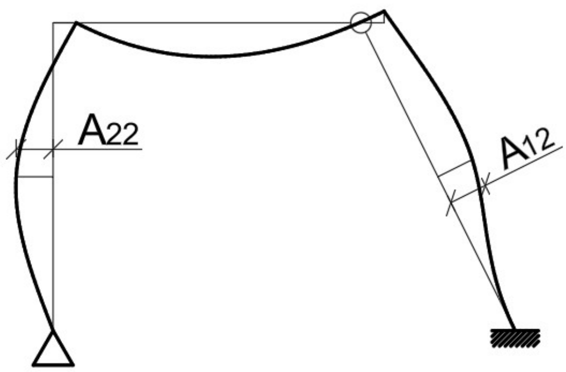
$$A_{12} := 1 \qquad A_{22} := \frac{-\left(m'_1 \cdot \delta'_{11} \cdot X_2 - 1\right)}{m'_2 \cdot \delta'_{12} \cdot X_2} = -0.967$$

Formy drgań własnych

I forma drgań własnych



II forma drgań własnych



Sprawdzenie ortogonalności:

Para 1 i 2: $m_1 \cdot A_{11} \cdot A_{12} + m_2 \cdot A_{21} \cdot A_{22} = 0\ \mathbf{kg}$

Sprawdzenie metodą Dunkerlaya i Reileigh'a:

$$\omega_D := \frac{1}{\sqrt{m_1 \cdot \delta_{11} + m_2 \cdot \delta_{22}}} = 27.272\ \frac{\mathbf{rad}}{\mathbf{s}}$$

$$\delta_1 := m_1 \cdot \delta_{11} + m_2 \cdot \delta_{12}$$

$$\delta_2 := m_1 \cdot \delta_{21} + m_2 \cdot \delta_{22}$$

$$\omega_R := \frac{\sqrt{m_1 \cdot \delta_1 + m_2 \cdot \delta_2}}{\sqrt{m_1 \cdot \delta_1^{\,2} + m_2 \cdot \delta_2^{\,2}}} = 29.535\ \frac{\mathbf{rad}}{\mathbf{s}}$$

$\omega_D = 27.272\ \frac{\mathbf{rad}}{\mathbf{s}} \le\ \omega_1 = 29.385\ \frac{\mathbf{rad}}{\mathbf{s}} \le\ \omega_R = 29.535\ \frac{\mathbf{rad}}{\mathbf{s}}$ Warunek spełniony

Drgania wymuszone

Sprawdzenie warunku rezonansu:

$$0.9\ \omega_1 = 26.447\ \frac{\mathrm{rad}}{\mathrm{s}}\ \le\ \theta = 31.416\ \frac{\mathrm{rad}}{\mathrm{s}}\ \le\ 1.1\ \omega_1 = 32.324\ \frac{\mathrm{rad}}{\mathrm{s}}$$

$$0.9\ \omega_2 = 65.923\ \frac{\mathrm{rad}}{\mathrm{s}}\ \le\ \theta = 31.416\ \frac{\mathrm{rad}}{\mathrm{s}}\ \le\ 1.1\ \omega_2 = 80.573\ \frac{\mathrm{rad}}{\mathrm{s}}$$

Warunek spełniony, rezonans zajdzie w pierwszej formie drgań własnych

Wymuszenie jest na kierunku 2, więc:

$$\delta_{1P} := \delta_{21} \cdot P_0 = 0.01324\ \mathrm{m}$$

$$\delta_{2P} := \delta_{22} \cdot P_0 = 0.02965\ \mathrm{m}$$

Wyznaczenie sił bezwładności:

$$\left(\delta_{11} - \frac{1}{m_1 \cdot \Theta^2}\right) \cdot B_1 + \delta_{12} \cdot B_2 + \delta_{1P} = 0$$

$$\delta_{11} \cdot B_1 + \left(\delta_{12} - \frac{1}{m_2 \cdot \Theta^2}\right) \cdot B_2 + \delta_{2P} = 0$$

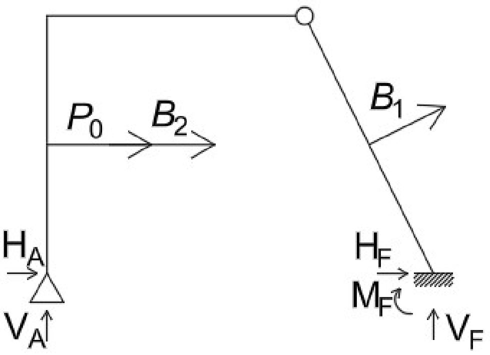
$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} := \begin{bmatrix} \delta_{11} - \dfrac{1}{m_1 \cdot \theta^2} & \delta_{12} \\ \delta_{21} & \delta_{22} - \dfrac{1}{m_2 \cdot \theta^2} \end{bmatrix}^{-1} \cdot -\begin{bmatrix} \delta_{1P} \\ \delta_{2P} \end{bmatrix}$$

$$B_1 = -56.009\ \mathrm{kN}$$

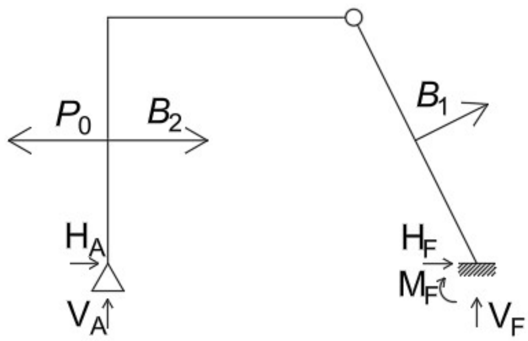
$$B_2 = -55.419\ \mathrm{kN}$$

Dwa warianty obciążenia statycznego:

D1



D2

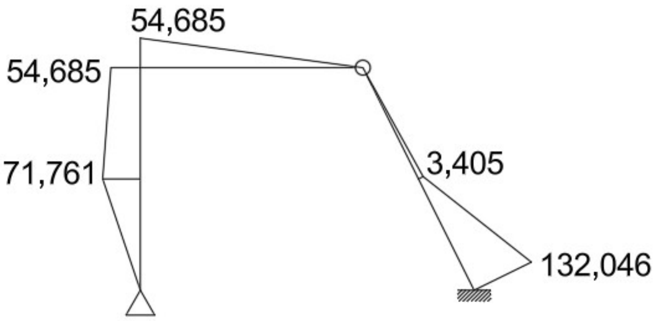


$$M_{D1} := M_{ost1} \cdot B_1 + M_{ost2} \cdot \left(B_2 + P_0\right)$$

$$M_{D2} := M_{ost1} \cdot B_1 + M_{ost2} \cdot \left(B_2 - P_0\right)$$

$$M_{D1} = \begin{bmatrix} 0 & -71.761 \\ -71.761 & -54.685 \\ -54.685 & 0 \\ 0 & -3.405 \\ -3.405 & -132.046 \end{bmatrix} \mathrm{kNm} \qquad M_{D2} = \begin{bmatrix} 0 & -100.105 \\ -100.105 & -67.372 \\ -67.372 & 0 \\ 0 & -15.89 \\ -15.89 & -157.015 \end{bmatrix} \mathrm{kNm}$$

M_{D1} [kNm]



Obliczenie tnących (metoda równoważenia prętów):

$$T_{AB} := \frac{71.761\ \mathbf{kNm}}{2\ \mathbf{m}} = 35.881\ \mathbf{kN}$$

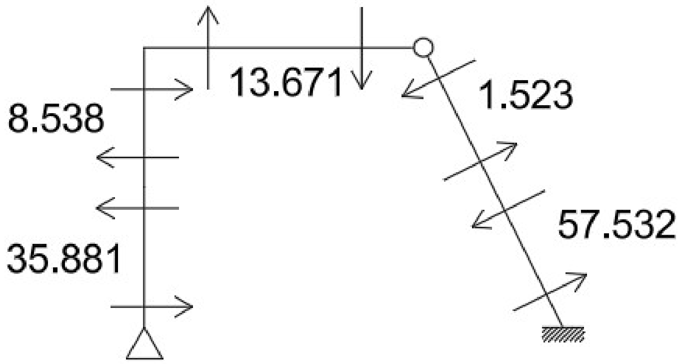
$$T_{BC} := \frac{71.761\ \mathbf{kNm} - 54.685\ \mathbf{kNm}}{2\ \mathbf{m}} = 8.538\ \mathbf{kN}$$

$$T_{CD} := \frac{54.685\ \mathbf{kNm}}{4\ \mathbf{m}} = 13.671\ \mathbf{kN}$$

$$T_{DE} := \frac{3.405\ \mathbf{kNm}}{2.236\ \mathbf{m}} = 1.523\ \mathbf{kN}$$

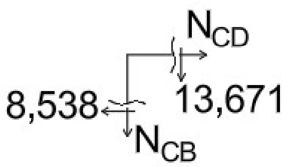
$$T_{EF} := \frac{132.046\ \mathbf{kNm} - 3.405\ \mathbf{kNm}}{2.236\ \mathbf{m}} = 57.532\ \mathbf{kN}$$

T_{D1} [kN]



Obliczenie Normalnych (metoda równoważenia węzłów):

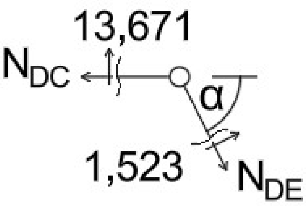
Węzeł C



$$\Sigma X = -8.538\ kN + N_{CD} = 0 \qquad N_{CD} := 8.538\ \mathbf{kN}$$

$$\Sigma Y = -13.671\ kN - N_{CB} = 0 \qquad N_{CB} := -13.671\ \mathbf{kN}$$

Węzeł D



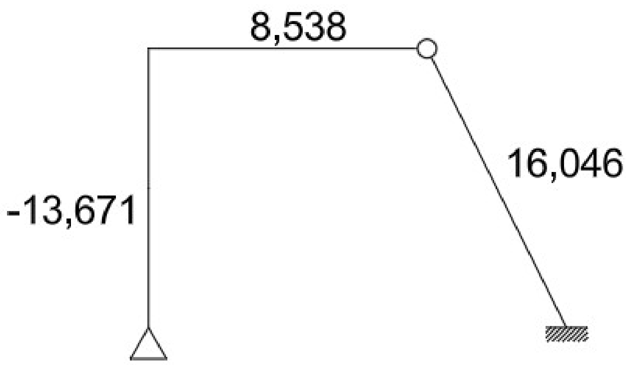
$$\Sigma Y = 13.671\ kN + 1.523\ kN \cdot cos\alpha - N_{DE} \cdot sin\alpha = 0$$

$$N_{DE} := \frac{13.671\ \mathbf{kN} + 1.523\ \mathbf{kN} \cdot cos\alpha}{sin\alpha} = 16.046\ \mathbf{kN}$$

Sprawdzenie:

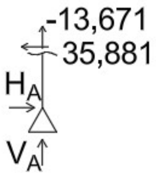
$$\Sigma X := -N_{CD} + N_{DE} \cdot cos\alpha + 1.523\ \mathbf{kN} \cdot sin\alpha = 0\ \mathbf{kN}$$

N_{D1} [kN]



Obliczenie Reakcji (metoda równoważenia węzłów):

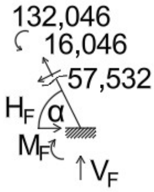
Węzeł A



$H_A := 35.881\ \mathrm{kN}$

$V_A := 13.671\ \mathrm{kN}$

Węzeł F



$\Sigma Y = V_F + 16.046\ kN \cdot sin\alpha - 57.572\ kN \cdot cos\alpha = 0$

$V_F := -16.046\ \mathrm{kN} \cdot sin\alpha + 57.572\ \mathrm{kN} \cdot cos\alpha = 11.395\ \mathrm{kN}$

$\Sigma X = H_F - 16.046\ kN \cdot cos\alpha - 57.572\ kN \cdot sin\alpha = 0$

$H_F := 16.046\ \mathrm{kN} \cdot cos\alpha + 57.572\ \mathrm{kN} \cdot sin\alpha = 58.67\ \mathrm{kN}$

$M_F := 132.046\ \mathrm{kNm}$

Sprawdzenie reakcji:

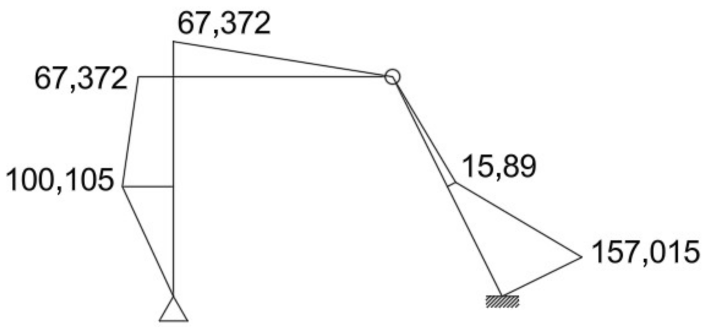
$\Sigma X := H_A + H_F + P_0 + B_2 + B_1 \cdot sin\alpha = 0.036\ \mathrm{kN}$

$\Sigma Y := V_A + V_F + B_1 \cdot cos\alpha = 0.018\ \mathrm{kN}$

$\Sigma M_D := V_A \cdot 4\ \mathrm{m} - H_A \cdot 4\ \mathrm{m} - P_0 \cdot 2\ \mathrm{m} - B_2 \cdot 2\ \mathrm{m} - B_1 \cdot 2.236\ \mathrm{m} - H_F \cdot 4\ \mathrm{m} - V_F \cdot 2\ \mathrm{m} + M_F = -0.191\ \mathrm{kNm}$

$\dfrac{0.191\ \mathrm{kNm}}{132.046\ \mathrm{kNm}} \cdot 100\% = 0.145\%$ Warunek spełniony

MD2 [kNm]



Obliczenie tnących (metoda równoważenia prętów):

$T_{AB} := \dfrac{100.105\ \mathrm{kNm}}{2\ \mathrm{m}} = 50.053\ \mathrm{kN}$

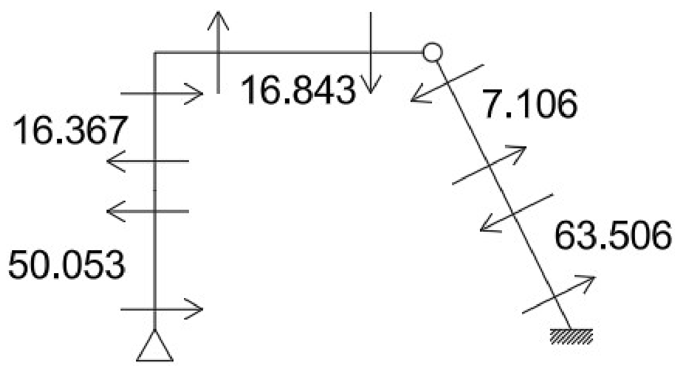
$T_{BC} := \dfrac{100.105\ \mathrm{kNm} - 67.372\ \mathrm{kNm}}{2\ \mathrm{m}} = 16.367\ \mathrm{kN}$

$T_{CD} := \dfrac{67.372\ \mathrm{kNm}}{4\ \mathrm{m}} = 16.843\ \mathrm{kN}$

$T_{DE} := \dfrac{15.89\ \mathrm{kNm}}{2.236\ \mathrm{m}} = 7.106\ \mathrm{kN}$

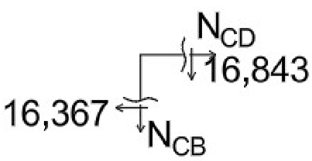
$T_{EF} := \dfrac{157.015\ \mathrm{kNm} - 15.015\ \mathrm{kNm}}{2.236\ \mathrm{m}} = 63.506\ \mathrm{kN}$

TD2 [kN]



Obliczenie Normalnych (metoda równoważenia węzłów):

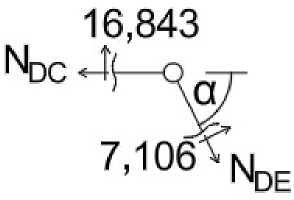
Węzeł C



$\Sigma X = -16.367\ kN + N_{CD} = 0$ $N_{CD} := 16.367\ \mathrm{kN}$

$\Sigma Y = -16.843\ kN - N_{CB} = 0$ $N_{CB} := -16.843\ \mathrm{kN}$

Węzeł D



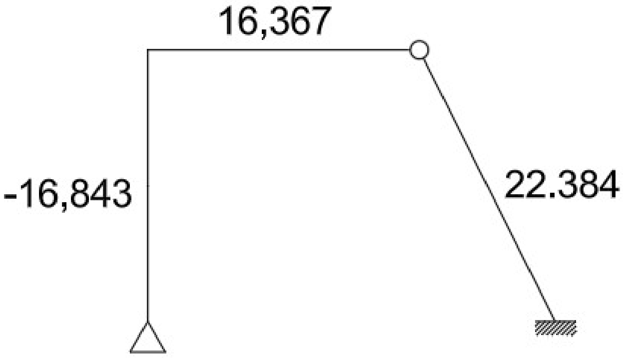
$$\Sigma Y = 16.843\ kN + 7.106\ kN \cdot cos\alpha - N_{DE} \cdot sin\alpha = 0$$

$$N_{DE} := \frac{16.843\ \mathrm{kN} + 7.106\ \mathrm{kN} \cdot cos\alpha}{sin\alpha} = 22.384\ \mathrm{kN}$$

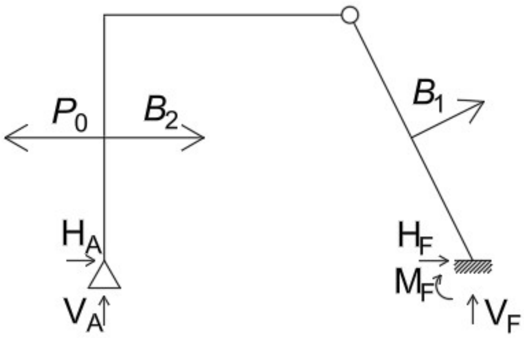
Sprawdzenie:

$$\Sigma X := -N_{CD} + N_{DE} \cdot cos\alpha + 7.106\ \mathrm{kN} \cdot sin\alpha = -0.001\ \mathrm{kN}$$

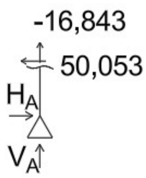
ND1 [kN]



Obliczenie Reakcji (metoda równoważenia węzłów):



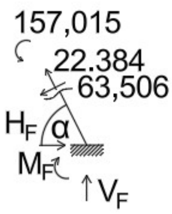
Węzeł A



$$H_A := 50.053\ \mathrm{kN}$$

$$V_A := 16.843\ \mathrm{kN}$$

Węzeł F



$$\Sigma Y = V_F + 22.384\ kN \cdot sin\alpha - 63.506\ kN \cdot cos\alpha = 0$$

$$V_F := -22.384\ \mathrm{kN} \cdot sin\alpha + 63.506\ \mathrm{kN} \cdot cos\alpha = 8.38\ \mathrm{kN}$$

$$\Sigma X = H_F - 22.384\ kN \cdot cos\alpha - 63.506\ kN \cdot sin\alpha = 0$$

$$H_F := 22.384\ \mathrm{kN} \cdot cos\alpha + 63.506\ \mathrm{kN} \cdot sin\alpha = 66.812\ \mathrm{kN}$$

$$M_F := 157.015\ \mathrm{kNm}$$

Sprawdzenie reakcji:

$$\Sigma X := H_A + H_F - P_0 + B_2 + B_1 \cdot sin\alpha = 0.35\ \mathrm{kN}$$

$$\Sigma Y := V_A + V_F + B_1 \cdot cos\alpha = 0.175\ \mathrm{kN}$$

$$\Sigma M_D := V_A \cdot 4\ \mathrm{m} - H_A \cdot 4\ \mathrm{m} + P_0 \cdot 2\ \mathrm{m} - B_2 \cdot 2\ \mathrm{m} - B_1 \cdot 2.236\ \mathrm{m} - H_F \cdot 4\ \mathrm{m} - V_F \cdot 2\ \mathrm{m} + M_F = -1.759\ \mathrm{kNm}$$

$$\frac{1.759\ \mathrm{kNm}}{157.015\ \mathrm{kNm}} \cdot 100\% = 1.12\%$$

Warunek spełniony