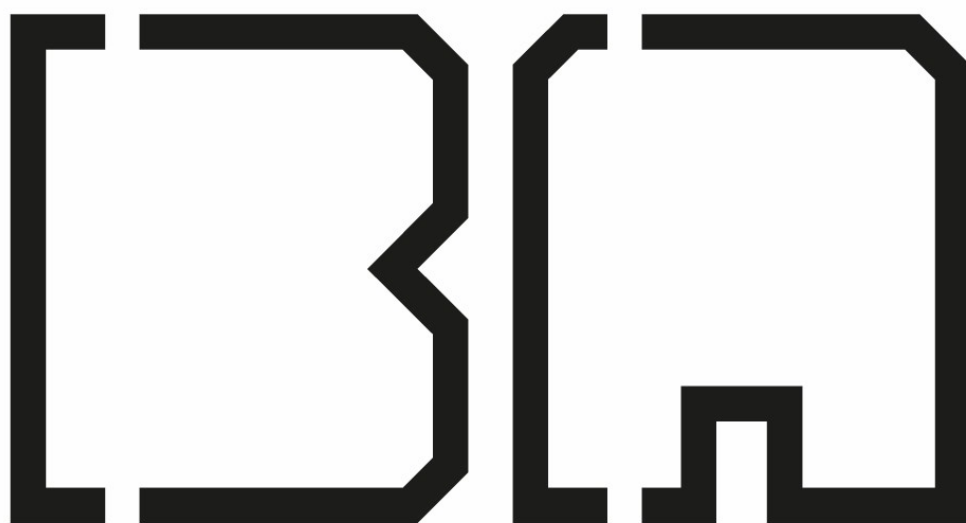


Politechnika Lubelska

Wydział Budownictwa i Architektury



Mechanika Teoretyczna

Projekt 1

Wykonał:

.....

Sprawdził:

dr inż. Jakub Gontarz

PRZYKŁADOWY TEMAT

Imię i Nazwisko:

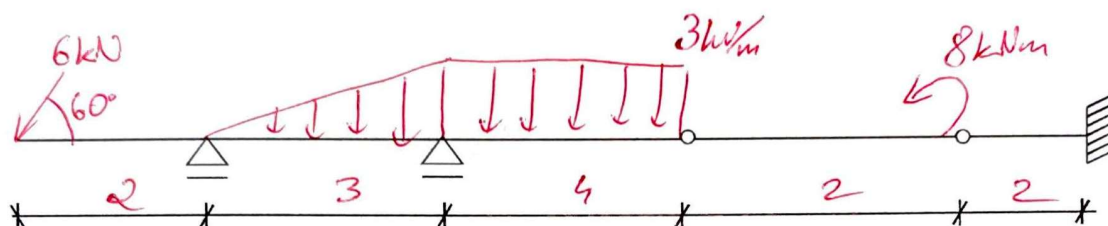
Grupa:

PROJEKT NR 1

MECHANIKA TEORETYCZNA

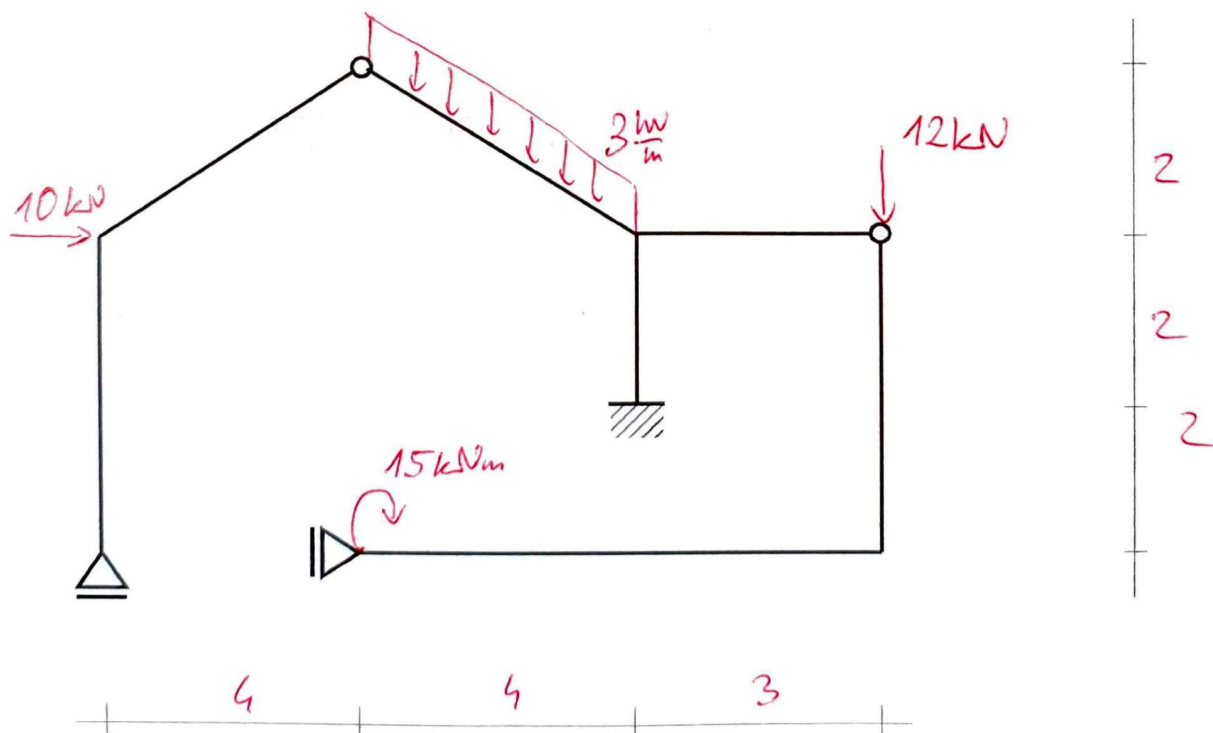
ZADANIE 1.

Obliczyć reakcje podporowe w belce.



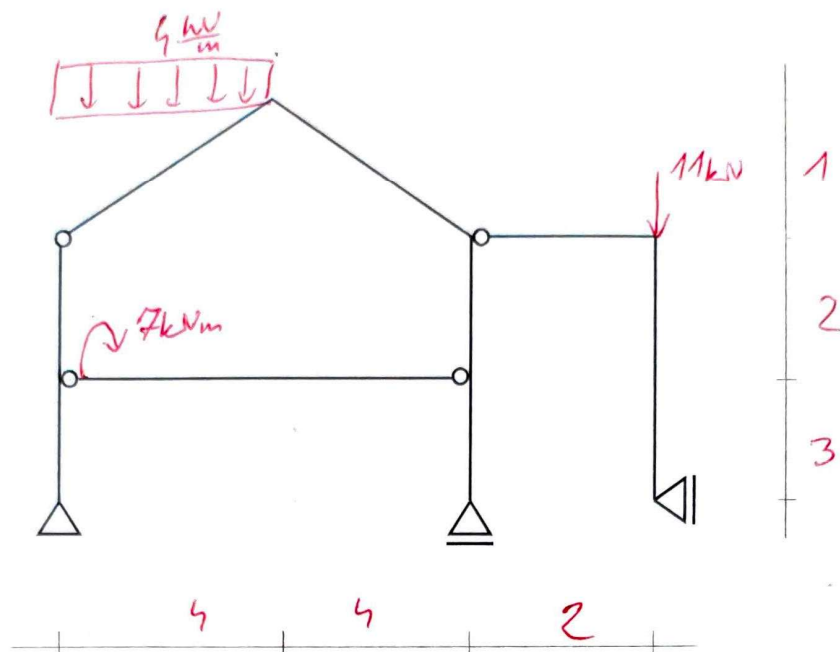
ZADANIE 2.

Obliczyć reakcje podporowe w ramie.



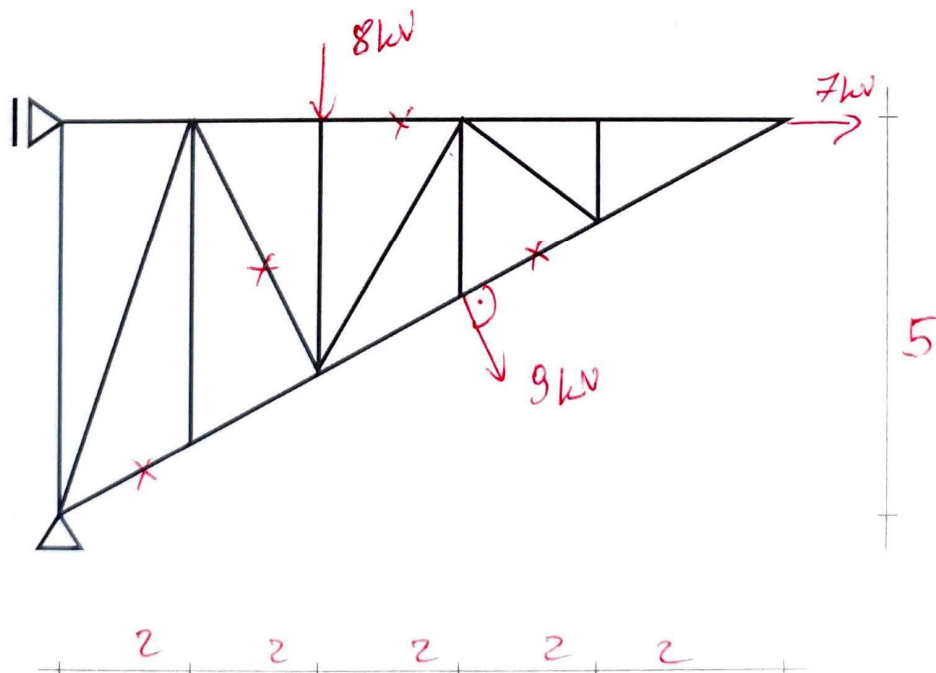
ZADANIE 3.

Obliczyć siły w ściągu.



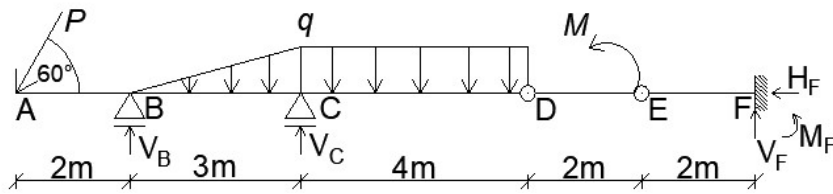
ZADANIE 4.

Obliczyć siły wewnętrzne w kratownicy metodą równoważenia węzłów.
Wykonać sprawdzenie metodą Rittera w zaznaczonych prętach.



G

Zadanie 1: Belka



$$\text{kNm} := \text{kN} \cdot \text{m}$$

$$P := 6 \text{ kN}$$

$$M := 8 \text{ kNm}$$

$$q := 3 \frac{\text{kN}}{\text{m}}$$

Reakcje:

$$\Sigma X = -P \cdot \cos(60^\circ) - H_F = 0$$

$$H_F := -P \cdot \cos(60^\circ) = -3 \text{ kN}$$

$$\Sigma M_D^P = -M - M_F - V_F \cdot 4 = 0$$

$$\Sigma M_E = -M_F - V_F \cdot 2 = 0$$

$$M_F = -M - V_F \cdot 4$$

$$\Sigma M_E = M + V_F \cdot 4 - V_F \cdot 2 = 0$$

$$V_F := \frac{-M}{2 \text{ m}} = -4 \text{ kN}$$

$$M_F := -M - V_F \cdot 4 \text{ m} = 8 \text{ kNm}$$

$$\Sigma M_B = -P \cdot \sin(60^\circ) \cdot 2 + \frac{1}{2} \cdot q \cdot 3 \cdot \frac{2}{3} \cdot 3 + q \cdot 4 \cdot 5 - M - V_F \cdot 11 - M_F - V_C \cdot 3 = 0$$

$$V_C := \frac{-P \cdot \sin(60^\circ) \cdot 2 \text{ m} + \frac{1}{2} \cdot q \cdot 3 \text{ m} \cdot \frac{2}{3} \cdot 3 \text{ m} + q \cdot 4 \text{ m} \cdot 5 \text{ m} - M - V_F \cdot 11 \text{ m} - M_F}{3 \text{ m}} = 28.869 \text{ kN}$$

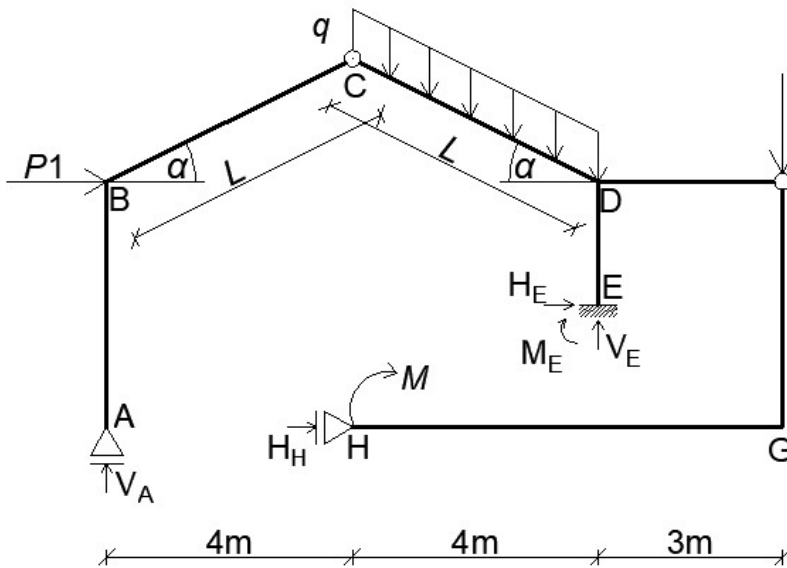
$$\Sigma Y = -P \cdot \sin(60^\circ) - \frac{1}{2} \cdot q \cdot 3 - q \cdot 4 + V_B + V_C + V_F = 0$$

$$V_B := P \cdot \sin(60^\circ) + \frac{1}{2} \cdot q \cdot 3 \text{ m} + q \cdot 4 \text{ m} - V_C - V_F = -3.173 \text{ kN}$$

Sprawdzenie:

$$\Sigma M_A := \frac{1}{2} \cdot q \cdot 3 \text{ m} \cdot \left(2 \text{ m} + \frac{2}{3} \cdot 3 \text{ m}\right) + q \cdot 4 \text{ m} \cdot 7 \text{ m} - M - V_B \cdot 2 \text{ m} - V_C \cdot 5 \text{ m} - M_F - V_F \cdot 13 \text{ m} = 0 \text{ kNm}$$

Zadanie 2: Rama



$$P1 := 10 \text{ kN}$$

$$P2 := 12 \text{ kN}$$

$$M := 15 \text{ kNm}$$

$$q := 3 \frac{\text{kN}}{\text{m}}$$

$$L := \sqrt{(4 \text{ m})^2 + (2 \text{ m})^2} = 4.472 \text{ m}$$

$$\sin \alpha := \frac{2 \text{ m}}{L} = 0.447$$

$$\cos \alpha := \frac{4 \text{ m}}{L} = 0.894$$

Reakcje:

$$\Sigma M_C^L = -P1 \cdot 2 + V_A \cdot 4 = 0 \quad V_A := \frac{P1 \cdot 2 \text{ m}}{4 \text{ m}} = 5 \text{ kN}$$

$$\Sigma M_F^D = M - H_H \cdot 4 = 0 \quad H_H := \frac{M}{4 \text{ m}} = 3.75 \text{ kN}$$

$$\Sigma X = P1 + H_E + H_H = 0 \quad H_E := -P1 - H_H = -13.75 \text{ kN}$$

$$\Sigma Y = V_A - q \cdot L - P2 + V_E = 0 \quad V_E := -V_A + q \cdot L + P2 = 20.416 \text{ kN}$$

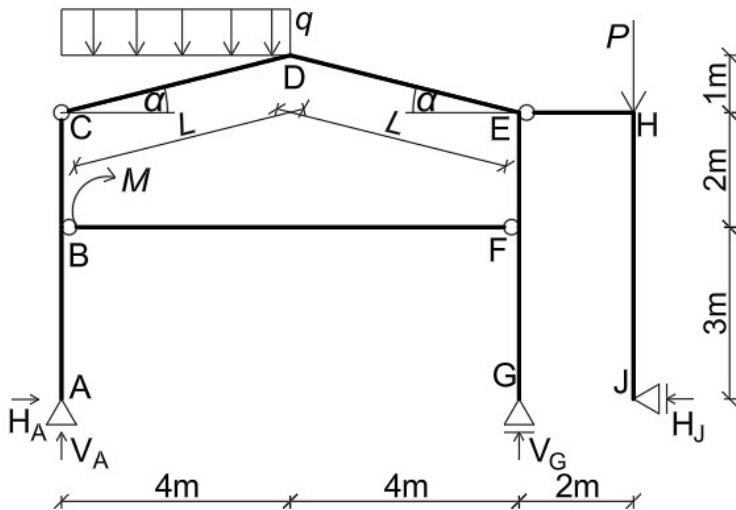
$$\Sigma M_C^P = q \cdot L \cdot 2 + P2 \cdot 7 + M - H_E \cdot 4 - V_E \cdot 4 + M_E - H_H \cdot 6 = 0$$

$$M_E := -q \cdot L \cdot 2 \text{ m} - P2 \cdot 7 \text{ m} - M + H_E \cdot 4 \text{ m} + V_E \cdot 4 \text{ m} + H_H \cdot 6 \text{ m} = -76.667 \text{ kNm}$$

Sprawdzenie:

$$\Sigma M_F := V_A \cdot 11 \text{ m} - q \cdot L \cdot 5 \text{ m} - H_E \cdot 2 \text{ m} + V_E \cdot 3 \text{ m} + M_E - H_H \cdot 4 \text{ m} + M = 0 \text{ kNm}$$

Zadanie 3: Siły w ściągu



$$P := 11 \text{ kN}$$

$$M := 7 \text{ kNm}$$

$$q := 4 \frac{\text{kN}}{\text{m}}$$

$$L := \sqrt{(4 \text{ m})^2 + (1 \text{ m})^2} = 4.123 \text{ m}$$

$$\sin \alpha := \frac{1 \text{ m}}{L} = 0.243$$

$$\cos \alpha := \frac{4 \text{ m}}{L} = 0.97$$

Reakcje:

$$\Sigma M_E^P = P \cdot 2 + H_J \cdot 5 = 0$$

$$H_J := -\frac{P \cdot 2 \text{ m}}{5 \text{ m}} = -4.4 \text{ kN}$$

$$\Sigma M_A = q \cdot 4 \cdot 2 + M - V_G \cdot 8 + P \cdot 10 = 0$$

$$V_G := \frac{q \cdot 4 \text{ m} \cdot 2 \text{ m} + M + P \cdot 10 \text{ m}}{8 \text{ m}} = 18.625 \text{ kN}$$

$$\Sigma X = H_A - H_J = 0$$

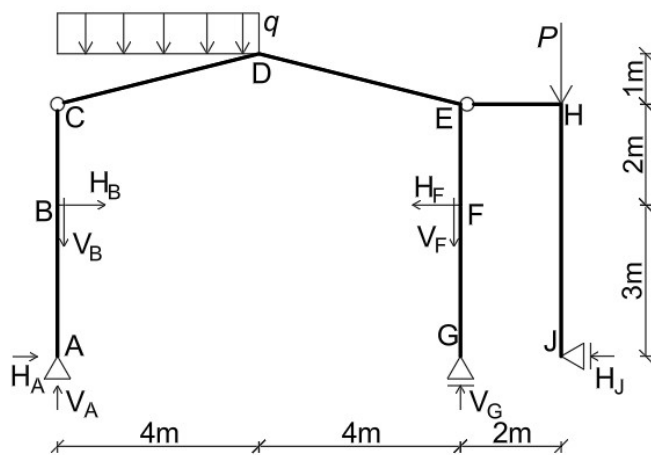
$$H_A := H_J = -4.4 \text{ kN}$$

$$\Sigma Y = V_A + V_G - q \cdot 4 - P = 0$$

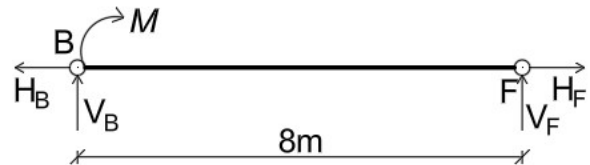
$$V_A := -V_G + q \cdot 4 \text{ m} + P = 8.375 \text{ kN}$$

$$\text{Spr.: } \Sigma M_D := V_A \cdot 4 \text{ m} - H_A \cdot 6 \text{ m} + M - q \cdot 4 \text{ m} \cdot 2 \text{ m} + P \cdot 6 \text{ m} - V_G \cdot 4 \text{ m} + H_J \cdot 6 \text{ m} = 0 \text{ kNm}$$

Rama:



Ściąg:



$$\text{Ściąg: } \Sigma M_B = M - V_F \cdot 8 = 0$$

$$V_F := \frac{M}{8 \text{ m}} = 0.875 \text{ kN}$$

$$\text{Ściąg: } \Sigma Y = V_B + V_F = 0$$

$$V_B := -V_F = -0.875 \text{ kN}$$

$$\text{Rama: } \Sigma M_C^L = -H_A \cdot 5 - H_B \cdot 2 = 0$$

$$H_B := \frac{-H_A \cdot 5 \text{ m}}{2 \text{ m}} = 11 \text{ kN}$$

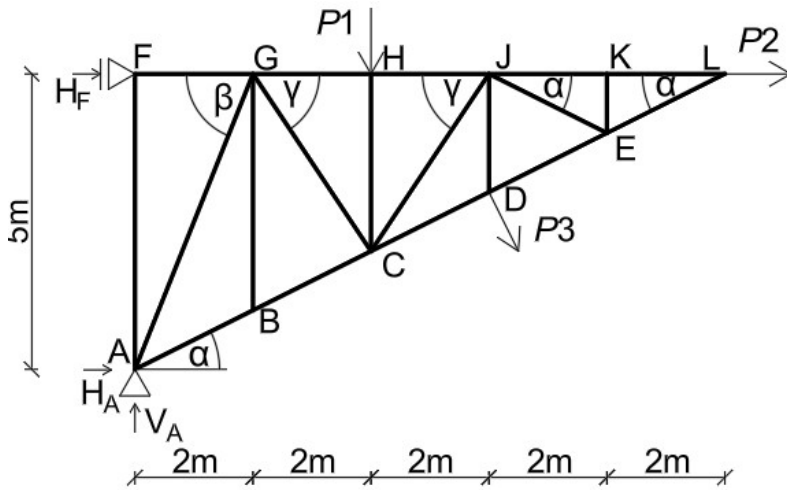
$$\text{Ściąg: } \Sigma X = -H_B + H_F = 0$$

$$H_F := H_B = 11 \text{ kN}$$

Spr.:

$$\text{Rama: } \Sigma M_C^P = q \cdot 4 \text{ m} \cdot 2 \text{ m} + P \cdot 10 \text{ m} + H_J \cdot 5 \text{ m} - V_G \cdot 8 \text{ m} + H_F \cdot 2 \text{ m} + V_F \cdot 8 \text{ m} = 0 \text{ kNm}$$

Zadanie 4: Kratownica



$$P1 := 8 \text{ kN}$$

$$P2 := 7 \text{ kN}$$

$$P3 := 9 \text{ kN}$$

$$\sin \alpha := \frac{5}{\sqrt{5^2 + 10^2}} = 0.447$$

$$\sin \beta := \frac{5}{\sqrt{5^2 + 2^2}} = 0.928$$

$$\sin \gamma := \frac{3}{\sqrt{3^2 + 2^2}} = 0.832$$

$$\cos \alpha := \frac{10}{\sqrt{5^2 + 10^2}} = 0.894$$

$$\cos \beta := \frac{2}{\sqrt{5^2 + 2^2}} = 0.371$$

$$\cos \gamma := \frac{2}{\sqrt{3^2 + 2^2}} = 0.555$$

Reakcje:

$$\Sigma M_A = H_F \cdot 5 + P1 \cdot 4 + P2 \cdot 5 + P3 \cdot \cos \alpha \cdot 6 + P3 \cdot \sin \alpha \cdot 3$$

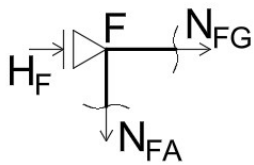
$$H_F := \frac{-P1 \cdot 4 \text{ m} - P2 \cdot 5 \text{ m} - P3 \cdot \cos \alpha \cdot 6 \text{ m} - P3 \cdot \sin \alpha \cdot 3 \text{ m}}{5 \text{ m}} = -25.475 \text{ kN}$$

$$\Sigma X = H_F + H_A + P2 + P3 \cdot \sin \alpha = 0 \quad H_A := -H_F - P2 - P3 \cdot \sin \alpha = 14.45 \text{ kN}$$

$$\Sigma Y = V_A - P1 - P3 \cdot \cos \alpha = 0 \quad V_A := P1 + P3 \cdot \cos \alpha = 16.05 \text{ kN}$$

Siły wewnętrzne:

Węzeł F



$$\Sigma Y = -N_{FA} = 0$$

$$N_{FA} := 0 \text{ kN}$$

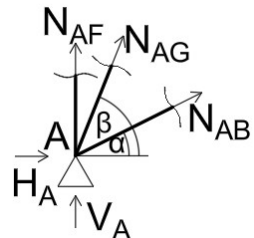
$$N_{AF} := N_{FA}$$

$$\Sigma X = H_F + N_{FG} = 0$$

$$N_{FG} := -H_F = 25.475 \text{ kN}$$

$$N_{GF} := N_{FG}$$

Węzeł A



układ równań:

$$\Sigma X = H_A + N_{AB} \cdot \cos \alpha + N_{AG} \cdot \cos \beta = 0$$

$$\Sigma Y = V_A + N_{AF} + N_{AB} \cdot \sin \alpha + N_{AG} \cdot \sin \beta = 0$$

$$N_{AB} = \frac{-H_A - N_{AG} \cdot \cos \beta}{\cos \alpha}$$

$$\Sigma Y = V_A + N_{AF} + \frac{-H_A - N_{AG} \cdot \cos \beta}{\cos \alpha} \cdot \sin \alpha + N_{AG} \cdot \sin \beta = 0$$

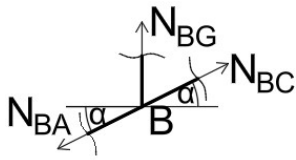
$$N_{AG} := \frac{-V_A - N_{AF} + H_A \cdot \frac{\sin \alpha}{\cos \alpha}}{\sin \beta - \cos \beta \cdot \frac{\sin \alpha}{\cos \alpha}} = -11.881 \text{ kN}$$

$$N_{AB} := \frac{-H_A - N_{AG} \cdot \cos \beta}{\cos \alpha} = -11.222 \text{ kN}$$

$$N_{GA} := N_{AG}$$

$$N_{BA} := N_{AB}$$

Węzeł B



$$\Sigma X = -N_{BA} \cdot \cos \alpha + N_{BC} \cdot \cos \alpha = 0$$

$$N_{BC} := N_{BA} = -11.222 \text{ kN}$$

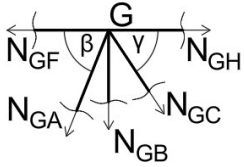
$$N_{CB} := N_{BC}$$

$$\Sigma Y = -N_{BA} \cdot \sin \alpha + N_{BC} \cdot \sin \alpha + N_{BG} = 0$$

$$N_{BG} := N_{BA} \cdot \sin \alpha - N_{BC} \cdot \sin \alpha = 0 \text{ kN}$$

$$N_{GB} := N_{BG}$$

Węzeł G



$$\Sigma Y = -N_{GA} \cdot \sin \beta - N_{GB} - N_{GC} \cdot \sin \gamma = 0$$

$$N_{GC} := \frac{-N_{GA} \cdot \sin \beta - N_{GB}}{\sin \gamma} = 13.258 \text{ kN}$$

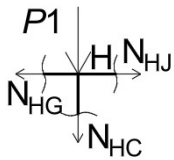
$$N_{CG} := N_{GC}$$

$$\Sigma X = -N_{GF} - N_{GA} \cdot \cos \beta + N_{GC} \cdot \cos \gamma + N_{GH} = 0$$

$$N_{GH} := N_{GF} + N_{GA} \cdot \cos \beta - N_{GC} \cdot \cos \gamma = 13.708 \text{ kN}$$

$$N_{HG} := N_{GH}$$

Węzeł H



$$\Sigma X = -N_{HG} + N_{HJ} = 0$$

$$N_{HJ} := N_{HG} = 13.708 \text{ kN}$$

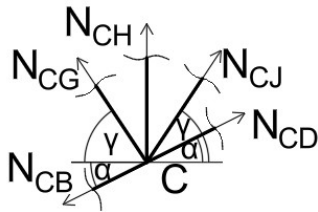
$$N_{JH} := N_{HJ}$$

$$\Sigma Y = -N_{HC} - P1 = 0$$

$$N_{HC} := -P1 = -8 \text{ kN}$$

$$N_{CH} := N_{HC}$$

Węzeł C



układ równań:

$$\Sigma X = -N_{CB} \cdot \cos \alpha - N_{CG} \cdot \cos \gamma + N_{CJ} \cdot \cos \gamma + N_{CD} \cdot \cos \alpha = 0$$

$$\Sigma Y = -N_{CB} \cdot \sin \alpha + N_{CG} \cdot \sin \gamma + N_{CH} + N_{CJ} \cdot \sin \gamma + N_{CD} \cdot \sin \alpha = 0$$

$$N_{CJ} = \frac{N_{CB} \cdot \cos \alpha + N_{CG} \cdot \cos \gamma - N_{CD} \cdot \cos \alpha}{\cos \gamma}$$

$$\Sigma Y = -N_{CB} \cdot \sin \alpha + N_{CG} \cdot \sin \gamma + N_{CH} + (N_{CB} \cdot \cos \alpha + N_{CG} \cdot \cos \gamma) \cdot \frac{\sin \gamma}{\cos \gamma} - N_{CD} \cdot \cos \alpha \cdot \frac{\sin \gamma}{\cos \gamma} + N_{CD} \cdot \sin \alpha = 0$$

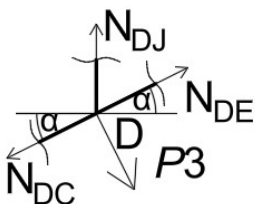
$$N_{CD} := \frac{N_{CB} \cdot \sin \alpha - N_{CG} \cdot \sin \gamma - N_{CH} - (N_{CB} \cdot \cos \alpha + N_{CG} \cdot \cos \gamma) \cdot \frac{\sin \gamma}{\cos \gamma}}{\sin \alpha - \cos \alpha \cdot \frac{\sin \gamma}{\cos \gamma}} = 4.5 \text{ kN}$$

$$N_{DC} := N_{CD}$$

$$N_{CJ} := \frac{N_{CB} \cdot \cos \alpha + N_{CG} \cdot \cos \gamma - N_{CD} \cdot \cos \alpha}{\cos \gamma} = -12.093 \text{ kN}$$

$$N_{JC} := N_{CJ}$$

Węzeł D



$$\Sigma X = -N_{DC} \cdot \cos \alpha + N_{DE} \cdot \cos \alpha + P3 \cdot \sin \alpha = 0$$

$$N_{DE} := \frac{N_{DC} \cdot \cos \alpha - P3 \cdot \sin \alpha}{\cos \alpha} = 0 \text{ kN}$$

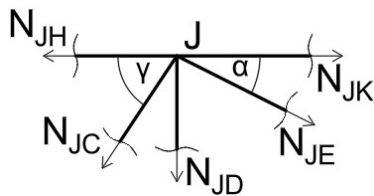
$$N_{ED} := N_{DE}$$

$$\Sigma Y = -N_{DC} \cdot \sin \alpha + N_{DE} \cdot \sin \alpha + N_{DJ} - P3 \cdot \cos \alpha = 0$$

$$N_{DJ} := N_{DC} \cdot \sin \alpha - N_{DE} \cdot \sin \alpha + P3 \cdot \cos \alpha = 10.062 \text{ kN}$$

$$N_{JD} := N_{DJ}$$

Węzeł J



$$\Sigma Y = -N_{JC} \cdot \sin \gamma - N_{JD} - N_{JE} \cdot \sin \alpha = 0$$

$$N_{JE} := \frac{-N_{JC} \cdot \sin \gamma - N_{JD}}{\sin \alpha} = 0 \text{ kN}$$

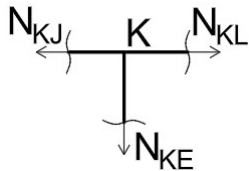
$$N_{EJ} := N_{JE}$$

$$\Sigma X = -N_{JH} - N_{JC} \cdot \cos \gamma + N_{JE} \cdot \sin \alpha + N_{JK} = 0$$

$$N_{JK} := N_{JH} + N_{JC} \cdot \cos \gamma - N_{JE} \cdot \sin \alpha = 7 \text{ kN}$$

$$N_{KJ} := N_{JK}$$

Węzeł K



$$\Sigma X = -N_{KJ} + N_{KL} = 0$$

$$N_{KL} := N_{KJ} = 7 \text{ kN}$$

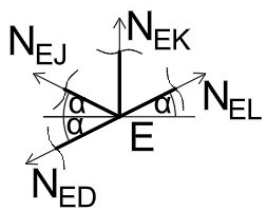
$$N_{LK} := N_{KL}$$

$$\Sigma Y = -N_{KE} = 0$$

$$N_{KE} := 0 \text{ kN}$$

$$N_{EK} := N_{KE}$$

Węzeł E



$$\Sigma X = -N_{ED} \cdot \cos \alpha - N_{EJ} \cdot \cos \alpha + N_{EL} \cdot \cos \alpha = 0$$

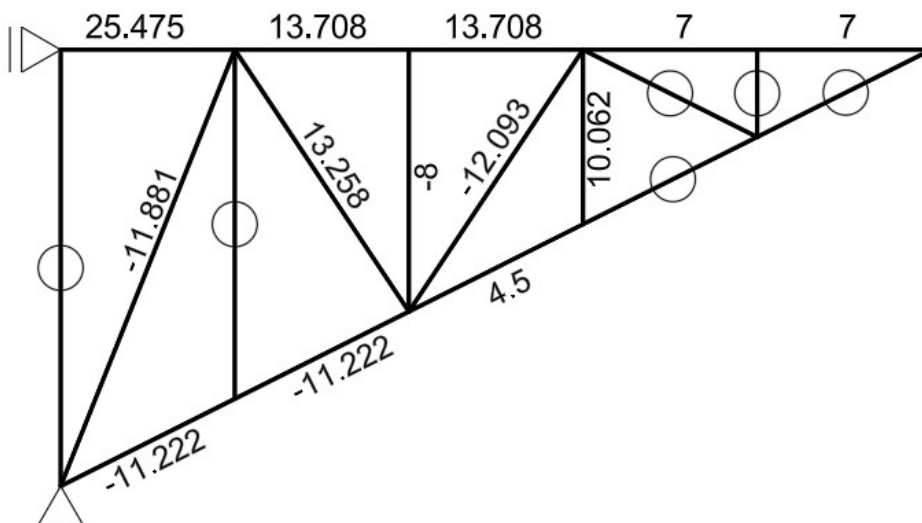
$$N_{EL} := N_{ED} + N_{EJ} = 0 \text{ kN}$$

Sprawdzenie:

$$\Sigma Y = -N_{ED} \cdot \sin \alpha + N_{EJ} \cdot \sin \alpha + N_{EK} + N_{EL} \cdot \sin \alpha = 0 \text{ kN}$$

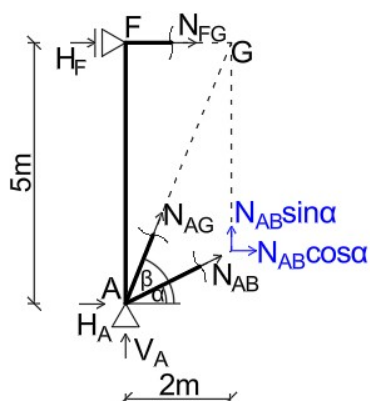
Warunek spełniony

N [kN]



Sprawdzenie metodą Rittera

N_{AB} :

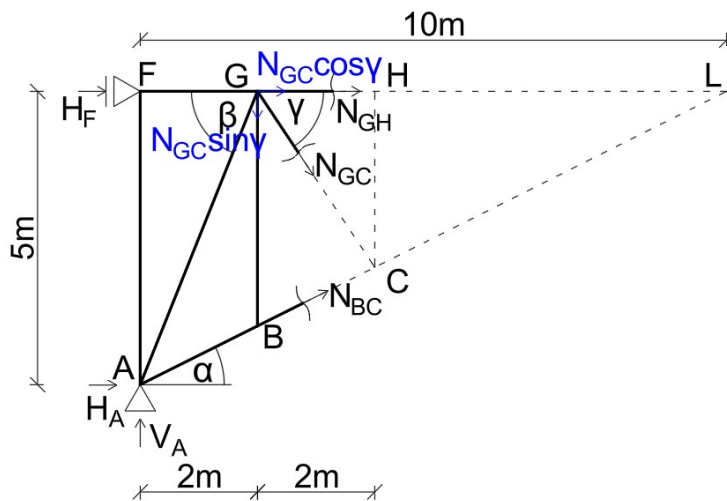


$$\Sigma M_G = V_A \cdot 2 - H_A \cdot 5 - N_{AB} \cdot \cos \alpha \cdot 4 = 0$$

$$N_{AB} := \frac{V_A \cdot 2 - H_A \cdot 5}{\cos \alpha \cdot 4} = -11.222 \text{ kN}$$

Warunek spełniony

N_{GC} :

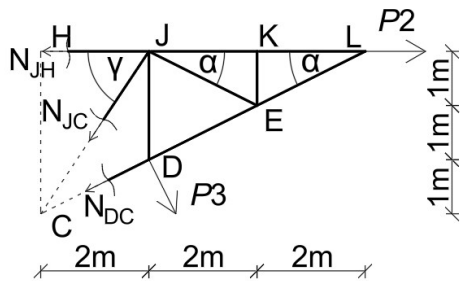


$$\Sigma M_L = V_A \cdot 10 - H_A \cdot 5 - N_{GC} \cdot \sin \gamma \cdot 8 = 0$$

$$N_{GC} := \frac{V_A \cdot 10 - H_A \cdot 5}{\sin \gamma \cdot 8} = 13.258 \text{ kN}$$

Warunek spełniony

N_{JH} :

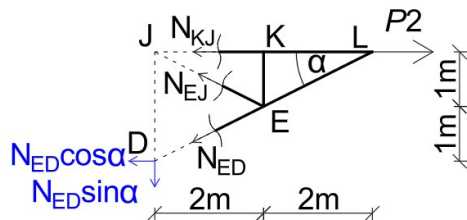


$$\Sigma M_C = -N_{JH} \cdot 3 + P3 \cdot \cos \alpha \cdot 2 + P3 \cdot \sin \alpha \cdot 1 + P2 \cdot 3 = 0$$

$$N_{JH} := \frac{P3 \cdot \cos \alpha \cdot 2 \text{ m} + P3 \cdot \sin \alpha \cdot 1 \text{ m} + P2 \cdot 3 \text{ m}}{3 \text{ m}} = 13.708 \text{ kN}$$

Warunek spełniony

N_{ED} :



$$\Sigma M_J = N_{ED} \cdot \cos \alpha \cdot 2 = 0$$

$$N_{ED} := 0 \text{ kN}$$

Warunek spełniony