Politechnika Lubelska

Wydział Budownictwa i Architektury

Mechanika Budowli II

Projekt 1: Metoda przemieszczeń

Wykonał: Sprawdził:

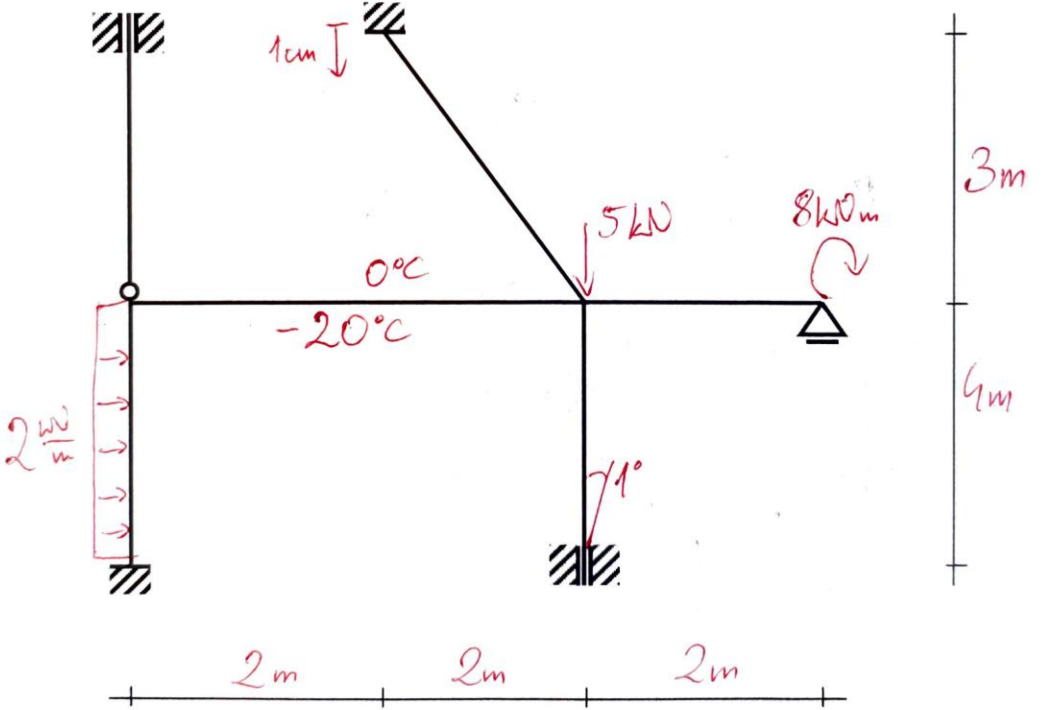
…………………….. dr inż. Jakub Gontarz

2025

ĆWICZENIE PROJEKTOWE NR 1 Z MECHANIKI BUDOWLI

PRZYKŁADOWY TEMAT

Ramę geometrycznie niewyznaczalną rozwiązać metodą przemieszczeń.

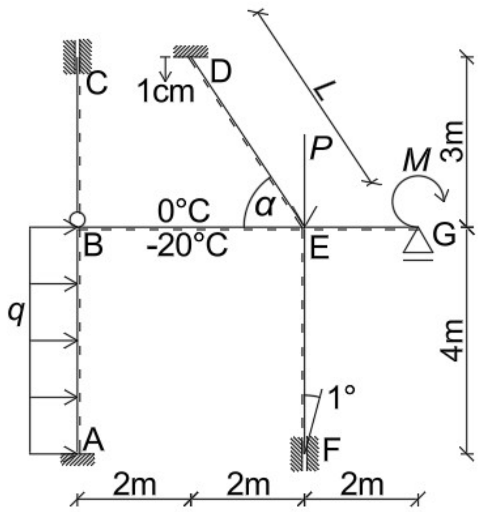


Przekrój: I 120

E = 210 GPa

$\alpha_t = 1{,}2 \cdot 10^{-5}$ 1/K

$kNm := kN \cdot m$



$E := 210\ GPa$

$\alpha_t := 1.2 \cdot 10^{-5}\ \frac{1}{K}$

$P := 5\ kN$

Przekrój: IPN 120

$M := 8\ kNm$

$J := 328\ cm^4$

$q := 2\ \frac{kN}{m}$

$EJ := E \cdot J = 688.8\ kN \cdot m^2$

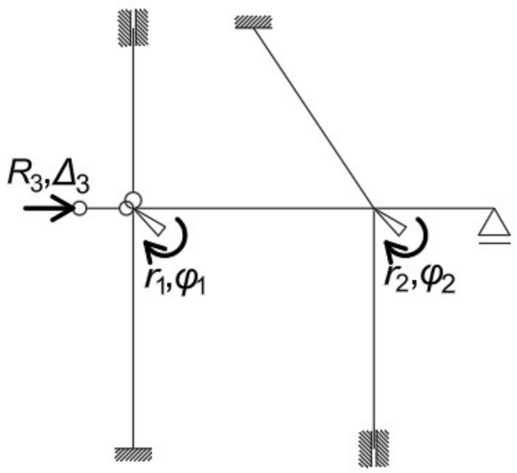
$h := 120\ mm$

Długość pręta ukośnego:

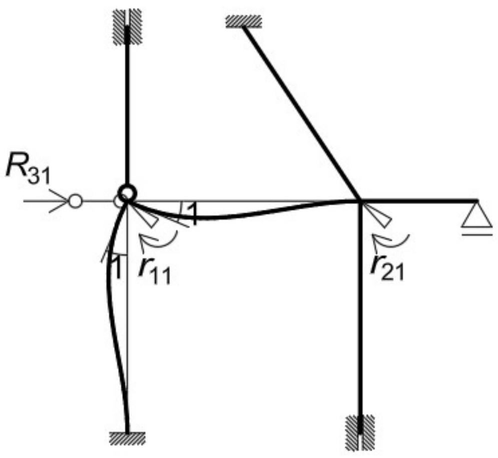
$L := \sqrt{(2\ m)^2 + (3\ m)^2} = 3.606\ m$

$sin\alpha := \frac{3\ m}{L} = 0.832$ $\qquad cos\alpha := \frac{2\ m}{L} = 0.555$

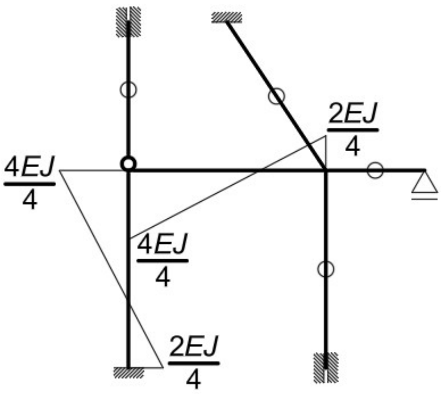
UPMP



Stan 1

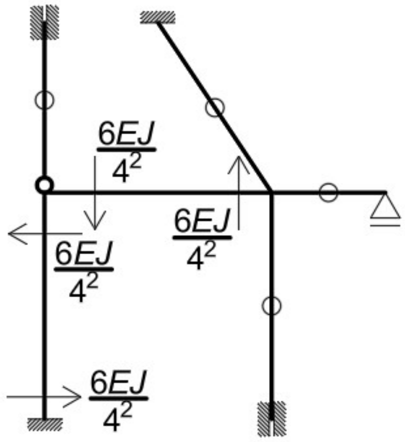


M1 [kNm]



$$M_1 := \begin{bmatrix} \frac{2\ EJ}{4\ \mathbf{m}} & \frac{-4\ EJ}{4\ \mathbf{m}} \\ 0 & 0 \\ \frac{4\ EJ}{4\ \mathbf{m}} & \frac{-2\ EJ}{4\ \mathbf{m}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 344.4 & -688.8 \\ 0 & 0 \\ 688.8 & -344.4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{kNm}$$

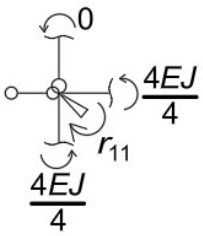
T1 [kN]



$$T_1 := \begin{bmatrix} \frac{-6\ EJ}{(4\ \mathbf{m})^2} & \frac{-6\ EJ}{(4\ \mathbf{m})^2} \\ 0 & 0 \\ \frac{-6\ EJ}{(4\ \mathbf{m})^2} & \frac{-6\ EJ}{(4\ \mathbf{m})^2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -258.3 & -258.3 \\ 0 & 0 \\ -258.3 & -258.3 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{kN}$$

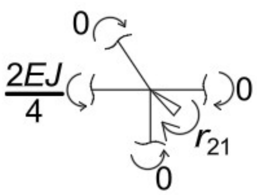
<u>Obliczenia reakcji</u>

Węzeł B



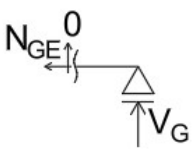
$$\Sigma M = r_{11} - \frac{4\ EJ}{4} - \frac{4\ EJ}{4} = 0 \qquad r_{11} := \frac{4\ EJ}{4\ \mathbf{m}} + \frac{4\ EJ}{4\ \mathbf{m}} = 2\ \frac{EJ}{\mathbf{m}} \qquad r_{11} = 1377.6\ \mathbf{kNm}$$

Węzeł E



$$\Sigma M = r_{21} - \frac{2\ EJ}{4} = 0 \qquad r_{21} := \frac{2\ EJ}{4\ \mathbf{m}} = 0.5\ \frac{EJ}{\mathbf{m}} \qquad r_{21} = 344.4\ \mathbf{kNm}$$

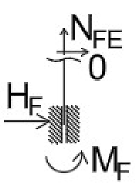
Węzeł G



$$\Sigma X = -N_{GE} = 0 \qquad N_{GE} := 0 \qquad N_{EG} := N_{GE}$$

Siła N jest równa 0 w tym pręcie
we wszystkich stanach

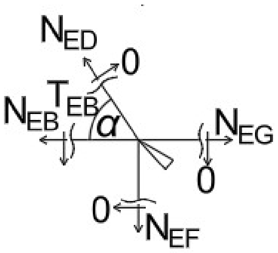
Węzeł F



$$\Sigma Y = N_{FE} = 0 \qquad N_{FE} := 0 \qquad N_{EF} := N_{FE}$$

Siła N jest równa 0 w tym pręcie
we wszystkich stanach

Węzeł E

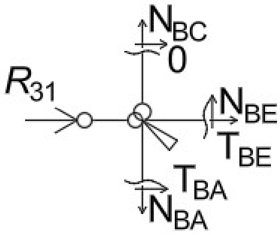


$$T_{EB} := \frac{6\ EJ}{(4\ \mathrm{m})^2}$$

$$\Sigma Y = -N_{EF} - T_{EB} + N_{ED} \cdot sin\alpha = 0 \qquad N_{ED} := \frac{N_{EF} + T_{EB}}{sin\alpha} = 0.451\ \frac{EJ}{\mathrm{m}^2}$$

$$\Sigma X = -N_{EB} + N_{EG} - N_{ED} \cdot cos\alpha = 0 \qquad N_{EB} := N_{EG} - N_{ED} \cdot cos\alpha = -0.25\ \frac{EJ}{\mathrm{m}^2} \qquad N_{BE} := N_{EB}$$

Węzeł B

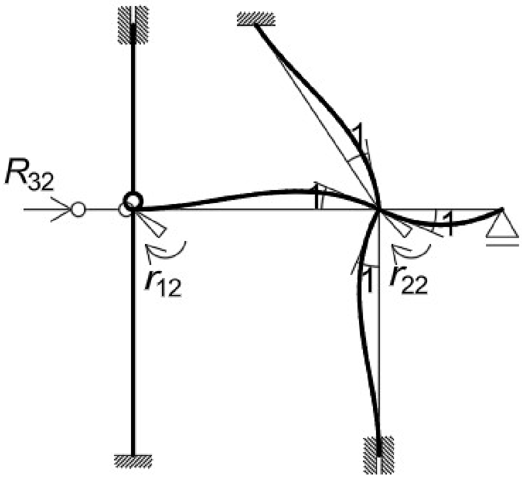


$$T_{BA} := \frac{6\ EJ}{(4\ \mathrm{m})^2}$$

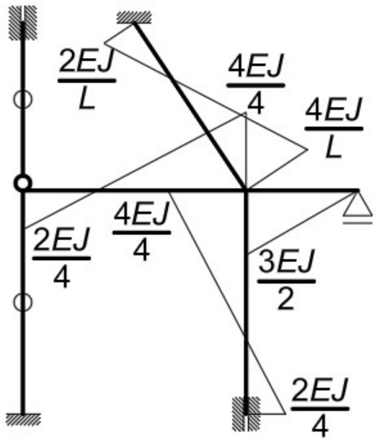
$$\Sigma X = R_{31} + N_{BE} + T_{BA} = 0 \qquad R_{31} := -N_{BE} - T_{BA} = -0.125\ \frac{EJ}{\mathrm{m}^2}$$

$$R_{31} = -86.1\ \mathrm{kN}$$

Stan 2

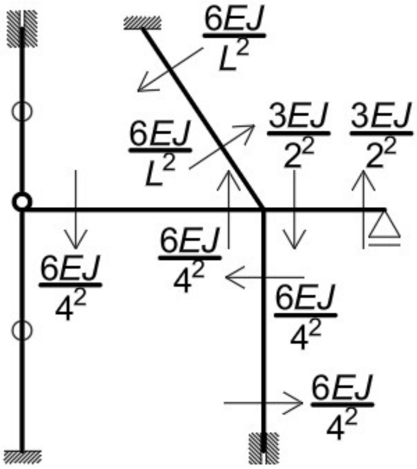


M2 [kN]



$$M_2 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{2\ EJ}{4\ \mathrm{m}} & \frac{-4\ EJ}{4\ \mathrm{m}} \\ \frac{2\ EJ}{L} & \frac{-4\ EJ}{L} \\ \frac{4\ EJ}{4\ \mathrm{m}} & \frac{-2\ EJ}{4\ \mathrm{m}} \\ \frac{3\ EJ}{2\ \mathrm{m}} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 344.4 & -688.8 \\ 382.077 & -764.155 \\ 688.8 & -344.4 \\ 1033.2 & 0 \end{bmatrix}\ \mathrm{kNm}$$

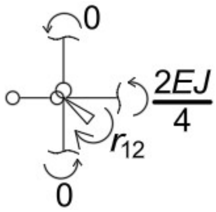
T2 [kN/m]



$$T_2 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{-6\ EJ}{(4\ \mathrm{m})^2} & \dfrac{-6\ EJ}{(4\ \mathrm{m})^2} \\ \dfrac{-6\ EJ}{L^2} & \dfrac{-6\ EJ}{L^2} \\ \dfrac{-6\ EJ}{(4\ \mathrm{m})^2} & \dfrac{-6\ EJ}{(4\ \mathrm{m})^2} \\ \dfrac{-3\ EJ}{(2\ \mathrm{m})^2} & \dfrac{-3\ EJ}{(2\ \mathrm{m})^2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -258.3 & -258.3 \\ -317.908 & -317.908 \\ -258.3 & -258.3 \\ -516.6 & -516.6 \end{bmatrix} \mathbf{kN}$$

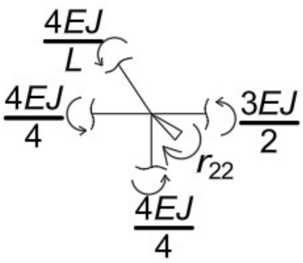
<u>Obliczenia reakcji</u>

Węzeł B



$$\Sigma M = r_{12} - \frac{2\ EJ}{4} = 0 \qquad r_{12} := \frac{2\ EJ}{4\ \mathrm{m}} = 0.5\ \frac{EJ}{\mathrm{m}} \qquad r_{12} = 344.4\ \mathbf{kNm}$$

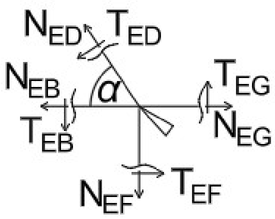
Węzeł E



$$\Sigma M = r_{22} - \frac{4\ EJ}{4} - \frac{4\ EJ}{4} - \frac{3\ EJ}{2} - \frac{4\ EJ}{L} = 0$$

$$r_{22} := \frac{4\ EJ}{4\ \mathrm{m}} + \frac{4\ EJ}{4\ \mathrm{m}} + \frac{3\ EJ}{2\ \mathrm{m}} + \frac{4\ EJ}{L} = 4.609\ \frac{EJ}{\mathrm{m}} \qquad r_{22} = 3174.955\ \mathbf{kNm}$$

Węzeł E



$$T_{EB} := \frac{6\ EJ}{(4\ \mathrm{m})^2} \qquad T_{ED} := \frac{6\ EJ}{L^2} \qquad T_{EF} := \frac{6\ EJ}{(4\ \mathrm{m})^2} \qquad T_{EG} := \frac{3\ EJ}{(2\ \mathrm{m})^2}$$

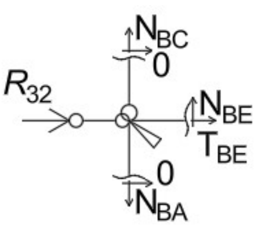
$$\Sigma Y = -N_{EF} - T_{EB} + T_{EG} + N_{ED} \cdot sin\alpha - T_{ED} \cdot cos\alpha = 0$$

$$N_{ED} := \frac{N_{EF} + T_{EB} - T_{EG} + T_{ED} \cdot cos\alpha}{sin\alpha} = -0.143\ \frac{EJ}{\mathrm{m}^2}$$

$$\Sigma X = -N_{EB} + N_{EG} - N_{ED} \cdot cos\alpha - T_{ED} \cdot sin\alpha + T_{EF} = 0$$

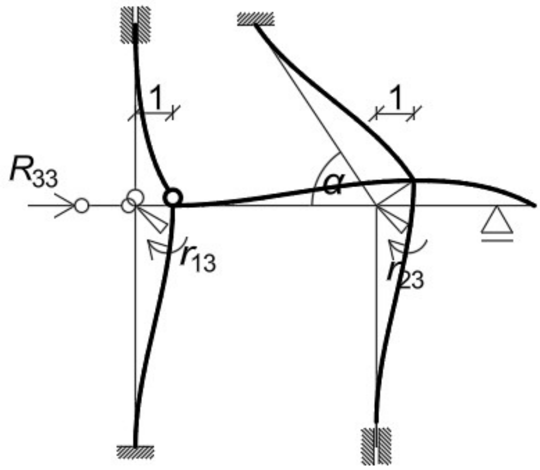
$$N_{EB} := N_{EG} - N_{ED} \cdot cos\alpha - T_{ED} \cdot sin\alpha + T_{EF} = 0.07\ \frac{EJ}{\mathrm{m}^2} \qquad N_{BE} := N_{EB}$$

Węzeł B

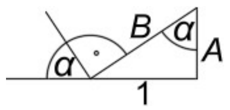


$$\Sigma X = R_{32} + N_{BE} = 0 \qquad R_{32} := -N_{BE} = -0.07\ \frac{EJ}{\mathrm{m}^2} \qquad R_{32} = -48.423\ \mathbf{kN}$$

Stan 3



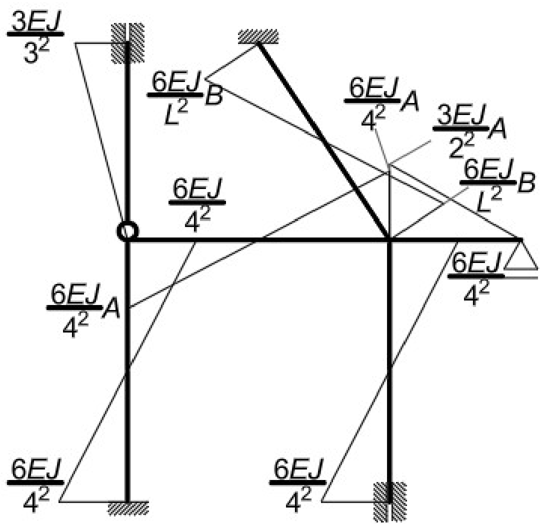
Obliczenie proporcji:



$$A := \frac{2\ \mathrm{m}}{3\ \mathrm{m}} = 0.667$$

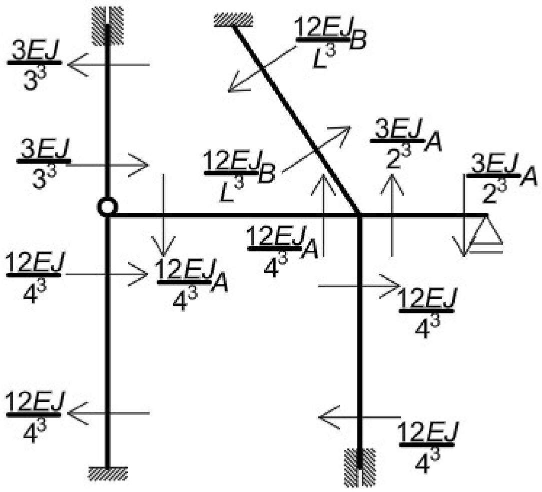
$$B := \frac{L}{3\ \mathrm{m}} = 1.202$$

M3 [kN]



$$M_3 := \begin{bmatrix} \frac{-6\ EJ}{(4\ \mathrm{m})^2} & \frac{6\ EJ}{(4\ \mathrm{m})^2} \\ 0 & \frac{-3\ EJ}{(3\ \mathrm{m})^2} \\ \frac{6\ EJ}{(4\ \mathrm{m})^2}\ A & \frac{-6\ EJ}{(4\ \mathrm{m})^2}\ A \\ \frac{6\ EJ}{L^2}\ B & \frac{-6\ EJ}{L^2}\ B \\ \frac{-6\ EJ}{(4\ \mathrm{m})^2} & \frac{6\ EJ}{(4\ \mathrm{m})^2} \\ \frac{-3\ EJ}{(2\ \mathrm{m})^2}\ A & 0 \end{bmatrix} = ?\ \mathrm{kN}$$

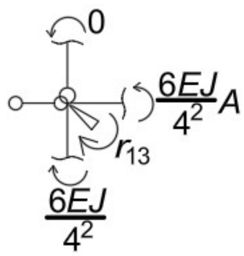
T3 [kN/m]



$$T_3 := \begin{bmatrix} \frac{12\ EJ}{(4\ \mathrm{m})^3} & \frac{12\ EJ}{(4\ \mathrm{m})^3} \\ \frac{-3\ EJ}{(3\ \mathrm{m})^3} & \frac{-3\ EJ}{(3\ \mathrm{m})^3} \\ \frac{-12\ EJ}{(4\ \mathrm{m})^3}\ A & \frac{-12\ EJ}{(4\ \mathrm{m})^3}\ A \\ \frac{-12\ EJ}{L^3}\ B & \frac{-12\ EJ}{L^3}\ B \\ \frac{12\ EJ}{(4\ \mathrm{m})^3} & \frac{12\ EJ}{(4\ \mathrm{m})^3} \\ \frac{3\ EJ}{(2\ \mathrm{m})^3}\ A & \frac{3\ EJ}{(2\ \mathrm{m})^3}\ A \end{bmatrix} = \begin{bmatrix} 129.15 & 129.15 \\ -76.533 & -76.533 \\ -86.1 & -86.1 \\ -211.938 & -211.938 \\ 129.15 & 129.15 \\ 172.2 & 172.2 \end{bmatrix} \frac{\mathrm{kN}}{\mathrm{m}}$$

Obliczenia reakcji

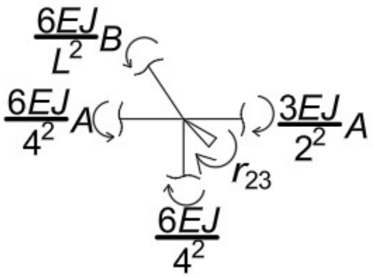
Węzeł B



$$\Sigma M = r_{13} - \frac{6\ EJ}{4^2} \cdot A + \frac{6\ EJ}{4^2} = 0$$

$$r_{13} := \frac{6\ EJ}{(4\ \mathrm{m})^2} \cdot A - \frac{6\ EJ}{(4\ \mathrm{m})^2} = -0.125\ \frac{EJ}{\mathrm{m}^2} \qquad r_{13} = -86.1\ \mathrm{kN}$$

Węzeł E

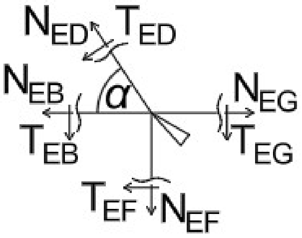


$$\Sigma M = r_{22} - \frac{6\ EJ}{L^2} \cdot B - \frac{6\ EJ}{4^2} \cdot A + \frac{6\ EJ}{4^2} + \frac{3\ EJ}{2^2} \cdot A = 0$$

$$r_{23} := \frac{6\ EJ}{L^2} \cdot B + \frac{6\ EJ}{(4\ \text{m})^2} \cdot A - \frac{6\ EJ}{(4\ \text{m})^2} - \frac{3\ EJ}{(2\ \text{m})^2} \cdot A = -0.07\ \frac{EJ}{\text{m}^2}$$

$$r_{23} = -48.423\ \text{kN}$$

Węzeł E



$$T_{EB} := \frac{12\ EJ}{(4\ \text{m})^3}\ A \qquad T_{ED} := \frac{12\ EJ}{L^3}\ B \qquad T_{EF} := \frac{12\ EJ}{(4\ \text{m})^3}$$

$$T_{EG} := \frac{3\ EJ}{(2\ \text{m})^3}\ A \qquad T_{BC} := \frac{3\ EJ}{(3\ \text{m})^3} \qquad T_{BA} := \frac{12\ EJ}{(4\ \text{m})^3}$$

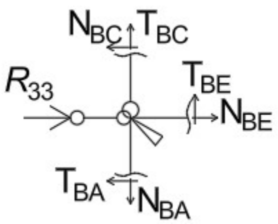
$$\Sigma Y = -N_{EF} - T_{EB} - T_{EG} + N_{ED} \cdot sin\alpha - T_{ED} \cdot cos\alpha = 0$$

$$N_{ED} := \frac{N_{EF} + T_{EB} + T_{EG} + T_{ED} \cdot cos\alpha}{sin\alpha} = 0.656\ \frac{EJ}{\text{m}^3}$$

$$\Sigma X = -N_{EB} + N_{EG} - N_{ED} \cdot cos\alpha - T_{ED} \cdot sin\alpha - T_{EF} = 0$$

$$N_{EB} := N_{EG} - N_{ED} \cdot cos\alpha - T_{ED} \cdot sin\alpha - T_{EF} = -0.807\ \frac{EJ}{\text{m}^3} \qquad N_{BE} := N_{EB}$$

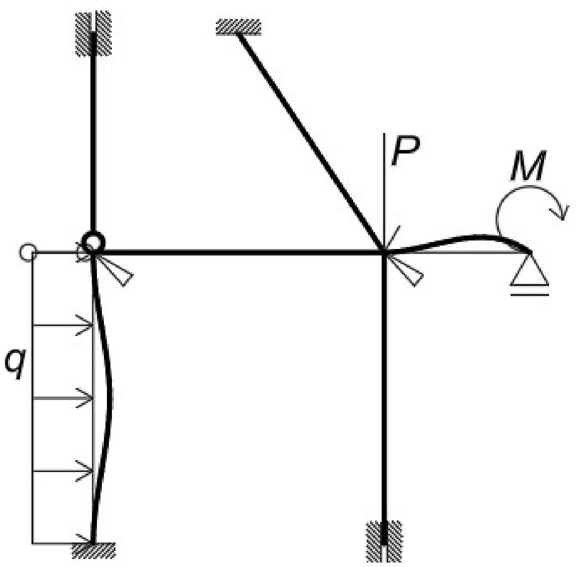
Węzeł B



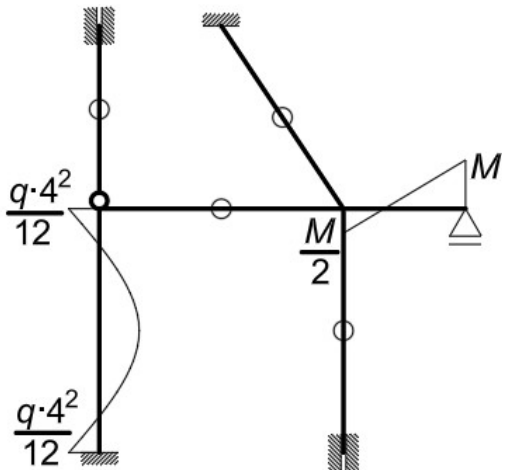
$$\Sigma X = R_{33} + N_{BE} - T_{BC} - T_{BA} = 0$$

$$R_{33} := -N_{BE} + T_{BC} + T_{BA} = 1.106\ \frac{EJ}{\text{m}^3} \qquad R_{33} = 761.752\ \frac{\text{kN}}{\text{m}}$$

Stan p

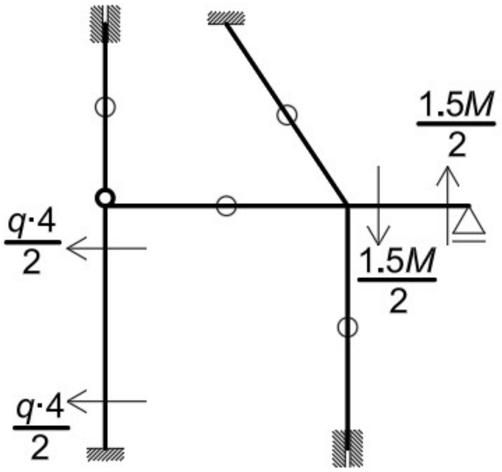


Mp [kNm]



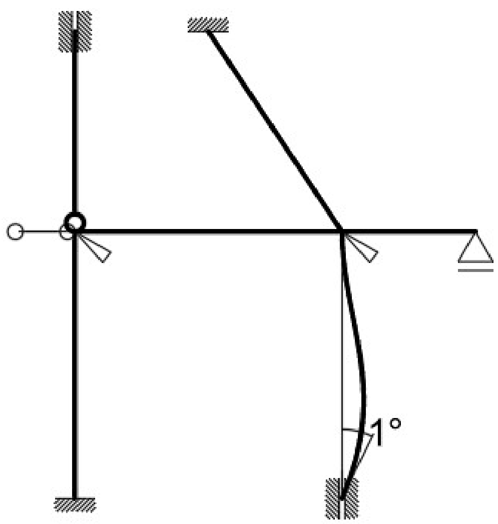
$$M_p := \begin{bmatrix} \frac{-q \cdot (4\ \mathrm{m})^2}{12} & \frac{-q \cdot (4\ \mathrm{m})^2}{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{M}{2} & -M \end{bmatrix} = \begin{bmatrix} -2.667 & -2.667 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 4 & -8 \end{bmatrix} \mathbf{kNm}$$

Tp [kN]

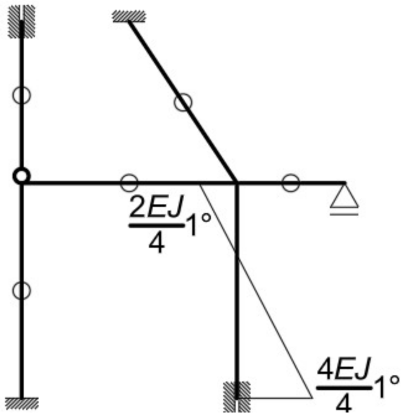


$$T_p := \begin{bmatrix} \frac{q \cdot 4\ \mathrm{m}}{2} & \frac{-q \cdot 4\ \mathrm{m}}{2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{-1.5\ M}{2\ \mathrm{m}} & \frac{-1.5\ M}{2\ \mathrm{m}} \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -6 & -6 \end{bmatrix} \mathbf{kN}$$

Stan φ

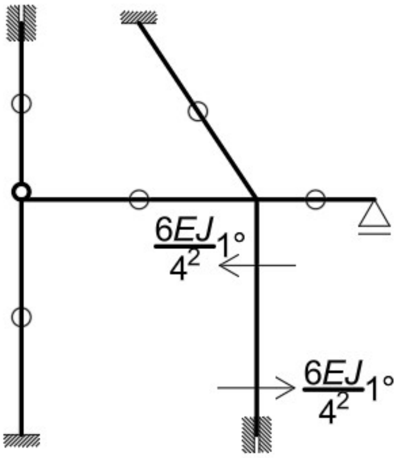


Mφ [kNm]



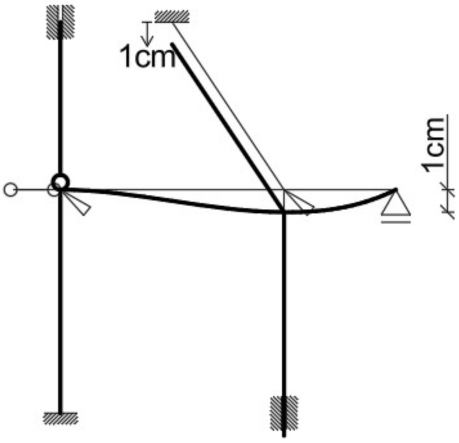
$$M_{\varphi} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{2\ EJ}{4\ \mathbf{m}}\ 1^{\circ} & \dfrac{-4\ EJ}{4\ \mathbf{m}}\ 1^{\circ} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 6.011 & -12.022 \\ 0 & 0 \end{bmatrix} \mathbf{kNm}$$

Tφ [kN]

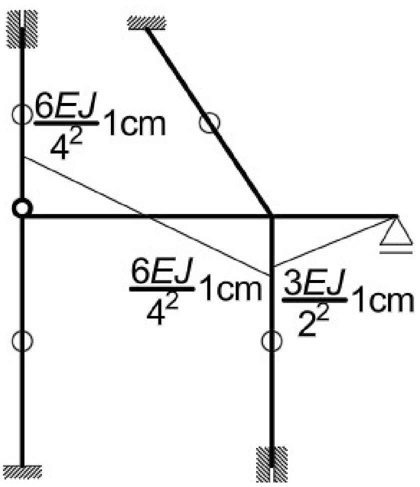


$$T_{\varphi} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{-6\ EJ}{(4\ \mathbf{m})^2}\ 1^{\circ} & \dfrac{-6\ EJ}{(4\ \mathbf{m})^2}\ 1^{\circ} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -4.508 & -4.508 \\ 0 & 0 \end{bmatrix} \mathbf{kN}$$

Stan Δ

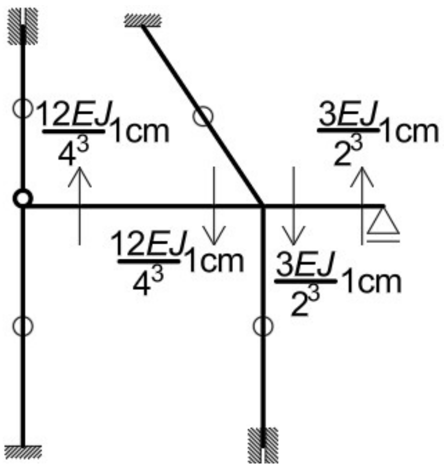


MΔ [kNm]



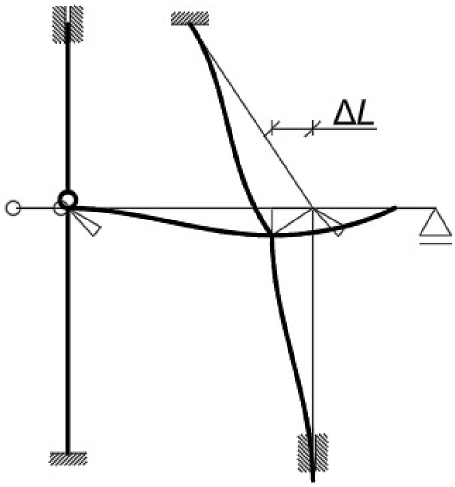
$$M_{\Delta} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{-6\ EJ}{(4\ \mathbf{m})^2}\ 1\ \mathbf{cm} & \dfrac{6\ EJ}{(4\ \mathbf{m})^2}\ 1\ \mathbf{cm} \\ 0 & 0 \\ 0 & 0 \\ \dfrac{3\ EJ}{(2\ \mathbf{m})^2}\ 1\ \mathbf{cm} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -2.583 & 2.583 \\ 0 & 0 \\ 0 & 0 \\ 5.166 & 0 \end{bmatrix} \mathbf{kNm}$$

TΔ [kN]



$$T_{\Delta} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{12\ EJ}{(4\ \mathrm{m})^3}\ 1\ \mathrm{cm} & \frac{12\ EJ}{(4\ \mathrm{m})^3}\ 1\ \mathrm{cm} \\ 0 & 0 \\ 0 & 0 \\ \frac{-3\ EJ}{(2\ \mathrm{m})^3}\ 1\ \mathrm{cm} & \frac{-3\ EJ}{(2\ \mathrm{m})^3}\ 1\ \mathrm{cm} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.292 & 1.292 \\ 0 & 0 \\ 0 & 0 \\ -2.583 & -2.583 \end{bmatrix} \mathrm{kN}$$

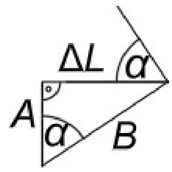
Stan t0



Obliczenie proporcji:

$$t_0 := \frac{20\ \mathrm{K} + 0\ \mathrm{K}}{2} = 10\ \mathrm{K}$$

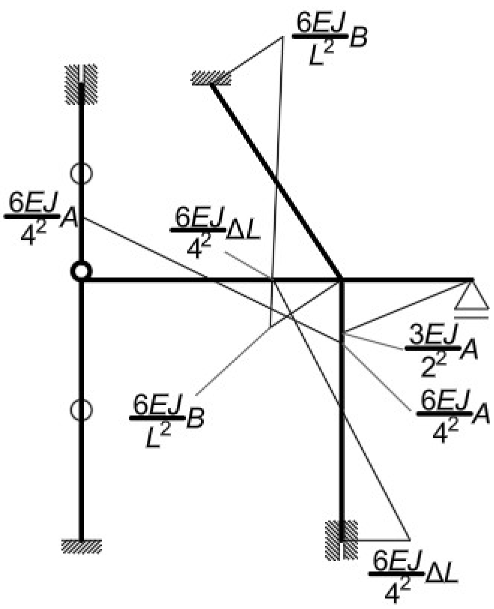
$$\Delta L := \alpha_t \cdot t_0 \cdot 4\ \mathrm{m} = 0.00048\ \mathrm{m}$$



$$\frac{A}{\Delta L} = \frac{2\ \mathrm{m}}{3\ \mathrm{m}} \qquad A := \frac{2\ \mathrm{m}}{3\ \mathrm{m}}\ \Delta L = 0.00032\ \mathrm{m}$$

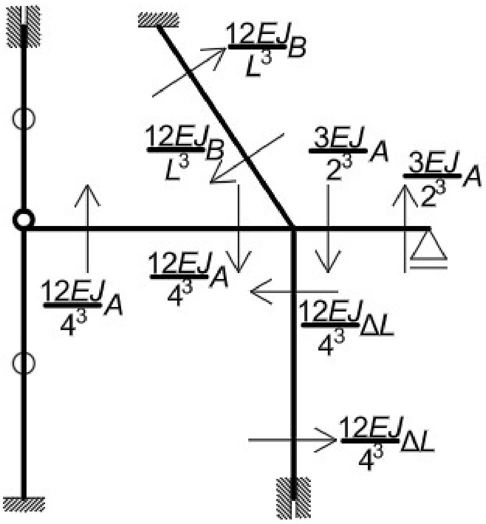
$$\frac{B}{\Delta L} = \frac{L}{3\ \mathrm{m}} \qquad B := \frac{L}{3\ \mathrm{m}}\ \Delta L = 0.000577\ \mathrm{m}$$

Mt0 [kNm]



$$M_{t0} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{-6\ EJ}{(4\ \mathrm{m})^2}\ A & \frac{6\ EJ}{(4\ \mathrm{m})^2}\ A \\ \frac{-6\ EJ}{L^2}\ B & \frac{6\ EJ}{L^2}\ B \\ \frac{6\ EJ}{(4\ \mathrm{m})^2}\ \Delta L & \frac{-6\ EJ}{(4\ \mathrm{m})^2}\ \Delta L \\ \frac{3\ EJ}{(2\ \mathrm{m})^2}\ A & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.083 & 0.083 \\ -0.183 & 0.183 \\ 0.124 & -0.124 \\ 0.165 & 0 \end{bmatrix} \mathrm{kNm}$$

Tt0 [kNm]

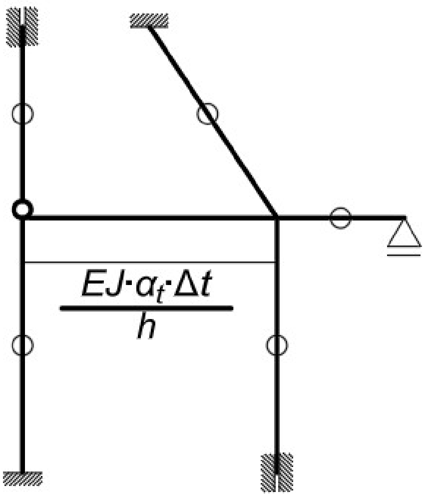


$$T_{t0} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{12\ EJ}{(4\ \mathrm{m})^3}\ A & \dfrac{12\ EJ}{(4\ \mathrm{m})^3}\ A \\ \dfrac{12\ EJ}{L^3}\ B & \dfrac{12\ EJ}{L^3}\ B \\ \dfrac{-12\ EJ}{(4\ \mathrm{m})^3}\ \Delta L & \dfrac{-12\ EJ}{(4\ \mathrm{m})^3}\ \Delta L \\ \dfrac{-3\ EJ}{(2\ \mathrm{m})^3}\ A & \dfrac{-3\ EJ}{(2\ \mathrm{m})^3}\ A \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.041 & 0.041 \\ 0.102 & 0.102 \\ -0.062 & -0.062 \\ -0.083 & -0.083 \end{bmatrix} \mathrm{kN}$$

Stan Δt

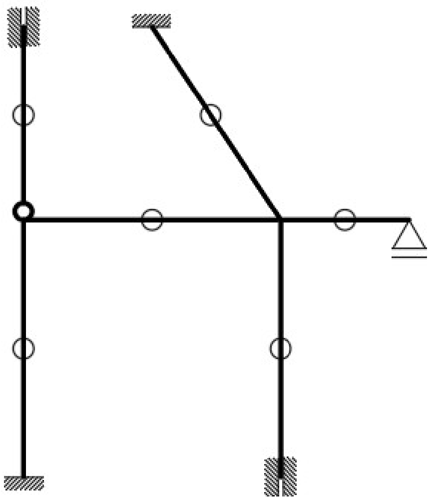
$\Delta t := |20\ \mathrm{K} - 0\ \mathrm{K}| = 20\ \mathrm{K}$

M Δt [kNm]



$$M_{\Delta t} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{EJ \cdot \alpha_t \cdot \Delta t}{h} & \dfrac{EJ \cdot \alpha_t \cdot \Delta t}{h} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.378 & 1.378 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathrm{kNm}$$

T Δt [kN]



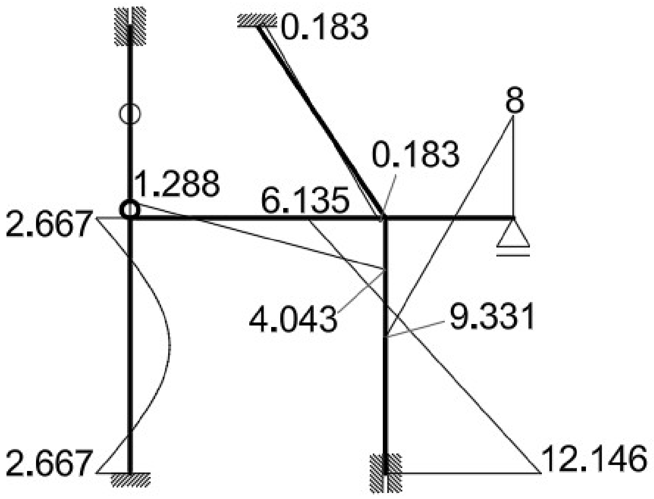
$$T_{\Delta t} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathrm{kN} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathrm{kN}$$

Stan rzeczywisty zsumowany:

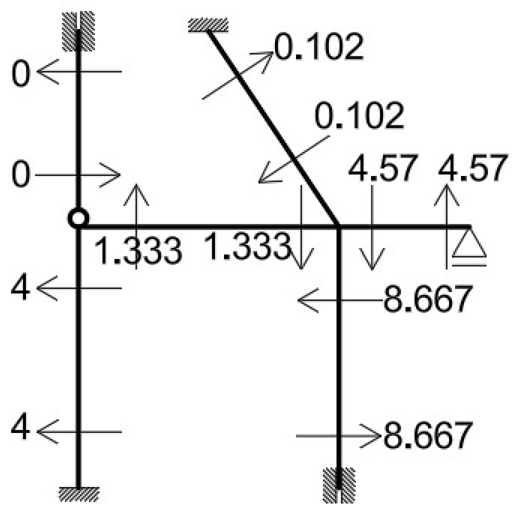
$$M_P := M_p + M_\varphi + M_\Delta + M_{t0} + M_{\Delta t} = \begin{bmatrix} -2.667 & -2.667 \\ 0 & 0 \\ -1.288 & 4.043 \\ -0.183 & 0.183 \\ 6.135 & -12.146 \\ 9.331 & -8 \end{bmatrix} \mathbf{kNm}$$

$$T_P := T_p + T_\varphi + T_\Delta + T_{t0} + T_{\Delta t} = \begin{bmatrix} 4 & -4 \\ 0 & 0 \\ 1.333 & 1.333 \\ 0.102 & 0.102 \\ -4.57 & -4.57 \\ -8.666 & -8.666 \end{bmatrix} \mathbf{kN}$$

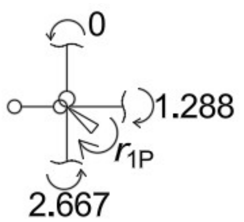
MP [kNm]



TP [kN]



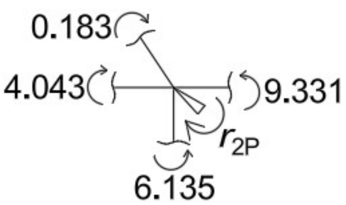
Węzeł B



$$\Sigma M = r_{1P} - 2.667\ \mathbf{kNm} + 1.288\ \mathbf{kNm} = 0$$

$$r_{1P} := 2.667\ \mathbf{kNm} - 1.288\ \mathbf{kNm} = 1.379\ \mathbf{kNm}$$

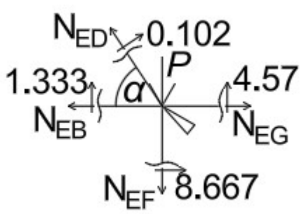
Węzeł E



$$\Sigma M = r_{2P} + 4.043\ \mathbf{kNm} + 0.183\ \mathbf{kNm} - 6.135\ \mathbf{kNm} - 9.331\ \mathbf{kNm} = 0$$

$$r_{2P} := -4.043\ \mathbf{kNm} - 0.183\ \mathbf{kNm} + 6.135\ \mathbf{kNm} + 9.331\ \mathbf{kNm} = 11.24\ \mathbf{kNm}$$

Węzeł E



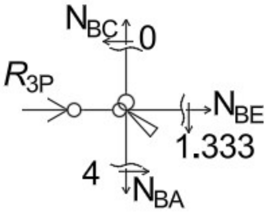
$$\Sigma Y = 1.333\ \mathbf{kN} + 8.666\ \mathbf{kN} - N_{EF} + N_{ED} \cdot sin\alpha + 0.102\ \mathbf{kN} \cdot cos\alpha - 5\ \mathbf{kN} = 0$$

$$N_{ED} := \frac{-1.333\ \mathbf{kN} - 8.666\ \mathbf{kN} + N_{EF} - 0.102\ \mathbf{kN} \cdot cos\alpha + 5\ \mathbf{kN}}{sin\alpha} = -6.076\ \mathbf{kN}$$

$$\Sigma X = -N_{EB} + N_{EG} - N_{ED} \cdot cos\alpha + 0.102\ \mathbf{kN} \cdot sin\alpha + 4.57\ \mathbf{kN} = 0$$

$$N_{EB} := N_{EG} - N_{ED} \cdot cos\alpha + 0.102\ \mathbf{kN} \cdot sin\alpha + 4.57\ \mathbf{kN} = 8.025\ \mathbf{kN} \qquad N_{BE} := N_{EB}$$

Węzeł B



$$\Sigma X = R_{3P} + N_{BE} + 4\ \text{kN} = 0$$

$$R_{3P} := -N_{BE} - 4\ \text{kN} = -12.025\ \text{kN}$$

Układ równań:

$$r_{11} \cdot \varphi_1 + r_{12} \cdot \varphi_2 + r_{13} \cdot \Delta_3 + r_{1P} = 0$$

$$r_{21} \cdot \varphi_1 + r_{22} \cdot \varphi_2 + r_{23} \cdot \Delta_3 + r_{2P} = 0$$

$$R_{31} \cdot \varphi_1 + R_{32} \cdot \varphi_2 + R_{33} \cdot \Delta_3 + R_{3P} = 0$$

Układ równań w postaci macierzowej:

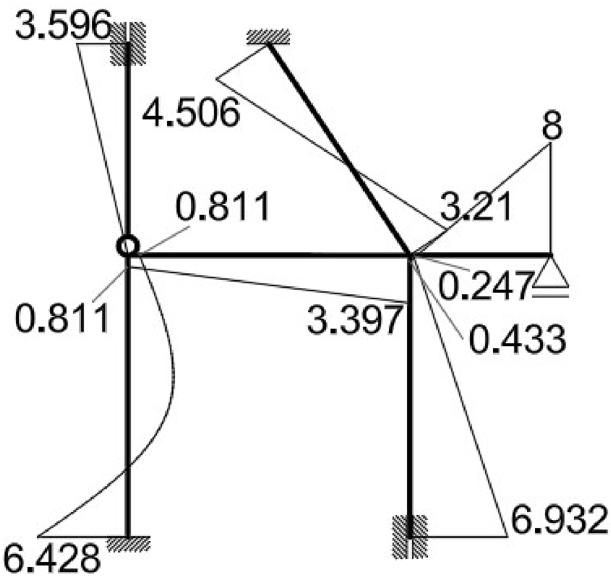
$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \Delta_3 \end{bmatrix} := \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}^{-1} \cdot - \begin{bmatrix} r_{1P} \\ r_{2P} \\ R_{3P} \end{bmatrix} = \begin{bmatrix} 0.000826 \\ -0.003391 \\ 0.015664\ \text{m} \end{bmatrix}$$

Wykresy ostateczne

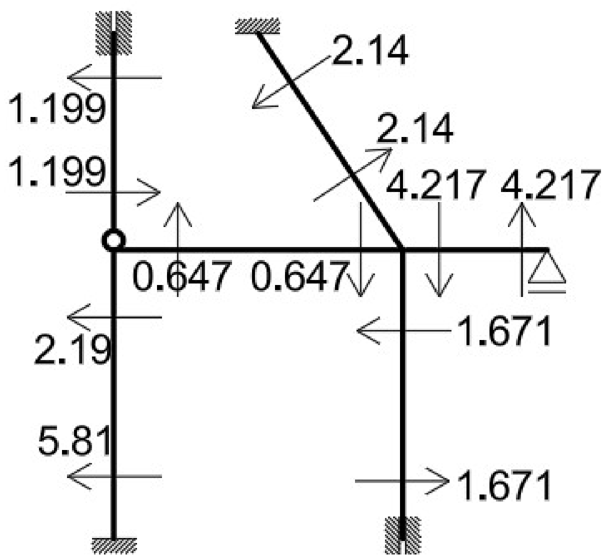
$$M_{ost} := M_1 \cdot \varphi_1 + M_2 \cdot \varphi_2 + M_3 \cdot \Delta_3 + M_P = \begin{bmatrix} -6.428 & 0.811 \\ 0 & -3.596 \\ 0.81 & 3.397 \\ 4.506 & -3.21 \\ -0.247 & -6.932 \\ 0.433 & -8 \end{bmatrix} \text{kNm}$$

$$T_{ost} := T_1 \cdot \varphi_1 + T_2 \cdot \varphi_2 + T_3 \cdot \Delta_3 + T_P = \begin{bmatrix} 5.81 & -2.19 \\ -1.199 & -1.199 \\ 0.647 & 0.647 \\ -2.14 & -2.14 \\ -1.671 & -1.671 \\ -4.217 & -4.217 \end{bmatrix} \text{kN}$$

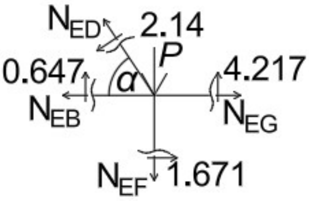
Most [kNm]



Tost [kN]



Węzeł E



$$\Sigma Y = 0.647\ \mathrm{kN} + 4.217\ \mathrm{kN} - N_{EF} + N_{ED} \cdot sin\alpha - 2.14\ \mathrm{kN} \cdot cos\alpha - P = 0$$

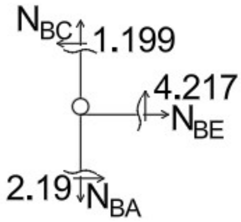
$$N_{ED} := \frac{-0.647\ \mathrm{kN} - 4.217\ \mathrm{kN} + N_{EF} + 2.14\ \mathrm{kN} \cdot cos\alpha + P}{sin\alpha} = 1.59\ \mathrm{kN}$$

$$\Sigma X = -N_{EB} + N_{EG} - N_{ED} \cdot cos\alpha - 2.14\ \mathrm{kN} \cdot sin\alpha + 1.671\ \mathrm{kN} = 0$$

$$N_{EB} := N_{EG} - N_{ED} \cdot cos\alpha - 2.14\ \mathrm{kN} \cdot sin\alpha + 1.671\ \mathrm{kN} = -0.992\ \mathrm{kN}$$

$$N_{BE} := N_{EB}$$

Węzeł B



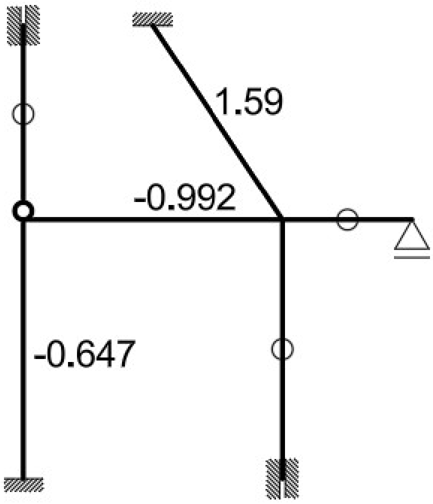
$$\Sigma Y = -0.647\ kN - N_{BA} = 0$$

$$N_{BA} := -0.647\ \mathrm{kN}$$

Sprawdzenie:

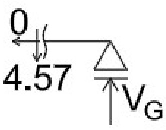
$$\Sigma X := -0.992\ \mathrm{kN} - 1.199\ \mathrm{kN} + 2.19\ \mathrm{kN} = -0.001\ \mathrm{kN}$$

Nost [kN]



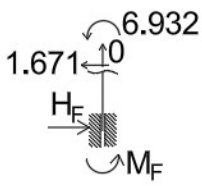
Obliczenie reakcji

Węzeł G



$$V_G := 4.57\ \mathrm{kN}$$

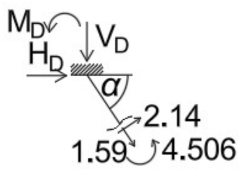
Węzeł F



$$M_F := -6.932\ \mathrm{kNm}$$

$$H_F := 1.671\ \mathrm{kN}$$

Węzeł G



$$M_D := -4.506\ \mathrm{kNm}$$

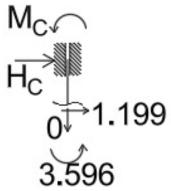
$$\Sigma X = H_D + 2.14\ \mathrm{kN} \cdot sin\alpha + 1.59\ \mathrm{kN} \cdot cos\alpha = 0$$

$$H_D := -2.14\ \mathrm{kN} \cdot sin\alpha - 1.59\ \mathrm{kN} \cdot cos\alpha = -2.663\ \mathrm{kN}$$

$$\Sigma Y = -V_D + 2.14\ \mathrm{kN} \cdot cos\alpha - 1.59\ \mathrm{kN} \cdot sin\alpha = 0$$

$$V_D := 2.14\ \mathrm{kN} \cdot cos\alpha - 1.59\ \mathrm{kN} \cdot sin\alpha = -0.136\ \mathrm{kN}$$

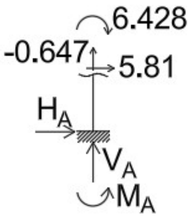
Węzeł C



$M_C := 3.596\ \mathbf{kNm}$

$H_C := -1.199\ \mathbf{kN}$

Węzeł A



$M_A := 6.428\ \mathbf{kNm}$

$V_A := 0.647\ \mathbf{kN}$

$H_A := -5.81\ \mathbf{kN}$

Rost