Politechnika Lubelska

Wydział Budownictwa i Architektury

Mechanika Budowli II

Projekt 3: Stateczność

Wykonał:

……..

Sprawdził:

dr inż. Jakub Gontarz

2025

PRZYKŁADOWY TEMAT

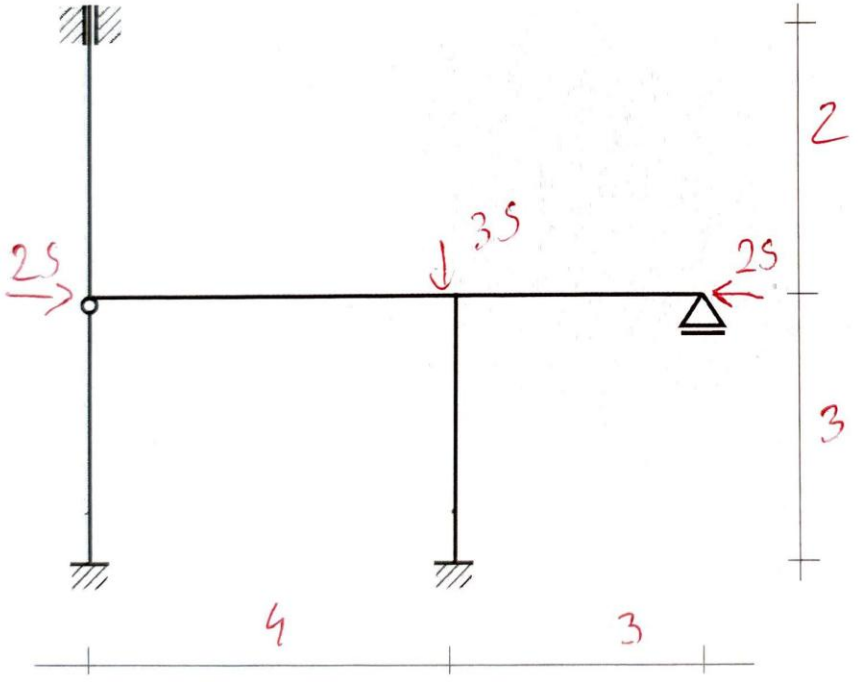
Student .....

Grupa .....

Rok akademicki .....

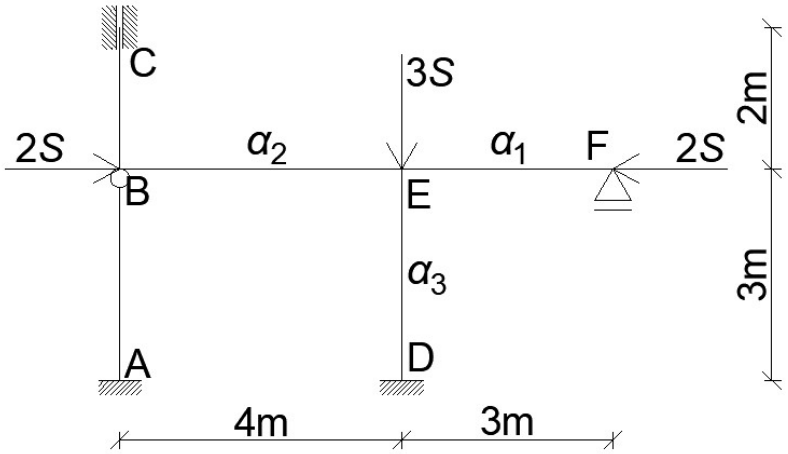
ĆWICZENIE PROJEKTOWE NR 4
Z MECHANIKI BUDOWLI

Wyznaczyć wartość siły krytycznej dla ramy obciążonej jak na rysunku.



Przekrój: IPE180

$E = 2.1 * 10^5$ MPa



$E := 210\ \mathbf{GPa}$ $\mathbf{kNm} := \mathbf{kN} \cdot \mathbf{m}$

Przekrój: IPE 180

$J := 1320\ \mathbf{cm}^4$

$EJ := E \cdot J = 2772\ \mathbf{kN} \cdot \mathbf{m}^2$

Współczynniki α:

$L := 3\ \mathbf{m}$ - długość odniesienia

$$\alpha^2 = \frac{S \cdot L^2}{EJ} \qquad \alpha_1{}^2 = \frac{2\ S \cdot L^2}{EJ} \qquad \alpha_2{}^2 = \frac{2\ S \cdot \left(\frac{4}{3} L\right)^2}{EJ} \qquad \alpha_3{}^2 = \frac{3\ S \cdot L^2}{EJ}$$

$$\alpha_1 = \sqrt{2}\ \alpha \quad \alpha_2 = \frac{4}{3}\sqrt{2}\ \alpha \qquad \alpha_2 = \sqrt{3}\ \alpha$$

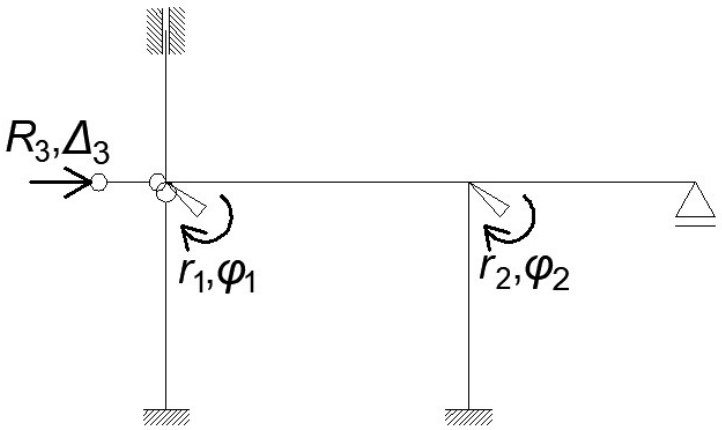
Funkcje kształtu:

$$d(\alpha) := 2\ (1 - \cos(\alpha)) - \alpha \cdot \sin(\alpha)$$

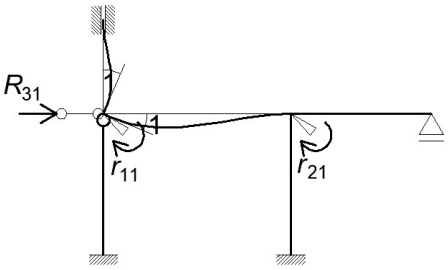
$$c(\alpha) := \frac{\alpha}{d(\alpha)} (\sin(\alpha) - \alpha \cdot \cos(\alpha)) \qquad s(\alpha) := \frac{\alpha}{d(\alpha)} (\alpha - \sin(\alpha)) \qquad r(\alpha) := \frac{\alpha^2}{d(\alpha)} (1 - \cos(\alpha))$$

$$c'(\alpha) := \alpha^2\ \frac{\sin(\alpha)}{\sin(\alpha) - \alpha \cdot \cos(\alpha)} \qquad b(\alpha) := \frac{\alpha^3}{d(\alpha)} \sin(\alpha)$$

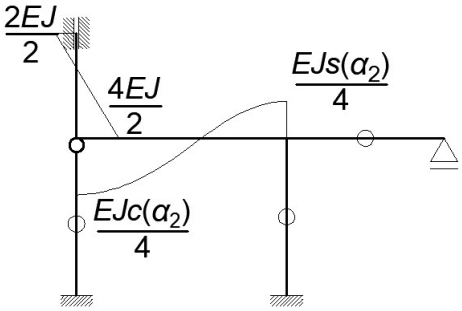
UPMP



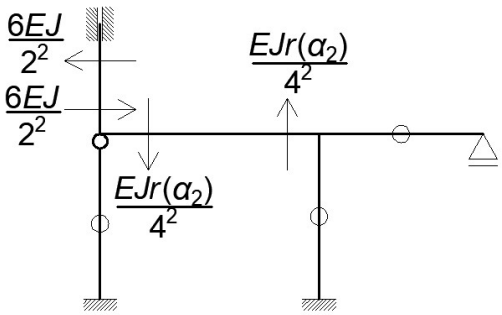
Stan 1



M1 [kNm]

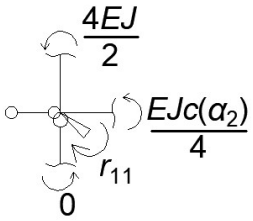


T1 [kN]



Obliczenia reakcji

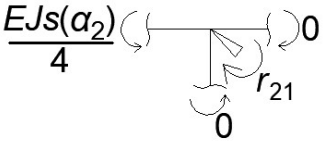
Węzeł B



$$\Sigma M = r_{11}(\alpha) - EJ \cdot \left(\frac{4}{2} + \frac{c\left(\frac{4}{3}\sqrt{2}\ \alpha\right)}{4} \right) = 0$$

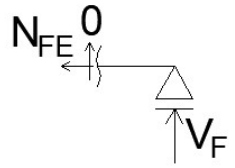
$$r_{11}(\alpha) := EJ \cdot \left(\frac{4}{2\ \mathbf{m}} + \frac{c\left(\frac{4}{3}\sqrt{2}\ \alpha\right)}{4\ \mathbf{m}} \right)$$

Węzeł E



$$\Sigma M = r_{21}(\alpha) - EJ \cdot \frac{s\left(\frac{4}{3}\sqrt{2}\ \alpha\right)}{4} = 0 \qquad r_{21}(\alpha) := EJ \cdot \frac{s\left(\frac{4}{3}\sqrt{2}\ \alpha\right)}{4\ \mathbf{m}}$$

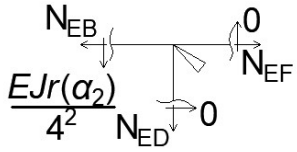
Węzeł F - Takie same obliczenia dla wszystkich stanów



$$\Sigma X = -N_{FE} = 0$$

$$N_{FE} := 0 \qquad N_{EF} := N_{FE}$$

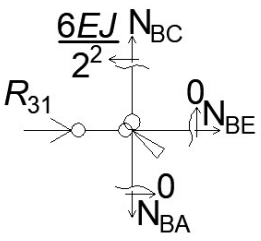
Węzeł E



$$\Sigma X = -N_{EB} + N_{EF} = 0$$

$$N_{EB} := N_{EF} = 0 \qquad N_{BE} := N_{EB}$$

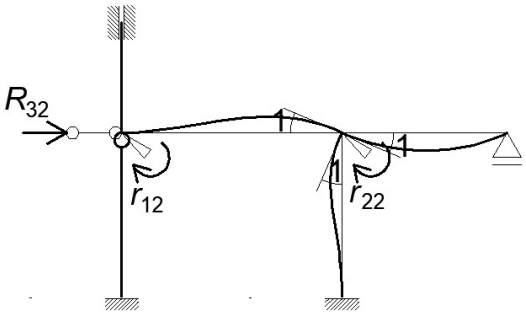
Węzeł B



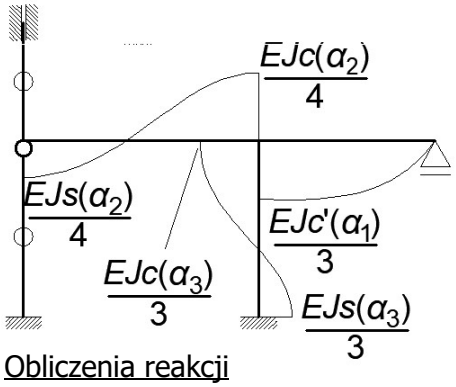
$$\Sigma X = R_{31}(\alpha) - \frac{6\ EJ}{2^2} = 0$$

$$R_{31}(\alpha) := \frac{6\ EJ}{(2\ \mathbf{m})^2}$$

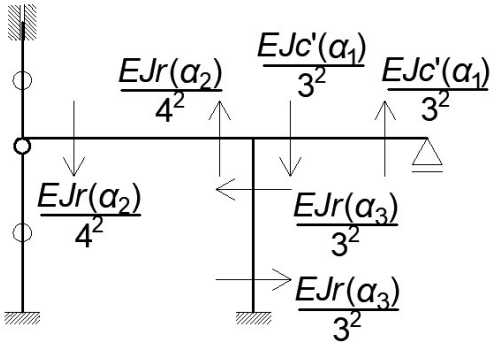
Stan 2



M2 [kNm]

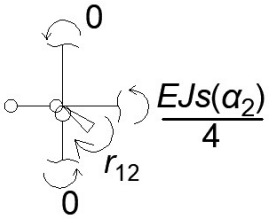


T2 [kN]



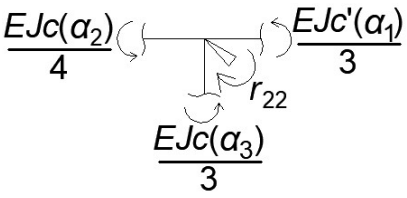
Obliczenia reakcji

Węzeł B



$$\Sigma M = r_{12}(\alpha) - EJ\,\frac{c\left(\frac{4}{3}\sqrt{2}\,\alpha\right)}{4} = 0 \qquad r_{12}(\alpha) := EJ\cdot\frac{s\left(\frac{4}{3}\sqrt{2}\,\alpha\right)}{4\ \mathbf{m}}$$

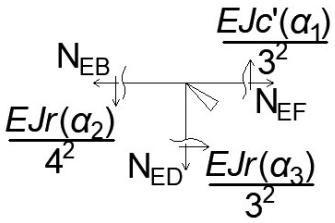
Węzeł E



$$\Sigma M = r_{22}(\alpha) - EJ\cdot\left(\frac{c\left(\frac{4}{3}\sqrt{2}\,\alpha\right)}{4} + \frac{c\left(\sqrt{3}\,\alpha\right)}{3} + \frac{c'\left(\sqrt{2}\,\alpha\right)}{3}\right) = 0$$

$$r_{22}(\alpha) := EJ\cdot\left(\frac{c\left(\frac{4}{3}\sqrt{2}\,\alpha\right)}{4\ \mathbf{m}} + \frac{c\left(\sqrt{3}\,\alpha\right)}{3\ \mathbf{m}} + \frac{c'\left(\sqrt{2}\,\alpha\right)}{3\ \mathbf{m}}\right)$$

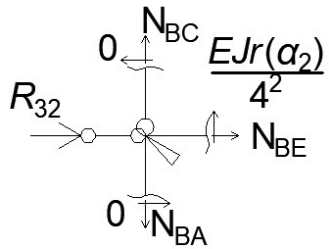
Węzeł E



$$\Sigma X = -N_{EB} + N_{EF} + EJ\cdot\frac{r\left(\sqrt{3}\,\alpha\right)}{(3\ m)^2} = 0$$

$$N_{EB}(\alpha) := EJ\cdot\frac{r\left(\sqrt{3}\,\alpha\right)}{(3\ \mathbf{m})^2} \qquad N_{BE} := N_{EB}$$

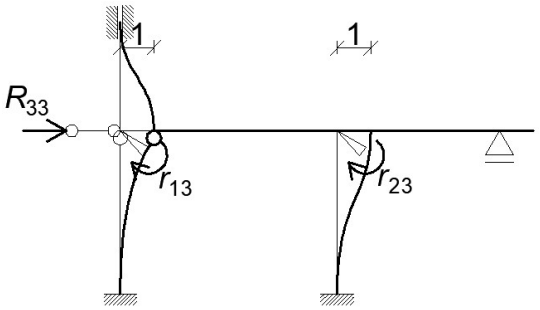
Węzeł B



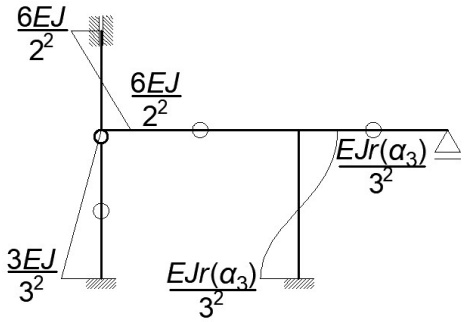
$$\Sigma X = R_{32}(\alpha) + N_{EB}(\alpha) = 0$$

$$R_{32}(\alpha) := -EJ\cdot\frac{r\left(\sqrt{3}\,\alpha\right)}{(3\ \mathbf{m})^2}$$

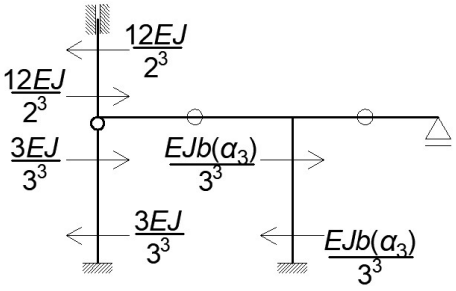
Stan 3



M3 [kN]

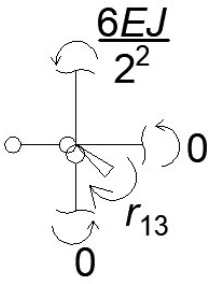


T3 [kN/m]



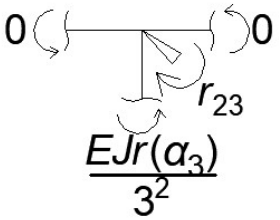
Obliczenia reakcji

Węzeł B



$$\Sigma M = r_{13}(\alpha) - \frac{6\ EJ}{2^2} = 0 \qquad r_{13}(\alpha) := \frac{6\ EJ}{(2\ \mathbf{m})^2}$$

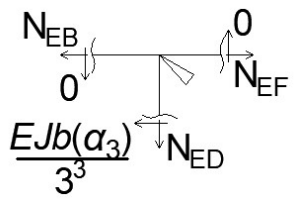
Węzeł E



$$\Sigma M = r_{23}(\alpha) + EJ \cdot \frac{r\left(\sqrt{3}\ \alpha\right)}{(3\ m)^2} = 0$$

$$r_{23}(\alpha) := -EJ \cdot \frac{r\left(\sqrt{3}\ \alpha\right)}{(3\ \mathbf{m})^2}$$

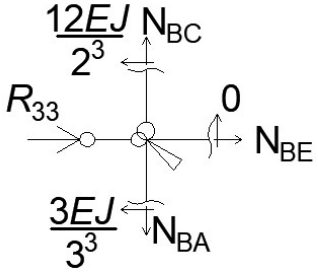
Węzeł E



$$\Sigma X = -N_{EB} + N_{EF} + EJ \cdot \frac{r\left(\sqrt{3}\ \alpha\right)}{(3\ m)^2} = 0$$

$$N_{EB}(\alpha) := -EJ \cdot \frac{b\left(\sqrt{3}\ \alpha\right)}{(3\ \mathbf{m})^3} \qquad N_{BE} := N_{EB}$$

Węzeł B



$$\Sigma X = R_{32}(\alpha) + EJ \cdot \left(\frac{12}{(2\ m)^3} + \frac{3}{(3\ m)^3} - N_{EB}(\alpha)\right) = 0$$

$$R_{33}(\alpha) := EJ \cdot \left(\frac{12}{(2\ \mathbf{m})^3} + \frac{3}{(3\ \mathbf{m})^3} + \frac{b\left(\sqrt{3}\ \alpha\right)}{(3\ \mathbf{m})^3}\right)$$

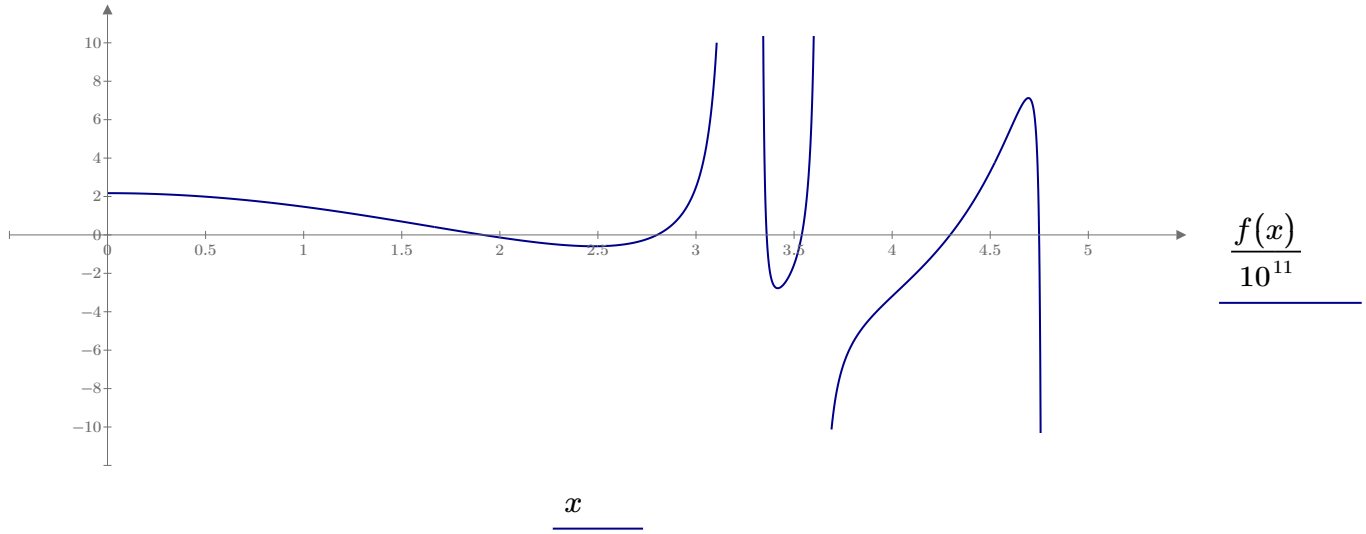
Obliczenie siły krytycznej

$$r_{11}(\alpha)\cdot\varphi_1+r_{12}(\alpha)\cdot\varphi_2+r_{13}(\alpha)\cdot\Delta_3=0$$

$$r_{21}(\alpha)\cdot\varphi_1+r_{22}(\alpha)\cdot\varphi_2+r_{23}(\alpha)\cdot\Delta_3=0$$

$$R_{31}(\alpha)\cdot\varphi_1+R_{32}(\alpha)\cdot\varphi_2+R_{33}(\alpha)\cdot\Delta_3=0$$

$$f(\alpha):=\left\|\begin{bmatrix}\frac{r_{11}(\alpha)}{\mathbf{kN\cdot m}} & \frac{r_{12}(\alpha)}{\mathbf{kN\cdot m}} & \frac{r_{13}(\alpha)}{\mathbf{kN}}\\ \frac{r_{21}(\alpha)}{\mathbf{kN\cdot m}} & \frac{r_{22}(\alpha)}{\mathbf{kN\cdot m}} & \frac{r_{23}(\alpha)}{\mathbf{kN}}\\ \frac{R_{31}(\alpha)}{\mathbf{kN}} & \frac{R_{32}(\alpha)}{\mathbf{kN}} & R_{33}(\alpha)\cdot\frac{\mathbf{m}}{\mathbf{kN}}\end{bmatrix}\right\| \qquad f(\alpha)=0$$



Obliczenie miejsca zerowego

$$c:=2 \qquad \alpha_{kr}:=\mathbf{root}\left(f(c),c\right)=1.914$$

Siła krytyczna

$$S_{kr}:=\frac{{\alpha_{kr}}^2\cdot EJ}{L^2}=1128.461\ \mathbf{kN} \qquad S_{soldis}:=1130.544\ \mathbf{kN}$$ - wynik z programu

Różnica: $\frac{S_{soldis}-S_{kr}}{S_{kr}}\cdot 100\%=0.185\%$