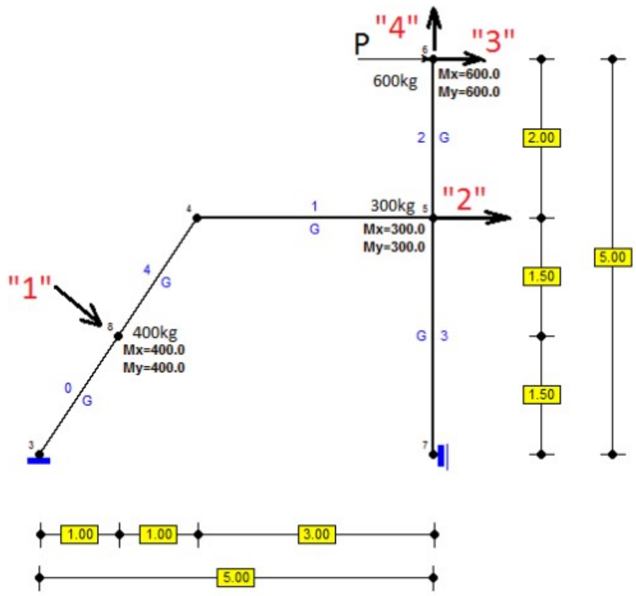
1. Drgania własne



Przekrój IPN200

$E := 205\ GPa \qquad J := 2140\ cm^4$

$EJ := E \cdot J = 4387\ kN \cdot m^2$

$P_0 := 8\ kN \qquad \Theta := 12\ \dfrac{rad}{s}$

$SSNU := R - 3 - P - 3 \cdot O = 2$

Rama dwukrotnie statycznie niewyznaczalna

$LSS := 4$ - liczba stopni swobody dynamicznej

$L_u := \sqrt{(2\ m)^2 + (3\ m)^2} = 3.606\ m$

- długość pręta ukośnego

Masy na kierunkach: zakładamy $m_0 := 100\ kg$ więc:

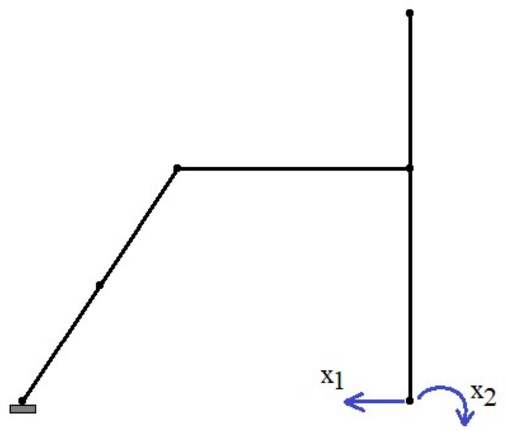
$m_1 := 400\ kg$ $\qquad m'_1 := 4$

$m_2 := 300\ kg$ $\qquad m'_2 := 3$

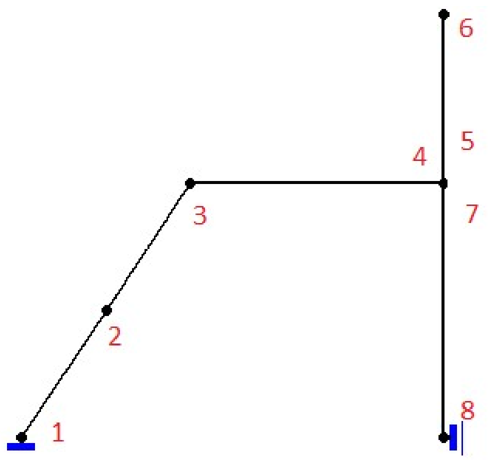
$m_3 := 600\ kg$ $\qquad m'_3 := 6$

$m_4 := 600\ kg + 300\ kg = 900\ kg$ $\qquad m'_4 := 9$

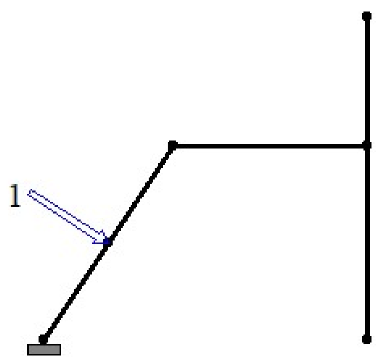
UPMS:



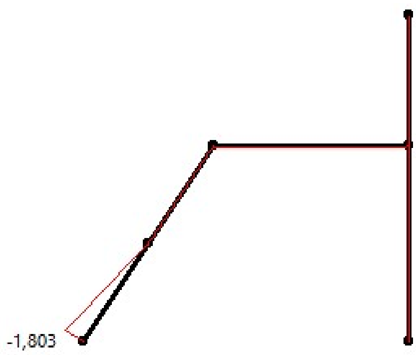
Przyjęta numeracja punktów obliczeń:



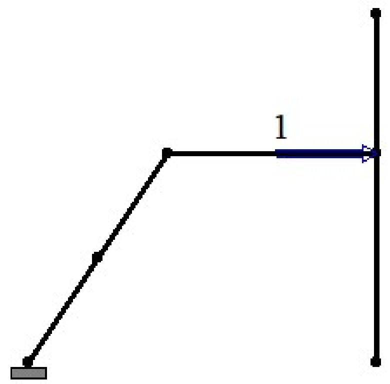
Stan P1



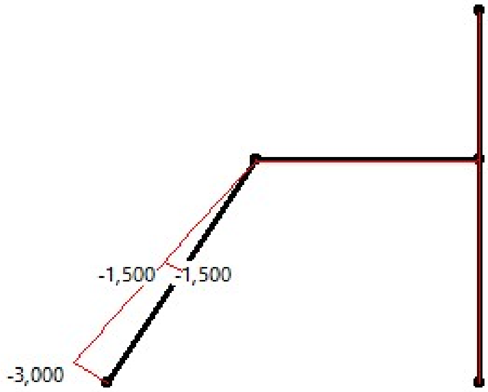
M P1 [m]



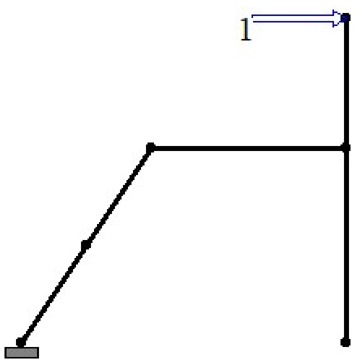
Stan P2



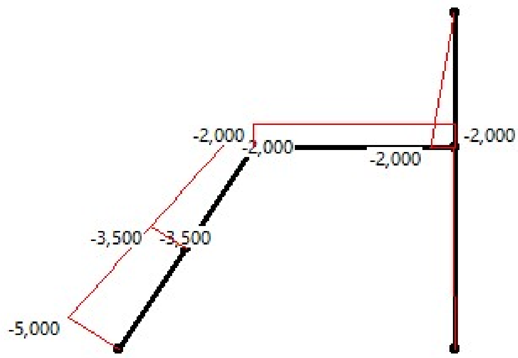
M P2 [m]



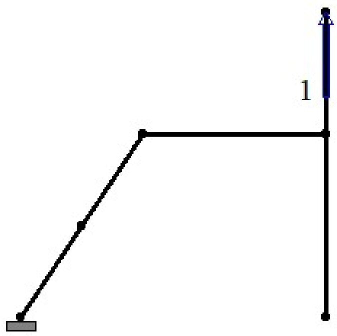
Stan P3



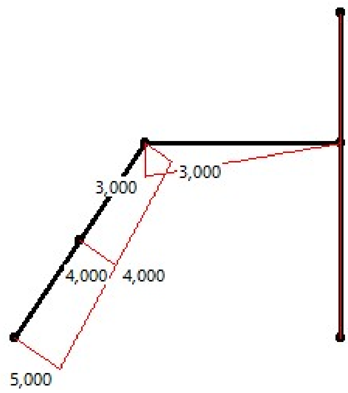
M P3 [m]



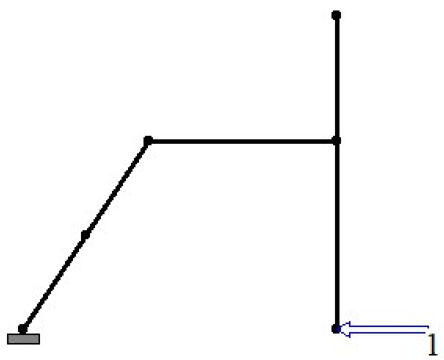
Stan P4



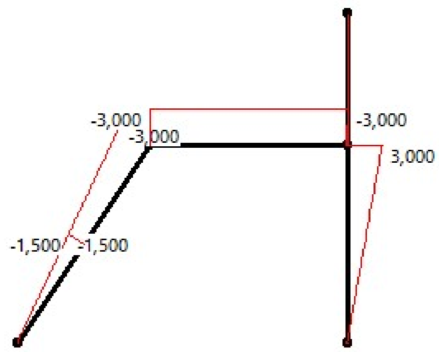
M P4 [m]



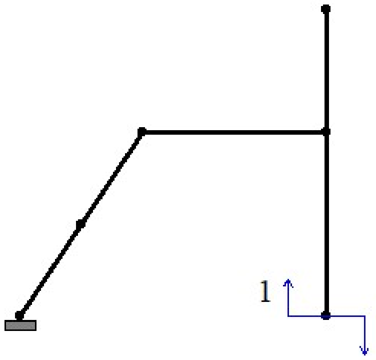
Stan x1=1



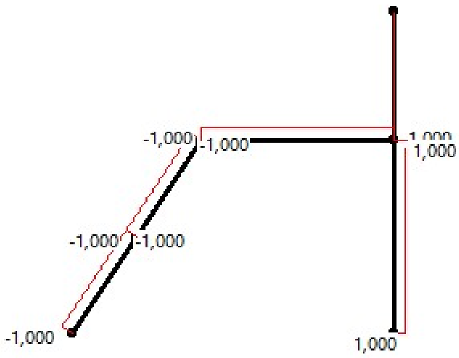
M x1 [m]



Stan x2=1



M x2 [-]



$$EJ\ \ \delta_{1p1} := \frac{1}{6} \cdot 1.803\ m \cdot 1.5\ m \cdot \frac{L_u}{2} = 0.813\ m^3$$

$$EJ\ \ \delta_{1p2} := \frac{1}{6} \cdot 3\ m \cdot 3\ m \cdot L_u = 5.408\ m^3$$

$$EJ\ \ \delta_{1p3} := 3\ m \cdot L_u \cdot \left(\frac{5\ m}{6} + \frac{2\ m}{3}\right) + 3\ m \cdot 2\ m \cdot 3\ m = 34.225\ m^3$$

$$EJ\ \ \delta_{1p4} := 3\ m \cdot L_u \cdot \left(\frac{-5\ m}{6} + \frac{-3\ m}{3}\right) + \frac{1}{2} \cdot 3\ m \cdot -3\ m \cdot 3\ m = -33.331\ m^3$$

$$EJ\ \ \delta_{2p1} := \frac{1}{2} \cdot 1.803\ m \cdot 1 \cdot \frac{L_u}{2} = 1.625\ m^2$$

$$EJ\ \ \delta_{2p2} := \frac{1}{2} \cdot 3\ m \cdot 1 \cdot L_u = 5.408\ m^2$$

$$EJ\ \ \delta_{2p3} := 1 \cdot L_u \cdot \frac{5\ m + 2\ m}{2} + 2\ m \cdot 1 \cdot 3\ m = 18.619\ m^2$$

$$EJ\ \ \delta_{2p4} := 1 \cdot L_u \cdot -\frac{5\ m + 3\ m}{2} + \frac{1}{2} \cdot 3\ m \cdot 1 \cdot -3\ m = -18.922\ m^2$$

$$EJ\ \ \delta_{11} := \frac{1}{3} \cdot 3\ m \cdot 3\ m \cdot L_u + 3\ m \cdot 3\ m \cdot 3\ m + \frac{1}{3} \cdot 3\ m \cdot 3\ m \cdot 3\ m = 46.817\ m^3$$

$$EJ\ \ \delta_{12} := \frac{1}{2} \cdot 3\ m \cdot 1 \cdot L_u + 3\ m \cdot 1 \cdot 3\ m + \frac{1}{2} \cdot 3\ m \cdot 1 \cdot 3\ m = 18.908\ m^2 \qquad \delta_{21} := \delta_{12}$$

$$EJ\ \ \delta_{22} := 1 \cdot 1 \cdot L_u + 1 \cdot 1 \cdot 3\ m + 1 \cdot 1 \cdot 3\ m = 9.606\ m$$

Układy równań:

$$\delta_{11} \cdot x_{11} + \delta_{12} \cdot x_{21} + \delta_{1p1} = 0$$
$$\delta_{21} \cdot x_{11} + \delta_{22} \cdot x_{21} + \delta_{2p1} = 0$$

$$\delta_{11} \cdot x_{12} + \delta_{12} \cdot x_{22} + \delta_{1p2} = 0$$
$$\delta_{21} \cdot x_{12} + \delta_{22} \cdot x_{22} + \delta_{2p2} = 0$$

$$\delta_{11} \cdot x_{13} + \delta_{13} \cdot x_{23} + \delta_{1p3} = 0$$
$$\delta_{21} \cdot x_{13} + \delta_{22} \cdot x_{23} + \delta_{2p3} = 0$$

$$\delta_{11} \cdot x_{14} + \delta_{13} \cdot x_{24} + \delta_{1p4} = 0$$
$$\delta_{21} \cdot x_{14} + \delta_{22} \cdot x_{24} + \delta_{2p4} = 0$$

Obliczenia:

$$\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} := \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\delta_{1p1} \\ -\delta_{2p1} \end{bmatrix}$$

$$\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} := \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\delta_{1p2} \\ -\delta_{2p2} \end{bmatrix}$$

$$\begin{bmatrix} x_{13} \\ x_{23} \end{bmatrix} := \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\delta_{1p3} \\ -\delta_{2p3} \end{bmatrix}$$

$$\begin{bmatrix} x_{14} \\ x_{24} \end{bmatrix} := \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\delta_{1p4} \\ -\delta_{2p4} \end{bmatrix}$$

Wyniki

$x_{11} = 0.249 \qquad x_{21} = -0.659\ m$

$x_{12} = 0.546 \qquad x_{22} = -1.638\ m$

$x_{13} = 0.253 \qquad x_{23} = -2.436\ m$

$x_{14} = -0.408 \qquad x_{24} = 2.774\ m$

Obliczenie momentów ostatecznych:

$$M_{ost1} := \begin{bmatrix} 0 \\ -1.5 \\ -3 \\ -3 \\ 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} m \cdot x_{11} + \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \cdot x_{21} + \begin{bmatrix} -1.803 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} m = \begin{bmatrix} -1.144 \\ 0.286 \\ -0.087 \\ -0.087 \\ 0 \\ 0 \\ 0.087 \\ -0.659 \end{bmatrix} m$$

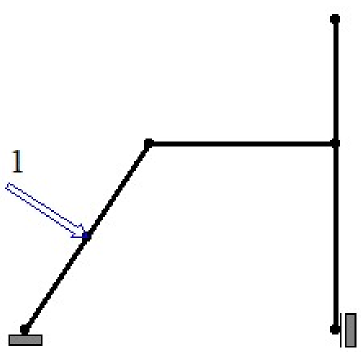
$$M_{ost2} := \begin{bmatrix} 0 \\ -1.5 \\ -3 \\ -3 \\ 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} m \cdot x_{12} + \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \cdot x_{22} + \begin{bmatrix} -3 \\ -1.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} m = \begin{bmatrix} -1.362 \\ -0.681 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1.638 \end{bmatrix} m$$

$$M_{ost3} := \begin{bmatrix} 0 \\ -1.5 \\ -3 \\ -3 \\ 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} m \cdot x_{13} + \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \cdot x_{23} + \begin{bmatrix} -5 \\ -3.5 \\ -2 \\ -2 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix} m = \begin{bmatrix} -2.564 \\ -1.443 \\ -0.322 \\ -0.322 \\ -2 \\ 0 \\ -1.678 \\ -2.436 \end{bmatrix} m$$

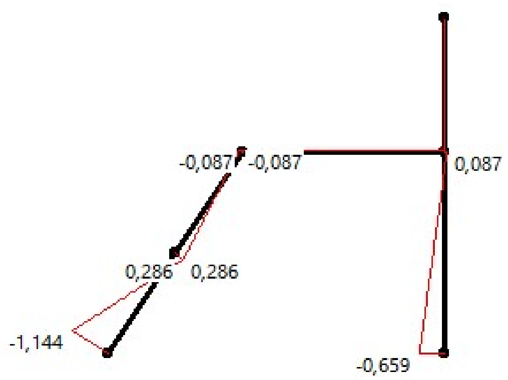
$$M_{ost4} := \begin{bmatrix} 0 \\ -1.5 \\ -3 \\ -3 \\ 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} m \cdot x_{14} + \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \cdot x_{24} + \begin{bmatrix} 5 \\ 4 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} m = \begin{bmatrix} 2.226 \\ 1.839 \\ 1.451 \\ -1.549 \\ 0 \\ 0 \\ 1.549 \\ 2.774 \end{bmatrix} m$$

Stany ostateczne:

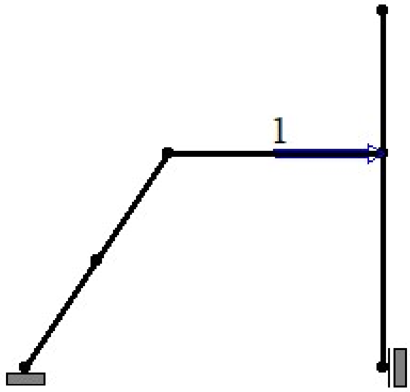
Stan 1



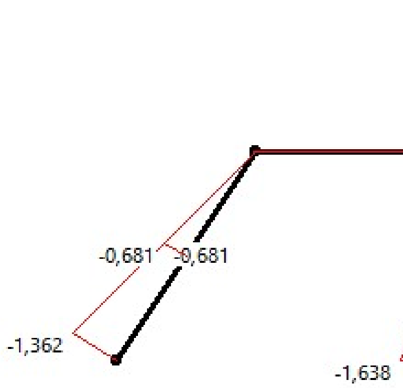
M ost1 [m]



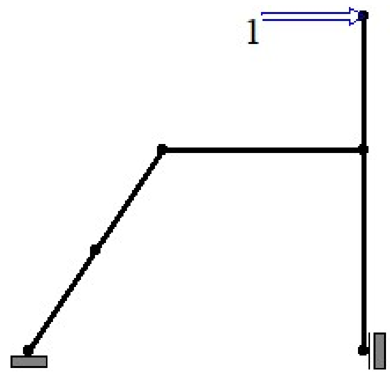
Stan 2



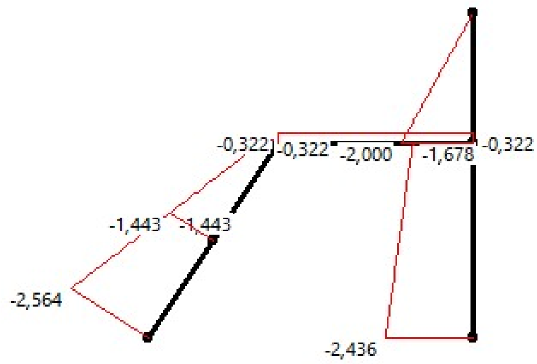
M ost2 [m]



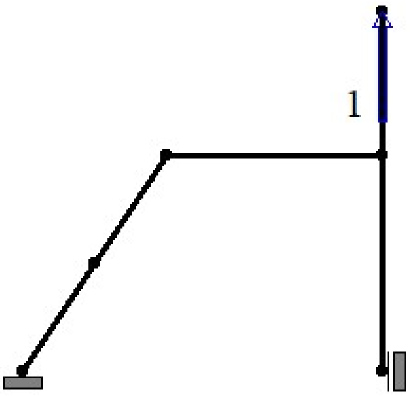
Stan 3



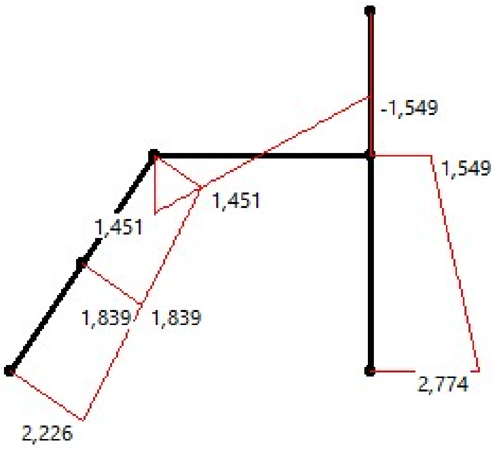
M ost3 [m]



Stan 4



M ost4 [m]



Obliczenie przemieszczeń na kierunkach swobody dynamicznej:

Oznaczenie: $EJ \cdot \delta_{ij} = \delta'_{ij}$

$$EJ \cdot \delta_{11} = \int_S M_{ost1} \cdot M_{p1} \, \mathrm{d}S \qquad \delta'_{11} := 1.803\ m \cdot \frac{L_u}{2} \cdot \left(\frac{1.144\ m}{3} - \frac{0.286\ m}{6} \right) \cdot \frac{1}{m^3} = 1.085$$

$$EJ \cdot \delta_{12} = \int_S M_{ost2} \cdot M_{p1} \, \mathrm{d}S \qquad \delta'_{12} := 1.803\ m \cdot \frac{L_u}{2} \cdot \left(\frac{1.362\ m}{3} + \frac{0.681\ m}{6} \right) \cdot \frac{1}{m^3} = 1.845$$

$$EJ \cdot \delta_{13} = \int_S M_{ost3} \cdot M_{p1} \, \mathrm{d}S \qquad \delta'_{13} := 1.803\ m \cdot \frac{L_u}{2} \cdot \left(\frac{2.564\ m}{3} + \frac{1.443\ m}{6} \right) \cdot \frac{1}{m^3} = 3.56$$

$$EJ \cdot \delta_{14} = \int_S M_{ost4} \cdot M_{p1} \, \mathrm{d}S \qquad \delta'_{14} := 1.803\ m \cdot \frac{L_u}{2} \cdot \left(\frac{-2.226\ m}{3} + \frac{-1.839\ m}{6} \right) \cdot \frac{1}{m^3} = -3.408$$

$$EJ\cdot\delta_{22}=\int_S M_{ost2}\cdot M_{p2}\,\mathrm{d}S \qquad \delta'_{22}:=\frac{1}{3}\cdot 3\ m\cdot L_u\cdot 1.362\ m\cdot\frac{1}{m^3}=4.911$$

$$EJ\cdot\delta_{23}=\int_S M_{ost3}\cdot M_{p2}\,\mathrm{d}S \qquad \delta'_{23}:=3\ m\cdot L_u\cdot\left(\frac{2.564\ m}{3}+\frac{0.322\ m}{6}\right)\cdot\frac{1}{m^3}=9.825$$

$$EJ\cdot\delta_{24}=\int_S M_{ost4}\cdot M_{p2}\,\mathrm{d}S \qquad \delta'_{24}:=3\ m\cdot L_u\cdot\left(\frac{-2.226\ m}{3}+\frac{-1.451\ m}{6}\right)\cdot\frac{1}{m^3}=-10.642$$

$$EJ\cdot\delta_{33}=\int_S M_{ost3}\cdot M_{p3}\,\mathrm{d}S$$

$$\delta'_{33}:=\left(5\ m\cdot L_u\cdot\left(\frac{2.564\ m}{3}+\frac{0.322\ m}{6}\right)+2\ m\cdot L_u\cdot\left(\frac{2.564\ m}{6}+\frac{0.322\ m}{3}\right)+2\ m\cdot 0.322\ m\cdot 3\ m+\frac{1}{3}\cdot 2\ m\cdot 2\ m\cdot 2\ m\right)\cdot\frac{1}{m^3}=24.829$$

$$EJ\cdot\delta_{34}=\int_S M_{ost3}\cdot M_{p4}\,\mathrm{d}S$$

$$\delta'_{34}:=\left(-5\ m\cdot L_u\cdot\left(\frac{2.564\ m}{3}+\frac{0.322\ m}{6}\right)-3\ m\cdot L_u\cdot\left(\frac{2.564\ m}{6}+\frac{0.322\ m}{3}\right)-\frac{1}{2}\cdot 3\ m\cdot 0.322\ m\cdot 3\ m\right)\cdot\frac{1}{m^3}=-23.608$$

$$EJ\cdot\delta_{44}=\int_S M_{ost4}\cdot M_{p4}\,\mathrm{d}S$$

$$\delta'_{44}:=\left(5\ m\cdot L_u\cdot\left(\frac{2.227\ m}{3}+\frac{1.451\ m}{6}\right)+3\ m\cdot L_u\cdot\left(\frac{2.227\ m}{6}+\frac{1.451\ m}{3}\right)+3\ m\cdot 3\ m\cdot\left(\frac{1.451\ m}{3}-\frac{1.549\ m}{6}\right)\right)\cdot\frac{1}{m^3}=29.018$$

$$\delta'_{21}:=\delta'_{12} \qquad \delta'_{31}:=\delta'_{13} \qquad \delta'_{41}:=\delta'_{14} \qquad \delta'_{32}:=\delta'_{23} \qquad \delta'_{42}:=\delta'_{24} \qquad \delta'_{43}:=\delta'_{34}$$

$$\delta_{11}:=\frac{\delta'_{11}\cdot m^3}{EJ} \qquad \delta_{12}:=\frac{\delta'_{12}\cdot m^3}{EJ} \qquad \delta_{13}:=\frac{\delta'_{13}\cdot m^3}{EJ} \qquad \delta_{14}:=\frac{\delta'_{14}\cdot m^3}{EJ}$$

$$\delta_{21}:=\frac{\delta'_{21}\cdot m^3}{EJ} \qquad \delta_{22}:=\frac{\delta'_{22}\cdot m^3}{EJ} \qquad \delta_{23}:=\frac{\delta'_{23}\cdot m^3}{EJ} \qquad \delta_{24}:=\frac{\delta'_{24}\cdot m^3}{EJ}$$

$$\delta_{31}:=\frac{\delta'_{31}\cdot m^3}{EJ} \qquad \delta_{32}:=\frac{\delta'_{32}\cdot m^3}{EJ} \qquad \delta_{33}:=\frac{\delta'_{33}\cdot m^3}{EJ} \qquad \delta_{34}:=\frac{\delta'_{34}\cdot m^3}{EJ}$$

$$\delta_{41}:=\frac{\delta'_{41}\cdot m^3}{EJ} \qquad \delta_{42}:=\frac{\delta'_{42}\cdot m^3}{EJ} \qquad \delta_{43}:=\frac{\delta'_{43}\cdot m^3}{EJ} \qquad \delta_{44}:=\frac{\delta'_{44}\cdot m^3}{EJ}$$

Układ równań opisujący ruch:

$$\left(m_1\cdot\delta_{11}-\frac{1}{\omega^2}\right)\cdot A_1+m_2\cdot\delta_{12}\cdot A_2+m_3\cdot\delta_{13}\cdot A_3+m_4\cdot\delta_{14}\cdot A_4=0$$

$$m_1\cdot\delta_{21}\cdot A_1+\left(m_2\cdot\delta_{22}-\frac{1}{\omega^2}\right)\cdot A_2+m_3\cdot\delta_{23}\cdot A_3+m_4\cdot\delta_{24}\cdot A_4=0$$

$$m_1\cdot\delta_{31}\cdot A_1+m_2\cdot\delta_{32}\cdot A_2+\left(m_3\cdot\delta_{33}-\frac{1}{\omega^2}\right)\cdot A_3+m_4\cdot\delta_{34}\cdot A_4=0$$

$$m_1\cdot\delta_{41}\cdot A_1+m_2\cdot\delta_{42}\cdot A_2+m_3\cdot\delta_{43}\cdot A_3+\left(m_4\cdot\delta_{44}-\frac{1}{\omega^2}\right)\cdot A_4=0$$

Podstawienie: $X = \frac{m_0 \cdot \omega^2}{EJ} \, m^3$

$$\left(m'_1 \cdot \delta'_{11} \cdot X - 1\right) \cdot A_1 + m'_2 \cdot \delta'_{12} \cdot X \cdot A_2 + m'_3 \cdot \delta'_{13} \cdot X \cdot A_3 + m'_4 \cdot \delta'_{14} \cdot X \cdot A_4 = 0$$

$$m'_1 \cdot \delta'_{21} \cdot X \cdot A_1 + \left(m'_2 \cdot \delta'_{22} \cdot X - 1\right) \cdot A_2 + m'_3 \cdot \delta'_{23} \cdot X \cdot A_3 + m'_4 \cdot \delta'_{24} \cdot X \cdot A_4 = 0$$

$$m'_1 \cdot \delta'_{31} \cdot X \cdot A_1 + m'_2 \cdot \delta'_{32} \cdot X \cdot A_2 + \left(m'_3 \cdot \delta'_{33} \cdot X - 1\right) \cdot A_3 + m'_4 \cdot \delta'_{34} \cdot X \cdot A_4 = 0$$

$$m'_1 \cdot \delta'_{41} \cdot X \cdot A_1 + m'_2 \cdot \delta'_{42} \cdot X \cdot A_2 + m'_3 \cdot \delta'_{43} \cdot X \cdot A_3 + \left(m'_4 \cdot \delta'_{44} \cdot X - 1\right) \cdot A_4 = 0$$

Stąd:

$$\begin{bmatrix} m'_1 \cdot \delta'_{11} \cdot X - 1 & m'_2 \cdot \delta'_{12} \cdot X & m'_3 \cdot \delta'_{13} \cdot X & m'_4 \cdot \delta'_{14} \cdot X \\ m'_1 \cdot \delta'_{21} \cdot X & m'_2 \cdot \delta'_{22} \cdot X - 1 & m'_3 \cdot \delta'_{23} \cdot X & m'_4 \cdot \delta'_{24} \cdot X \\ m'_1 \cdot \delta'_{31} \cdot X & m'_2 \cdot \delta'_{32} \cdot X & m'_3 \cdot \delta'_{33} \cdot X - 1 & m'_4 \cdot \delta'_{34} \cdot X \\ m'_1 \cdot \delta'_{41} \cdot X & m'_2 \cdot \delta'_{42} \cdot X & m'_3 \cdot \delta'_{43} \cdot X & m'_4 \cdot \delta'_{44} \cdot X - 1 \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = 0$$

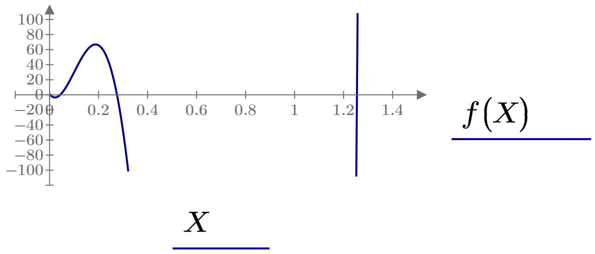
Odrzucamy rozwiązanie trywialne:

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \neq 0$$

Więc:

$$f(X) := \left\| \begin{bmatrix} m'_1 \cdot \delta'_{11} \cdot X - 1 & m'_2 \cdot \delta'_{12} \cdot X & m'_3 \cdot \delta'_{13} \cdot X & m'_4 \cdot \delta'_{14} \cdot X \\ m'_1 \cdot \delta'_{21} \cdot X & m'_2 \cdot \delta'_{22} \cdot X - 1 & m'_3 \cdot \delta'_{23} \cdot X & m'_4 \cdot \delta'_{24} \cdot X \\ m'_1 \cdot \delta'_{31} \cdot X & m'_2 \cdot \delta'_{32} \cdot X & m'_3 \cdot \delta'_{33} \cdot X - 1 & m'_4 \cdot \delta'_{34} \cdot X \\ m'_1 \cdot \delta'_{41} \cdot X & m'_2 \cdot \delta'_{42} \cdot X & m'_3 \cdot \delta'_{43} \cdot X & m'_4 \cdot \delta'_{44} \cdot X - 1 \end{bmatrix} \right\| = 0$$

Szukanie miejsc zerowych wyznacznika, ustawione od najmniejszego do największego:



$X_1 := 0 \qquad X_2 := 0.05 \qquad X_3 := 0.3 \qquad X_4 := 1.2$

$X_1 := \mathbf{root}\left(f\left(X_1\right), X_1\right) = 0.002 \qquad X_3 := \mathbf{root}\left(f\left(X_3\right), X_3\right) = 0.275$

$X_2 := \mathbf{root}\left(f\left(X_2\right), X_2\right) = 0.043 \qquad X_4 := \mathbf{root}\left(f\left(X_4\right), X_4\right) = 1.255$

Obliczenie amplitud drgań własnych:

Są 4 stopnie swobody, więc będą 4 układy równań do (4 formy drgań):

Przy założeniu, że A1=1 układ 4 równań zamienia się na układ 3 równań, z wyrazami wolnymi:

$$\left(m'_1 \cdot \delta'_{11} \cdot X - 1\right) \cdot 1 + m'_2 \cdot \delta'_{12} \cdot X \cdot A_2 + m'_3 \cdot \delta'_{13} \cdot X \cdot A_3 + m'_4 \cdot \delta'_{14} \cdot X \cdot A_4 = 0$$

$$m'_1 \cdot \delta'_{21} \cdot X \cdot 1 + \left(m'_2 \cdot \delta'_{22} \cdot X - 1\right) \cdot A_2 + m'_3 \cdot \delta'_{23} \cdot X \cdot A_3 + m'_4 \cdot \delta'_{24} \cdot X \cdot A_4 = 0$$

$$m'_1 \cdot \delta'_{31} \cdot X \cdot 1 + m'_2 \cdot \delta'_{32} \cdot X \cdot A_2 + \left(m'_3 \cdot \delta'_{33} \cdot X - 1\right) \cdot A_3 + m'_4 \cdot \delta'_{34} \cdot X \cdot A_4 = 0$$

- I forma drgań własnych:

$A_{11} := 1$

$$\begin{bmatrix} A_{21} \\ A_{31} \\ A_{41} \end{bmatrix} := \begin{bmatrix} m'_2 \cdot \delta'_{12} \cdot X_1 & m'_3 \cdot \delta'_{13} \cdot X_1 & m'_4 \cdot \delta'_{14} \cdot X_1 \\ m'_2 \cdot \delta'_{22} \cdot X_1 - 1 & m'_3 \cdot \delta'_{23} \cdot X_1 & m'_4 \cdot \delta'_{24} \cdot X_1 \\ m'_2 \cdot \delta'_{32} \cdot X_1 & m'_3 \cdot \delta'_{33} \cdot X_1 - 1 & m'_4 \cdot \delta'_{34} \cdot X_1 \end{bmatrix}^{-1} \cdot - \begin{bmatrix} \left(m'_1 \cdot \delta'_{11} \cdot X_1 - 1\right) \cdot A_{11} \\ m'_1 \cdot \delta'_{21} \cdot X_1 \cdot A_{11} \\ m'_1 \cdot \delta'_{31} \cdot X_1 \cdot A_{11} \end{bmatrix} = \begin{bmatrix} 2.957 \\ 6.84 \\ -7.66 \end{bmatrix}$$

$A_{11} = 1$ $\quad A_{21} = 2.957$ $\quad A_{31} = 6.84$ $\quad A_{41} = -7.66$

- II forma drgań własnych:

$A_{12} := 1$

$$\begin{bmatrix} A_{22} \\ A_{32} \\ A_{42} \end{bmatrix} := \begin{bmatrix} m'_2 \cdot \delta'_{12} \cdot X_2 & m'_3 \cdot \delta'_{13} \cdot X_2 & m'_4 \cdot \delta'_{14} \cdot X_2 \\ m'_2 \cdot \delta'_{22} \cdot X_2 - 1 & m'_3 \cdot \delta'_{23} \cdot X_2 & m'_4 \cdot \delta'_{24} \cdot X_2 \\ m'_2 \cdot \delta'_{32} \cdot X_2 & m'_3 \cdot \delta'_{33} \cdot X_2 - 1 & m'_4 \cdot \delta'_{34} \cdot X_2 \end{bmatrix}^{-1} \cdot - \begin{bmatrix} \left(m'_1 \cdot \delta'_{11} \cdot X_2 - 1\right) \cdot A_{12} \\ m'_1 \cdot \delta'_{21} \cdot X_2 \cdot A_{12} \\ m'_1 \cdot \delta'_{31} \cdot X_2 \cdot A_{12} \end{bmatrix} = \begin{bmatrix} 0.676 \\ 6.26 \\ 3.871 \end{bmatrix}$$

$A_{12} = 1$ $\quad A_{22} = 0.676$ $\quad A_{32} = 6.26$ $\quad A_{42} = 3.871$

- III forma drgań własnych:

$A_{13} := 1$

$$\begin{bmatrix} A_{23} \\ A_{33} \\ A_{43} \end{bmatrix} := \begin{bmatrix} m'_2 \cdot \delta'_{12} \cdot X_3 & m'_3 \cdot \delta'_{13} \cdot X_3 & m'_4 \cdot \delta'_{14} \cdot X_3 \\ m'_2 \cdot \delta'_{22} \cdot X_3 - 1 & m'_3 \cdot \delta'_{23} \cdot X_3 & m'_4 \cdot \delta'_{24} \cdot X_3 \\ m'_2 \cdot \delta'_{32} \cdot X_3 & m'_3 \cdot \delta'_{33} \cdot X_3 - 1 & m'_4 \cdot \delta'_{34} \cdot X_3 \end{bmatrix}^{-1} \cdot - \begin{bmatrix} \left(m'_1 \cdot \delta'_{11} \cdot X_3 - 1\right) \cdot A_{13} \\ m'_1 \cdot \delta'_{21} \cdot X_3 \cdot A_{13} \\ m'_1 \cdot \delta'_{31} \cdot X_3 \cdot A_{13} \end{bmatrix} = \begin{bmatrix} 1.125 \\ -0.229 \\ 0.066 \end{bmatrix}$$

$A_{13} = 1$ $\quad A_{23} = 1.125$ $\quad A_{33} = -0.229$ $\quad A_{43} = 0.066$

- IV forma drgań własnych:

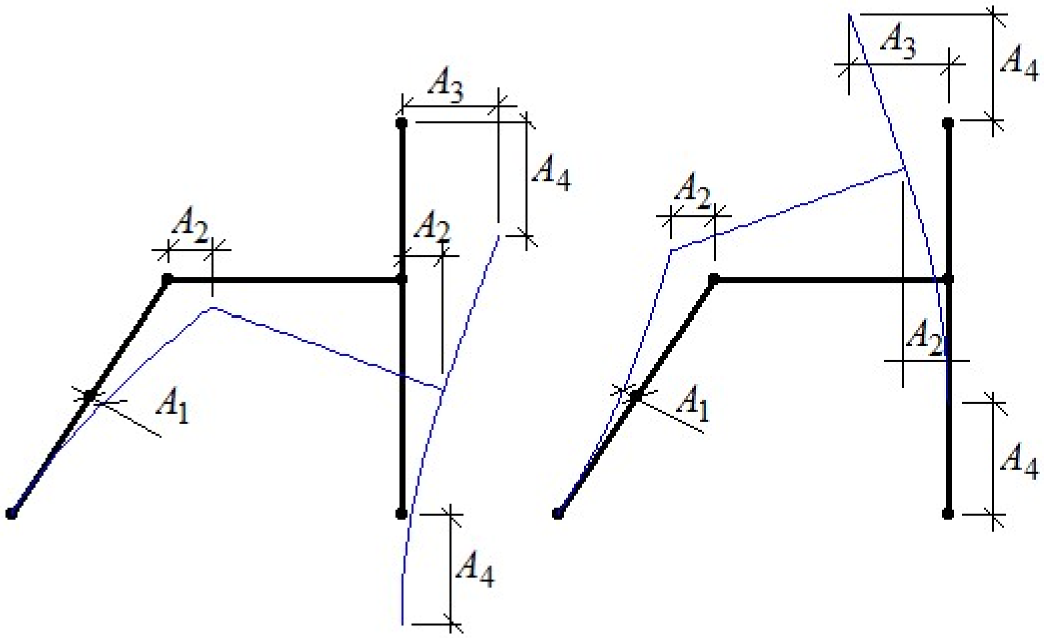
$A_{14} := 1$

$$\begin{bmatrix} A_{24} \\ A_{34} \\ A_{44} \end{bmatrix} := \begin{bmatrix} m'_2 \cdot \delta'_{12} \cdot X_4 & m'_3 \cdot \delta'_{13} \cdot X_4 & m'_4 \cdot \delta'_{14} \cdot X_4 \\ m'_2 \cdot \delta'_{22} \cdot X_4 - 1 & m'_3 \cdot \delta'_{23} \cdot X_4 & m'_4 \cdot \delta'_{24} \cdot X_4 \\ m'_2 \cdot \delta'_{32} \cdot X_4 & m'_3 \cdot \delta'_{33} \cdot X_4 - 1 & m'_4 \cdot \delta'_{34} \cdot X_4 \end{bmatrix}^{-1} \cdot - \begin{bmatrix} \left(m'_1 \cdot \delta'_{11} \cdot X_4 - 1\right) \cdot A_{14} \\ m'_1 \cdot \delta'_{21} \cdot X_4 \cdot A_{14} \\ m'_1 \cdot \delta'_{31} \cdot X_4 \cdot A_{14} \end{bmatrix} = \begin{bmatrix} -1.161 \\ 0.026 \\ -0.076 \end{bmatrix}$$

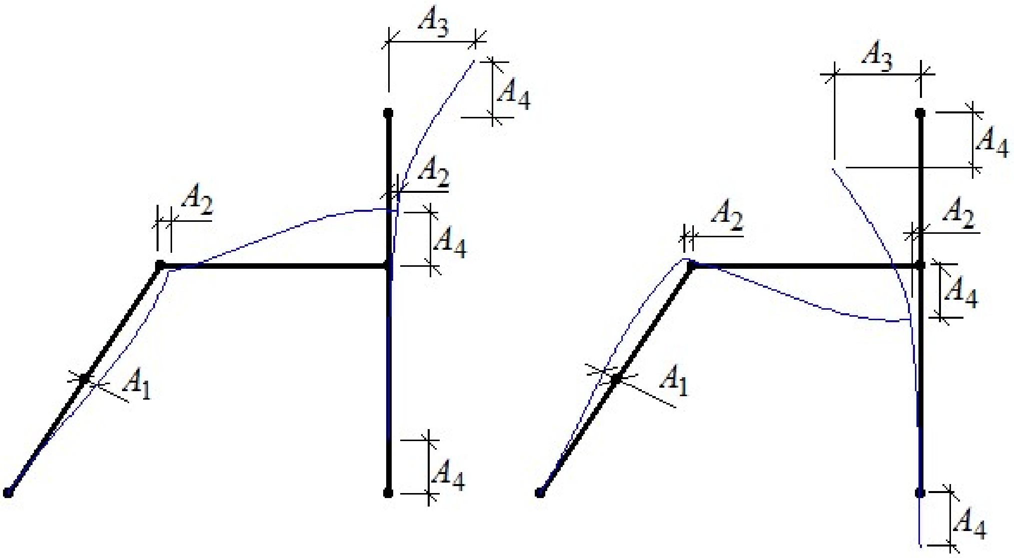
$A_{14} = 1$ $\quad A_{24} = -1.161$ $\quad A_{34} = 0.026$ $\quad A_{44} = -0.076$

Formy drgań własnych:

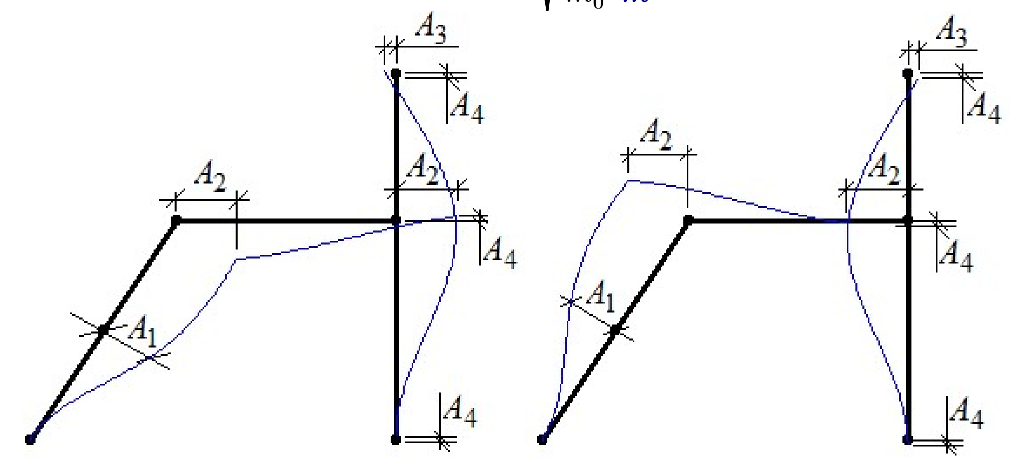
I forma drgań własnych: $\quad \omega_1 := \sqrt{\dfrac{X_1 \cdot EJ}{m_0 \cdot m^3}} = 10.45 \, \dfrac{rad}{s}$



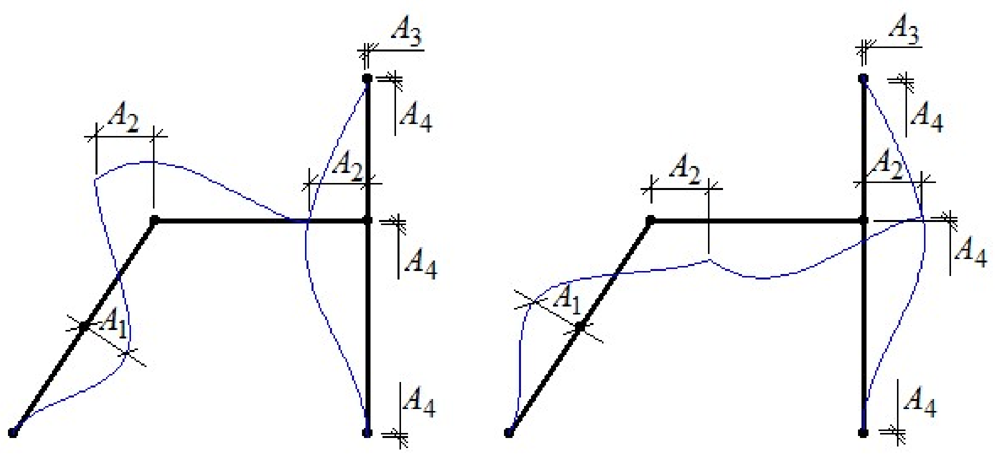
II forma drgań własnych: $\omega_2 := \sqrt{\dfrac{X_2 \cdot EJ}{m_0 \cdot m^3}} = 43.641 \ \dfrac{rad}{s}$



III forma drgań własnych: $\omega_3 := \sqrt{\dfrac{X_3 \cdot EJ}{m_0 \cdot m^3}} = 109.86 \ \dfrac{rad}{s}$



IV forma drgań własnych: $\omega_4 := \sqrt{\dfrac{X_4 \cdot EJ}{m_0 \cdot m^3}} = 234.601 \ \dfrac{rad}{s}$



Sprawdzenie ortogonalności:

Para 1 i 2: $m_1 \cdot A_{11} \cdot A_{12} + m_2 \cdot A_{21} \cdot A_{22} + m_3 \cdot A_{31} \cdot A_{32} + m_4 \cdot A_{41} \cdot A_{42} = 0\ kg$

Para 1 i 3: $m_1 \cdot A_{11} \cdot A_{13} + m_2 \cdot A_{21} \cdot A_{23} + m_3 \cdot A_{31} \cdot A_{33} + m_4 \cdot A_{41} \cdot A_{43} = 0\ kg$

Para 1 i 4: $m_1 \cdot A_{11} \cdot A_{14} + m_2 \cdot A_{21} \cdot A_{24} + m_3 \cdot A_{31} \cdot A_{34} + m_4 \cdot A_{41} \cdot A_{44} = 0\ kg$

Para 2 i 3: $m_1 \cdot A_{12} \cdot A_{13} + m_2 \cdot A_{22} \cdot A_{23} + m_3 \cdot A_{32} \cdot A_{33} + m_4 \cdot A_{42} \cdot A_{43} = 0\ kg$

Para 2 i 4: $m_1 \cdot A_{12} \cdot A_{14} + m_2 \cdot A_{22} \cdot A_{24} + m_3 \cdot A_{32} \cdot A_{34} + m_4 \cdot A_{42} \cdot A_{44} = 0\ kg$

Para 3 i 4: $m_1 \cdot A_{13} \cdot A_{14} + m_2 \cdot A_{23} \cdot A_{24} + m_3 \cdot A_{33} \cdot A_{34} + m_4 \cdot A_{43} \cdot A_{44} = 0\ kg$

Sprawdzenie metodą Dunkerlaya i Reileigh'a:

$$\omega_D := \frac{1}{\sqrt{m_1 \cdot \delta_{11} + m_2 \cdot \delta_{22} + m_3 \cdot \delta_{33} + m_4 \cdot \delta_{44}}} = 10.11\ \frac{rad}{s}$$

$$\delta_1 := m_1 \cdot \delta_{11} + m_2 \cdot \delta_{12} + m_3 \cdot \delta_{13} + m_4 \cdot \delta_{14}$$

$$\delta_2 := m_1 \cdot \delta_{21} + m_2 \cdot \delta_{22} + m_3 \cdot \delta_{23} + m_4 \cdot \delta_{24}$$

$$\delta_3 := m_1 \cdot \delta_{31} + m_2 \cdot \delta_{32} + m_3 \cdot \delta_{33} + m_4 \cdot \delta_{34}$$

$$\delta_4 := m_1 \cdot \delta_{41} + m_2 \cdot \delta_{42} + m_3 \cdot \delta_{43} + m_4 \cdot \delta_{44}$$

$$\omega_R := \frac{\sqrt{m_1 \cdot \delta_1 + m_2 \cdot \delta_2 + m_3 \cdot \delta_3 + m_4 \cdot \delta_4}}{\sqrt{m_1 \cdot \delta_1^{\ 2} + m_2 \cdot \delta_2^{\ 2} + m_3 \cdot \delta_3^{\ 2} + m_4 \cdot \delta_4^{\ 2}}} = 20.597\ \frac{rad}{s}$$

$\omega_D = 10.11\ \frac{rad}{s} \le \omega_1 = 10.45\ \frac{rad}{s} \le \omega_R = 20.597\ \frac{rad}{s}$ warunek spełniony

2. Drgania wymuszone

Sprawdzenie warunku rezonansu:

$0.9\ \omega_1 = 9.405\ \frac{rad}{s} \le \Theta = 12\ \frac{rad}{s} \le 1.1\ \omega_1 = 11.495\ \frac{rad}{s}$

$0.9\ \omega_2 = 39.277\ \frac{rad}{s} \le \Theta = 12\ \frac{rad}{s} \le 1.1\ \omega_2 = 48.005\ \frac{rad}{s}$

$0.9\ \omega_3 = 98.874\ \frac{rad}{s} \le \Theta = 12\ \frac{rad}{s} \le 1.1\ \omega_3 = 120.846\ \frac{rad}{s}$

$0.9\ \omega_4 = 211.141\ \frac{rad}{s} \le \Theta = 12\ \frac{rad}{s} \le 1.1\ \omega_4 = 258.061\ \frac{rad}{s}$

Wniosek: rezonans nie zajdzie.

Wymuszenie jest na kierunku 3, więc:

$\delta_{1P} := \delta_{31} \cdot P_0 = 0.006\ m$ $\delta_{3P} := \delta_{33} \cdot P_0 = 0.045\ m$

$\delta_{2P} := \delta_{32} \cdot P_0 = 0.018\ m$ $\delta_{4P} := \delta_{34} \cdot P_0 = -0.043\ m$

Wyznaczenie sił bezwładności:

$$\left(\delta_{11}-\frac{1}{m_1\cdot\Theta^2}\right)\cdot B_1+\delta_{12}\cdot B_2+\delta_{13}\cdot B_3+\delta_{14}\cdot B_4+\delta_{1P}=0$$

$$\delta_{11}\cdot B_1+\left(\delta_{12}-\frac{1}{m_2\cdot\Theta^2}\right)\cdot B_2+\delta_{13}\cdot B_3+\delta_{14}\cdot B_4+\delta_{2P}=0$$

$$\delta_{11}\cdot B_1+\delta_{12}\cdot B_2+\left(\delta_{13}-\frac{1}{m_3\cdot\Theta^2}\right)\cdot B_3+\delta_{14}\cdot B_4+\delta_{3P}=0$$

$$\delta_{11}\cdot B_1+\delta_{12}\cdot B_2+\delta_{13}\cdot B_3+\left(\delta_{14}-\frac{1}{m_4\cdot\Theta^2}\right)\cdot B_4+\delta_{4P}=0$$

$$\begin{bmatrix}B_1\\B_2\\B_3\\B_4\end{bmatrix}:=\begin{bmatrix}\delta_{11}-\frac{1}{m_1\cdot\Theta^2} & \delta_{12} & \delta_{13} & \delta_{14}\\ \delta_{21} & \delta_{22}-\frac{1}{m_2\cdot\Theta^2} & \delta_{23} & \delta_{24}\\ \delta_{31} & \delta_{32} & \delta_{33}-\frac{1}{m_3\cdot\Theta^2} & \delta_{34}\\ \delta_{41} & \delta_{42} & \delta_{43} & \delta_{44}-\frac{1}{m_4\cdot\Theta^2}\end{bmatrix}^{-1}\cdot-\begin{bmatrix}\delta_{1P}\\\delta_{2P}\\\delta_{3P}\\\delta_{4P}\end{bmatrix}$$

$B_1=-1.046\ kN$

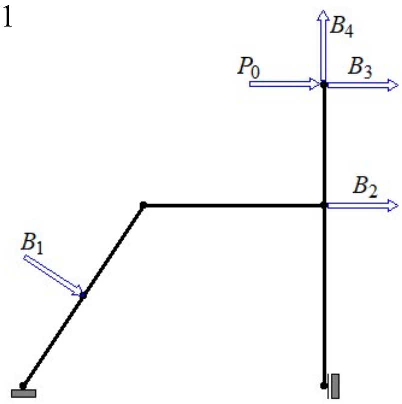
$B_2=-2.381\ kN$

$B_3=-10.661\ kN$

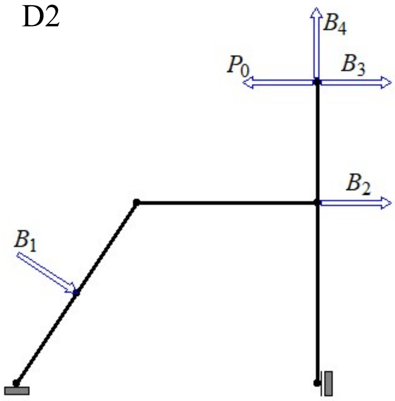
$B_4=18.983\ kN$

Dwa warianty obciążenia statycznego:

D1



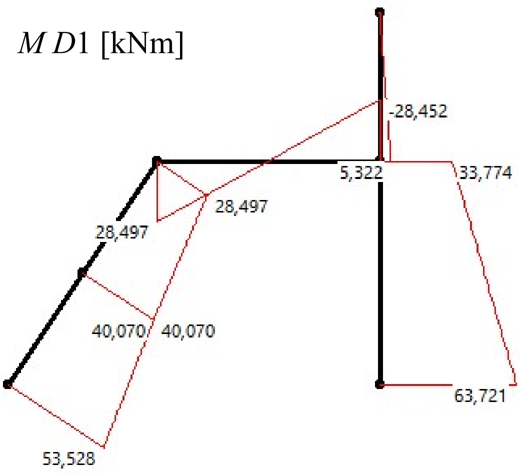
D2

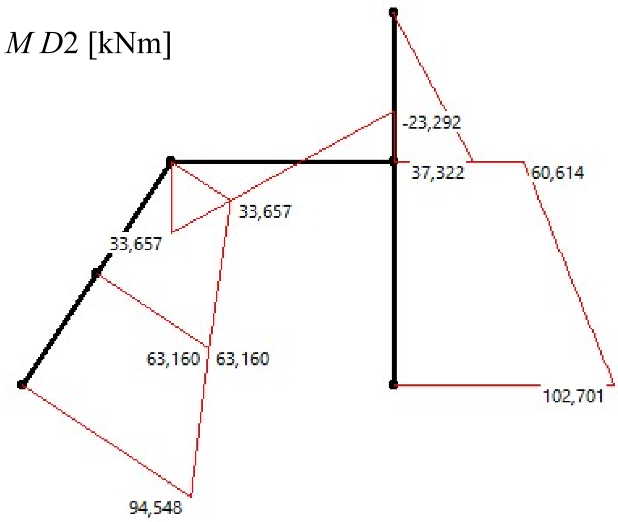


$$M_{D1}:=M_{ost1}\cdot B_1+M_{ost2}\cdot B_2+M_{ost3}\cdot\left(B_3+P_0\right)+M_{ost4}\cdot B_4$$

$$M_{D2}:=M_{ost1}\cdot B_1+M_{ost2}\cdot B_2+M_{ost3}\cdot\left(B_3-P_0\right)+M_{ost4}\cdot B_4$$

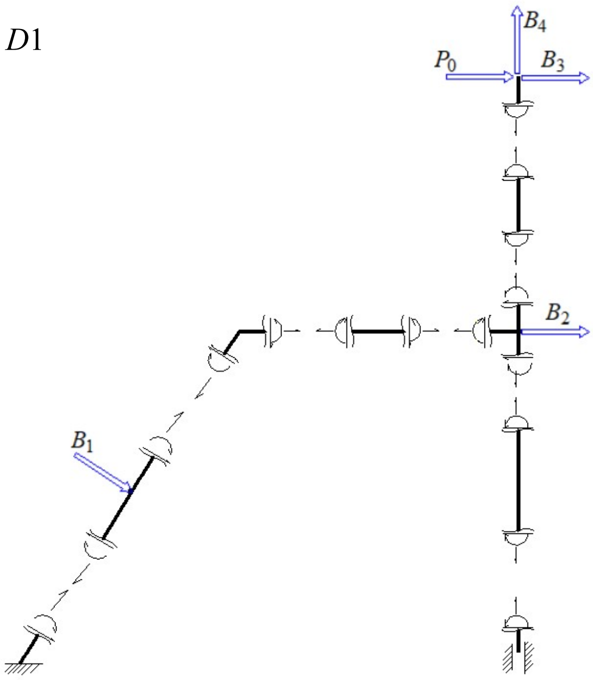
$$M_{D1}=\begin{bmatrix}53.528\\40.069\\28.497\\-28.451\\5.322\\0\\33.773\\63.72\end{bmatrix}kN\cdot m \qquad M_{D2}=\begin{bmatrix}94.548\\63.159\\33.657\\-23.291\\37.322\\0\\60.614\\102.7\end{bmatrix}kN\cdot m$$

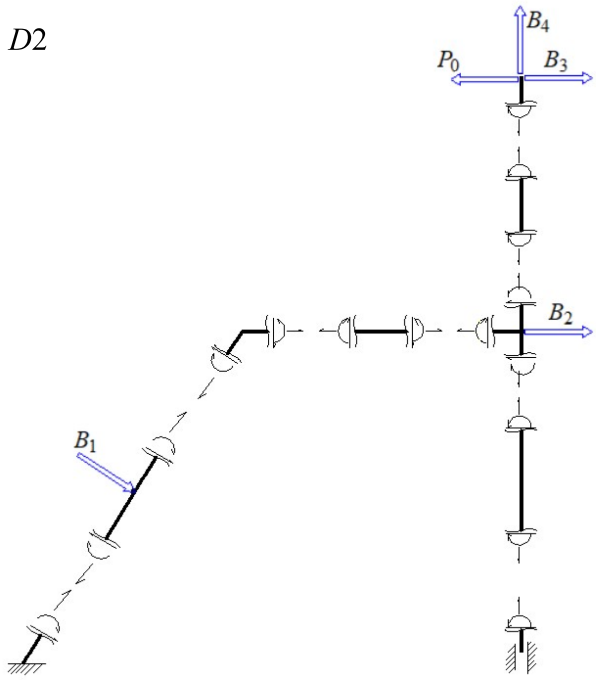


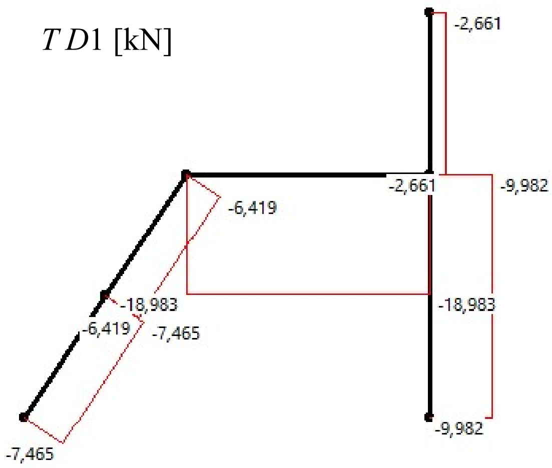


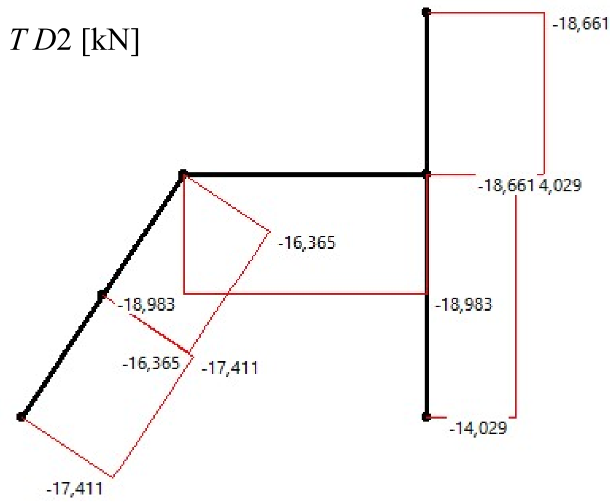
Tnące i normalne ostateczne można obliczyć z poniższego schematu (metoda równoważenia prętów do liczenia tnących i metoda równoważenia węzłów do liczenia normalnych):

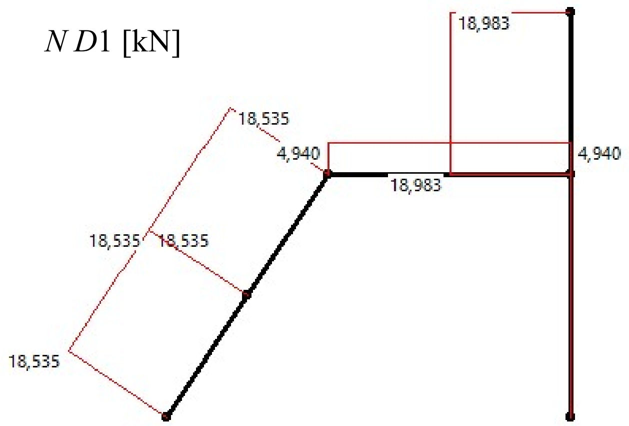
UWAGA! Proszę zamieścić pełne obliczenia w swoim projekcie. Poniżej jest przedstawiona tylko ogólna idea tej metody.

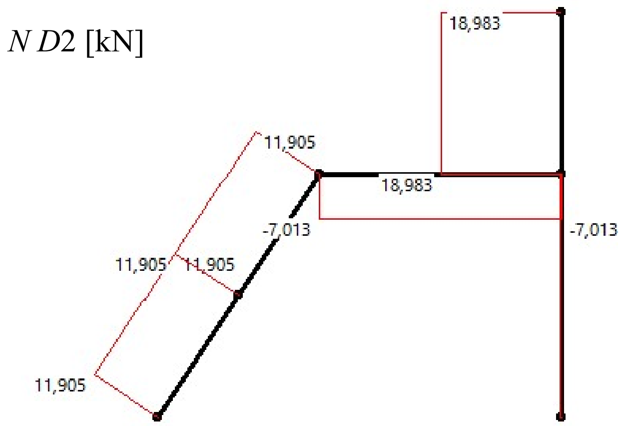









N D1 [kN]
18,983
18,535
4,940
4,940
18,983
18,535
18,535
18,535


N D2 [kN]
18,983
11,905
18,983
-7,013
-7,013
11,905
11,905
11,905