

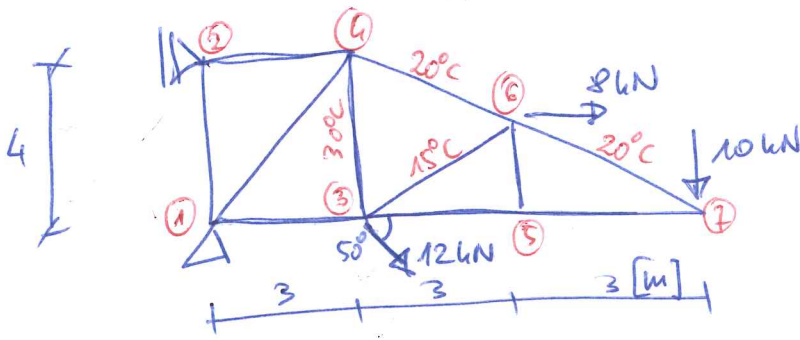
Wyznaczenie prędowności / obrotów w kierunku.

materiał:

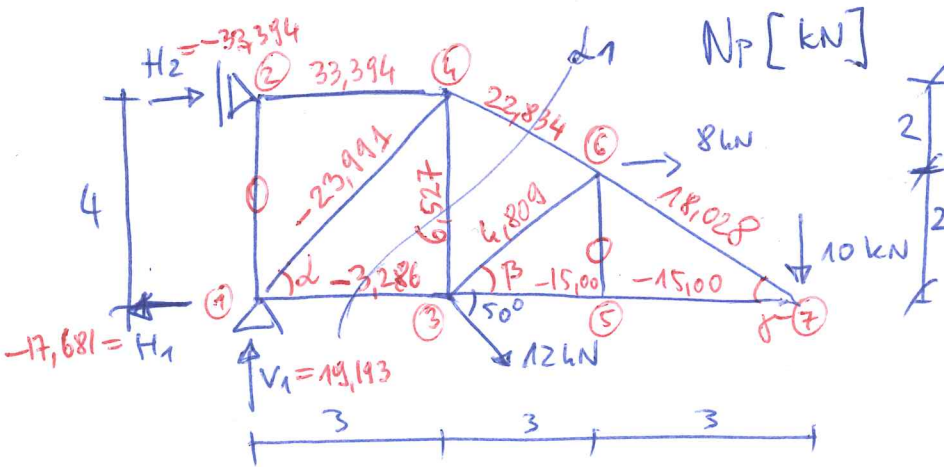
$E = 210 \text{ GPa}$  (stal)  
 $\alpha = 1,2 \cdot 10^{-5} \frac{1}{^\circ\text{C}}$

profil:

$\phi 54,0 \times 3,6 \text{ mm}$   
 $A = 5,7 \text{ cm}^2$



1 Rozwiązanie kraty w stanie obc. zastępczej.



zestawienie  $\alpha, \beta, \gamma$

$\cos \alpha = \frac{3}{5}$      $\sin \alpha = \frac{4}{5}$

$\cos \beta = \frac{3}{\sqrt{3^2+2^2}} = 0,8321$

$\sin \beta = \frac{2}{\sqrt{3^2+2^2}} = 0,5547$

$\cos \gamma = \frac{6}{\sqrt{6^2+4^2}} = 0,8321$

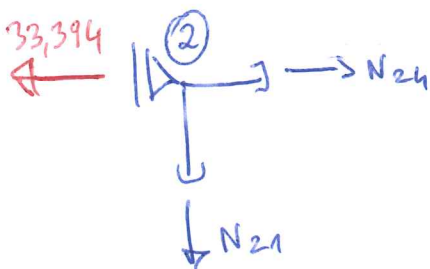
$\sin \gamma = \frac{4}{\sqrt{6^2+4^2}} = 0,5547$

Kratę można rozwiązać metodą równowagi węzłów, albo metodą Rittera, albo Taucz te dwie metody. Można sprawdzić met. równowagi węzłów metodą Rittera.

Rozwiązanie met. równ. węzłów:  $(2) \rightarrow (1) \rightarrow (4) \rightarrow (7) \rightarrow (5) \rightarrow (6) \rightarrow (3)$

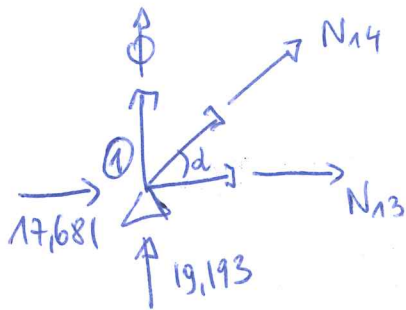
$$\begin{cases} \sum M_1 = 4 \cdot H_2 + 8 \cdot 2 + 10 \cdot 9 + 12 \sin 50^\circ \cdot 3 = 0 \Rightarrow H_2 = -33,394 \text{ kN} & 1 \times \text{spw} \quad 2 \times \text{spw.} \\ \sum X = H_2 - H_1 + 8 + 12 \cos 50^\circ = 0 \Rightarrow H_1 = -17,681 \text{ kN} \\ \sum Y = V_1 - 10 - 12 \sin 50^\circ = 0 \Rightarrow V_1 = 19,193 \text{ kN} \end{cases}$$

Spw  $\sum M_6 = V_1 \cdot 6 + H_1 \cdot 2 + H_2 \cdot 2 + 10 \cdot 3 - 12 \cos 50^\circ \cdot 2 - 12 \sin 50^\circ \cdot 3 = 0$

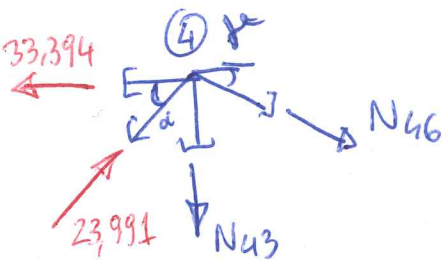


$\sum X = N_{24} - 33,394 = 0 \Rightarrow N_{24} = 33,394 \text{ kN}$

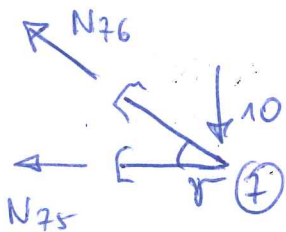
$\sum Y = N_{21} = 0$



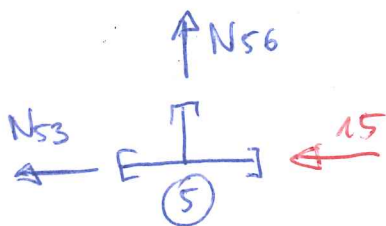
$$\begin{cases} \sum Y = 19,193 + N_{14} \cdot \sin \alpha = 0 \Rightarrow N_{14} = -23,991 \text{ kN} \\ \sum X = 17,681 + N_{13} + N_{14} \cos \alpha = 0 \Rightarrow N_{13} = -3,286 \text{ kN} \end{cases}$$



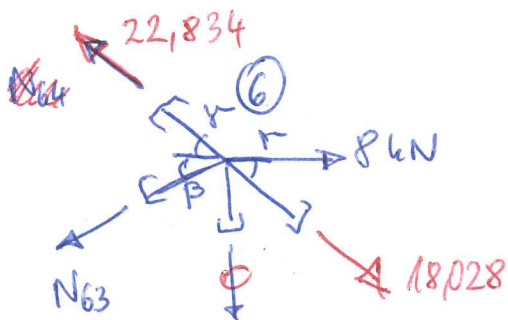
$$\begin{cases} \sum X = N_{46} \cdot \cos \gamma - 33,394 + 23,991 \cdot \cos \alpha = 0 \Rightarrow N_{46} = 22,834 \text{ kN} \\ \sum Y = -N_{43} + 23,991 \cdot \sin \alpha - N_{46} \cdot \sin \gamma = 0 \Rightarrow N_{43} = 6,527 \text{ kN} \end{cases}$$



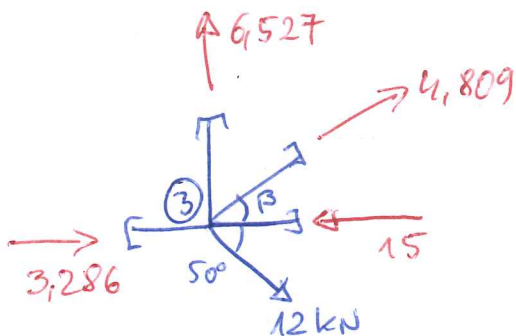
$$\begin{cases} \sum Y = -10 + N_{76} \sin \gamma = 0 \Rightarrow N_{76} = 18,028 \text{ kN} \\ \sum X = -N_{75} - N_{76} \cos \gamma = 0 \Rightarrow N_{75} = -15 \text{ kN} \end{cases}$$



$$\begin{cases} \sum Y = N_{56} = 0 \\ \sum X = -N_{53} - 15 = 0 \Rightarrow N_{53} = -15 \text{ kN} \end{cases}$$



$$\sum X = -22,834 \cos \gamma + 8 + 18,028 \cos \gamma - N_{63} \cdot \cos \beta = 0 \Rightarrow N_{63} = 4,809 \text{ kN}$$



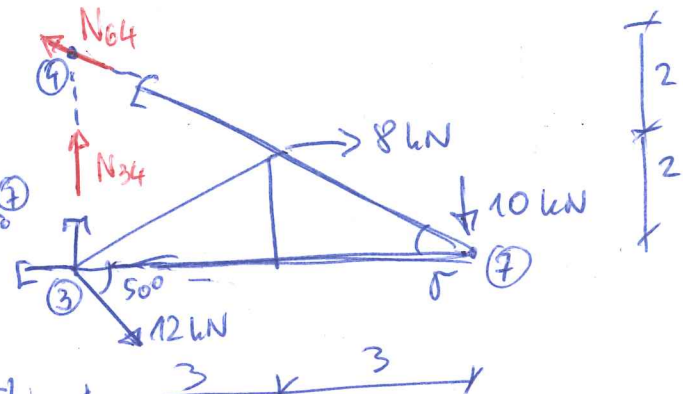
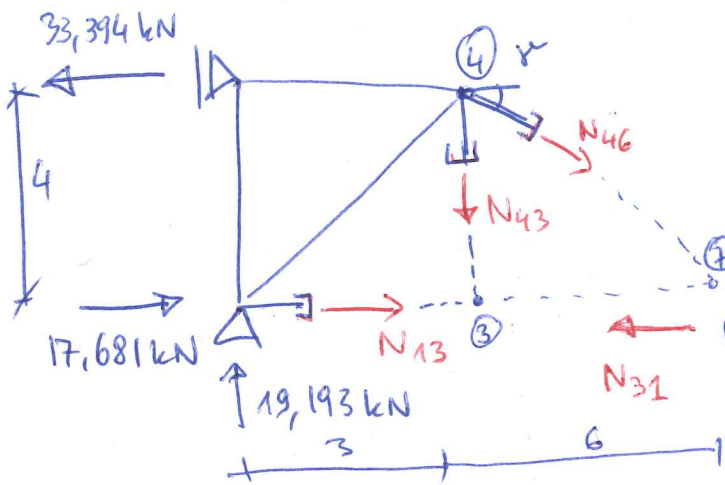
$$\begin{cases} \sum Y = -18,028 \cdot \sin \gamma + 22,834 \sin \gamma + \cancel{N_{63}} - 4,809 \sin \beta = 0 \\ \sum X = 3,286 - 15 + 4,809 \cdot \cos \beta + 12 \cos 50^\circ = 0 \\ \sum Y = 6,527 + 4,809 \sin \beta - 12 \sin 50^\circ = 0 \end{cases}$$

Obliczenie wykonano poprawnie

Sugeruję po węzle np. nr 4 zrobić sprawdzenie met. Ritzera - przełój  $d_1-d_1$

"lewa"

"prawa"



$$\sum M_4^L = -17,681 \cdot 4 + 19,193 \cdot 3 - N_{13} \cdot 4 = 0$$

$$\Rightarrow N_{13} = -3,286 \text{ kN}$$

$$\sum M_3^L = 19,193 \cdot 3 - 33,394 \cdot 4 + N_{46} \cos 8^\circ \cdot 4 = 0$$

$$\Rightarrow N_{46} = 22,834 \text{ kN}$$

$$\sum M_7^L = 19,193 \cdot 9 - 33,394 \cdot 4 - N_{43} \cdot 6 = 0$$

$$\Rightarrow N_{43} = 6,527 \text{ kN}$$

$$\sum M_4^P = N_{31} \cdot 4 - 12 \cos 50^\circ \cdot 4 - 8 \cdot 2 + 10 \cdot 6 = 0$$

$$\Rightarrow N_{31} = -3,287 \text{ kN}$$

$$\sum M_3^P = -N_{64} \cos 8^\circ \cdot 2 - N_{64} \sin 8^\circ \cdot 3 + 8 \cdot 2 + 10 \cdot 6 = 0$$

$$\Rightarrow N_{64} = 22,835 \text{ kN}$$

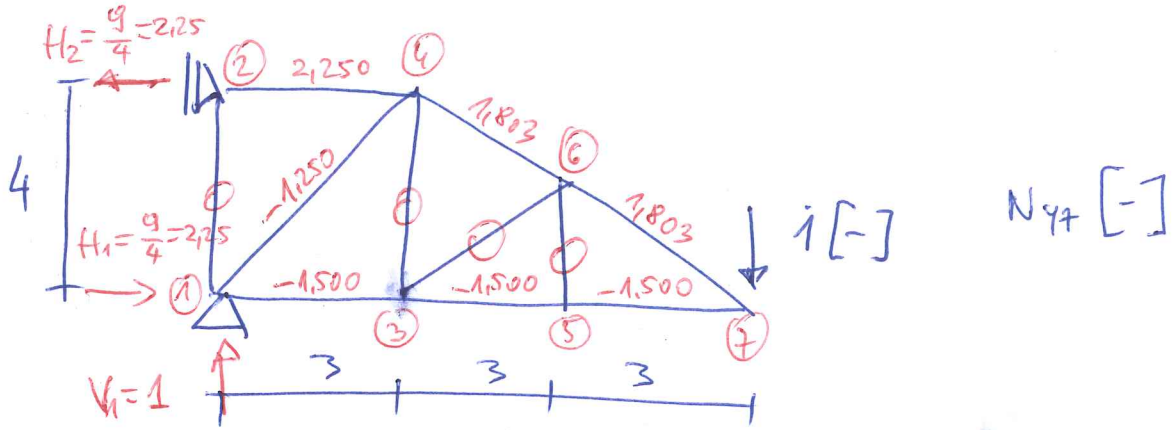
$$\sum M_7^P = N_{34} \cdot 6 - 12 \sin 50^\circ \cdot 6 + 8 \cdot 2 = 0$$

$$\Rightarrow N_{34} = 6,527 \text{ kN}$$

Sugeruję zrobić sprawdzenie met. Ritzera w trzeciej obliczeń met. równowazenie węzłów.



② Wyznaczenie pręmienników płaszczyzny punktu ⑦ - 47

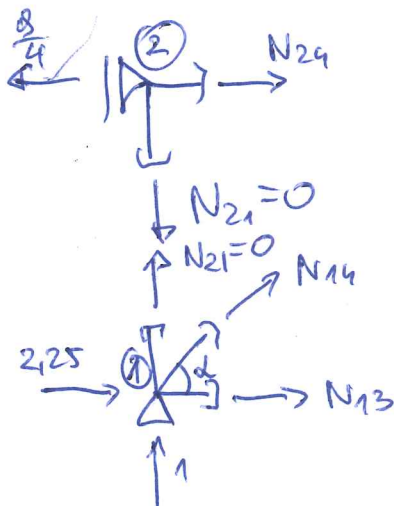


Pręty 56, 36, 34, 12 są zerowe, więc nie uwzględniamy ich w dalszym obliczeniach sił normalnych.

$$\sum Y = V_1 - 1 = 0 \Rightarrow V_1 = 1 [-]$$

$$\sum M_1 = -H_2 \cdot 4 + 1 \cdot 9 = 0 \Rightarrow H_2 = \frac{9}{4} [-]$$

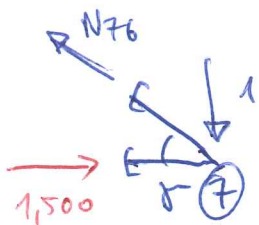
$$\sum X = -H_2 + H_1 = 0 \Rightarrow H_1 = \frac{9}{4} [-]$$



$$\sum X = N_{24} - \frac{9}{4} = 0 \Rightarrow N_{24} = \frac{9}{4} [-]$$

$$\begin{aligned} \sum Y &= N_{14} \cdot \sin \alpha + 1 = 0 \Rightarrow N_{14} = -1,250 [-] \\ \sum X &= N_{13} + N_{14} \cdot \cos \alpha + 2,25 = 0 \Rightarrow N_{13} = -1,500 [-] \end{aligned}$$

2 równości węzła 3 oraz 5 wygoda, więc:  $N_{35} = -1,500 [-]$   
 $N_{57} = -1,500 [-]$

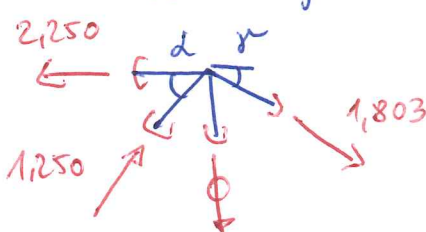


$$\sum Y = -1 + N_{76} \sin \gamma = 0 \Rightarrow N_{76} = 1,803 [-]$$

spw.

$$\sum X = 1,500 - 1,803 \cos \gamma = 0$$

2 równości węzła 6 wygoda, więc  $N_{64} = 1,803 [-]$

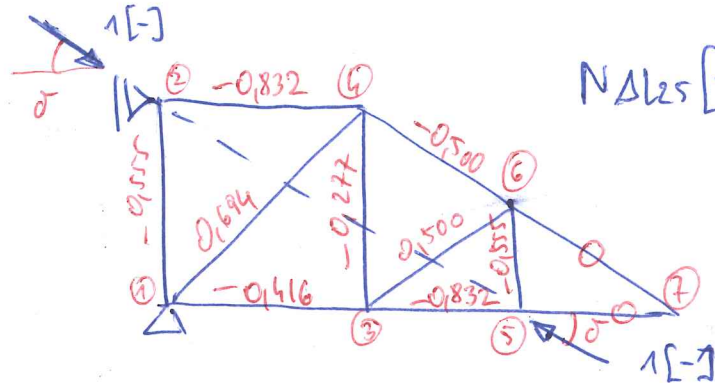


$$\sum X = 1,803 \cos \gamma - 2,250 + 1,250 \cos \alpha = 0$$

$$\sum Y = 1,205 \sin \alpha - 1,803 \sin \gamma = 0$$

siły zostały wyznaczone poprawnie

③ Wyznaczenie zmiany odległości między punktami 2 i 5 → Δl<sub>25</sub>



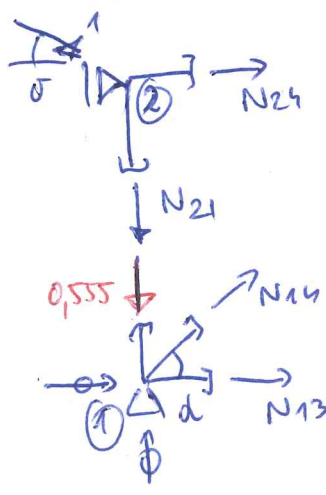
$$N_{\Delta l_{25}} [-] \quad \cos \delta = \frac{6}{\sqrt{4^2 + 6^2}} = 0,8321$$

$$\sin \delta = \frac{4}{\sqrt{4^2 + 6^2}} = 0,5564$$

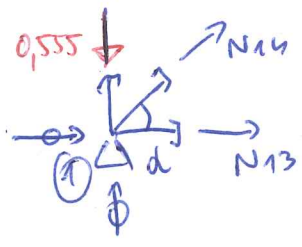
$$\delta = \beta = \gamma$$

δ = 33,69° – wartości potrzebne do sprawdzenia w Solch'sie lub RM-Winkel

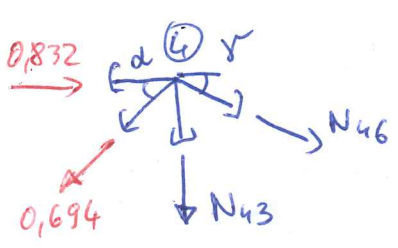
Reszty równa w strefie Δl są równe 0!



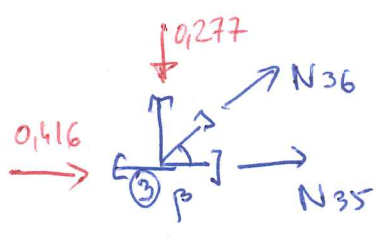
$$\begin{cases} \sum X = N_{24} + 1 \cos \delta = 0 \\ \sum Y = -N_{21} - 1 \sin \delta = 0 \end{cases} \Rightarrow \begin{cases} N_{24} = -0,832 [-] \\ N_{21} = -0,555 [-] \end{cases}$$



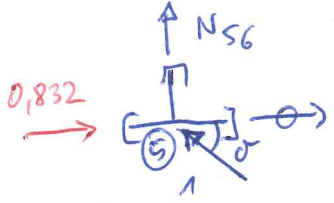
$$\begin{cases} \sum Y = -0,555 + N_{14} \sin \delta = 0 \Rightarrow N_{14} = 0,694 [-] \\ \sum X = N_{13} + N_{14} \cos \delta = 0 \Rightarrow N_{13} = -0,416 [-] \end{cases}$$



$$\begin{cases} \sum X = N_{46} \cos \delta + 0,832 - 0,694 \cos \delta = 0 \Rightarrow N_{46} = -0,500 [-] \\ \sum Y = -N_{43} - N_{46} \sin \delta - 0,694 \sin \delta = 0 \Rightarrow N_{43} = -0,277 [-] \end{cases}$$

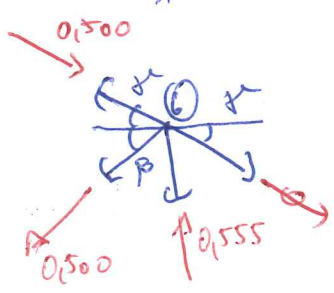


$$\begin{cases} \sum Y = -0,277 + N_{36} \sin \beta = 0 \Rightarrow N_{36} = 0,500 [-] \\ \sum X = N_{35} + N_{36} \cos \beta + 0,416 = 0 \Rightarrow N_{35} = -0,832 [-] \end{cases}$$



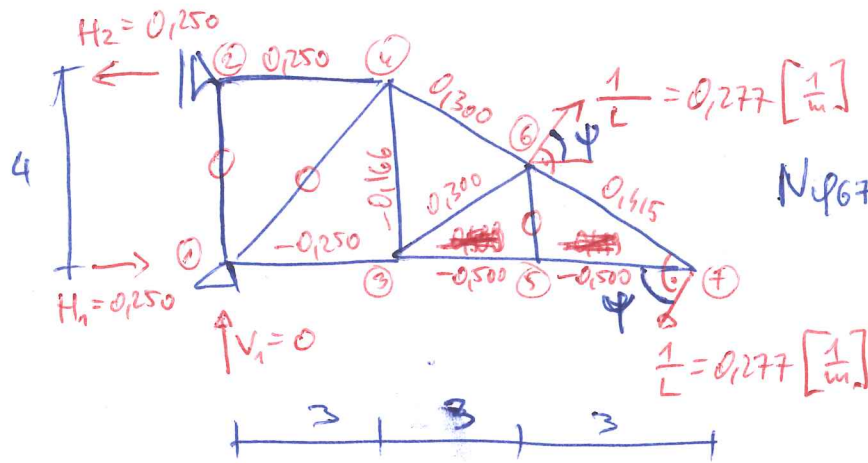
$$\sum Y = N_{56} + 1 \sin \delta = 0 \Rightarrow N_{56} = -0,555 [-]$$

$$\sum X = 0,832 - 1 \cos \delta = 0$$



$$\begin{cases} \sum X = 0,500 \cos \delta - 0,500 \cos \beta = 0 \\ \sum Y = 0,555 - 0,500 \sin \delta - 0,500 \sin \beta = 0 \end{cases}$$

④ Wyznaczenie kątów obrotu pręty 6-7 →  $\psi_{67}$

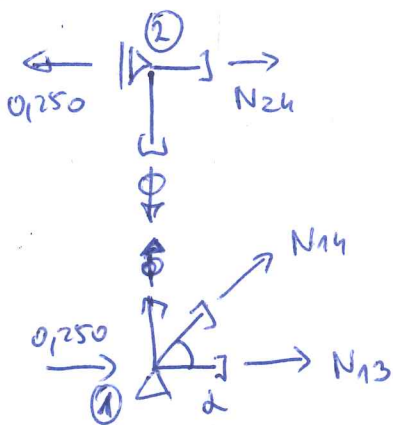


$$\sin \psi = \cos \gamma = 0,8321$$

$$N_{\psi 67} \left[ \frac{1}{m} \right] \cos \psi = \sin \gamma = 0,5547$$

$\psi = 56,31^\circ$  - potrzebne do sprawdzenia w Soldis lub RM-Win.

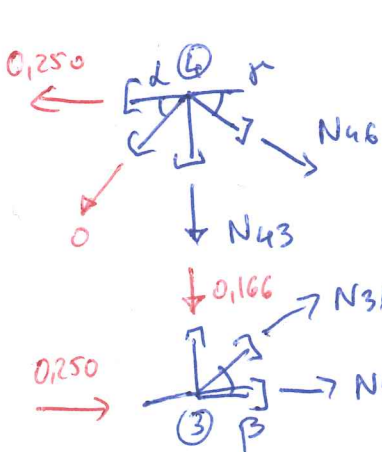
$$\begin{cases} \sum M_A = -4H_2 + 0,277 \cos \psi \cdot 2 - 0,277 \sin \psi \cdot 6 + 0,277 \cdot \sin \psi \cdot 9 = 0 \Rightarrow H_2 = 0,250 \frac{1}{m} \\ \sum X = -H_2 + H_1 + 0,277 \cdot \cos \psi - 0,277 \cos \psi = 0 \Rightarrow H_1 = 0,250 \frac{1}{m} \\ \sum Y = V_1 + 0,277 \sin \psi - 0,277 \sin \psi = 0 \Rightarrow V_1 = 0 \end{cases}$$



$$\sum X = N_{24} - 0,250 = 0 \Rightarrow N_{24} = 0,250 \frac{1}{m}$$

$$\sum Y = N_{14} \cdot \sin \alpha = 0 \Rightarrow N_{14} = 0$$

$$\sum X = N_{13} + 0,250 = 0 \Rightarrow N_{13} = -0,250 \frac{1}{m}$$

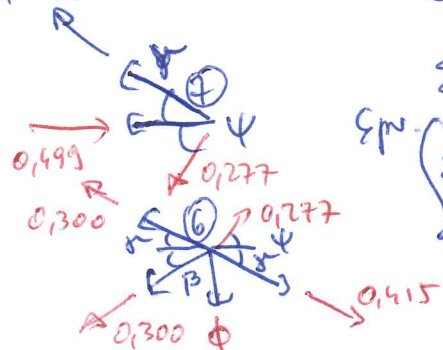


$$\begin{cases} \sum X = N_{46} \cos \gamma - 0,250 = 0 \Rightarrow N_{46} = 0,300 \frac{1}{m} \\ \sum Y = -N_{43} - N_{46} \cdot \sin \gamma = 0 \Rightarrow N_{43} = -0,166 \frac{1}{m} \end{cases}$$

$$\begin{cases} \sum Y = -0,166 + N_{36} \cdot \sin \beta = 0 \Rightarrow N_{36} = 0,300 \frac{1}{m} \\ \sum X = N_{35} + N_{36} \cos \beta + 0,250 = 0 \Rightarrow N_{35} = -0,500 \frac{1}{m} \end{cases}$$

z węzła ⑤  $N_{35} = N_{57} = -0,500 \frac{1}{m}$

$$\sum Y = N_{76} \cdot \sin \gamma - 0,277 \sin \psi = 0 \Rightarrow N_{76} = 0,415$$

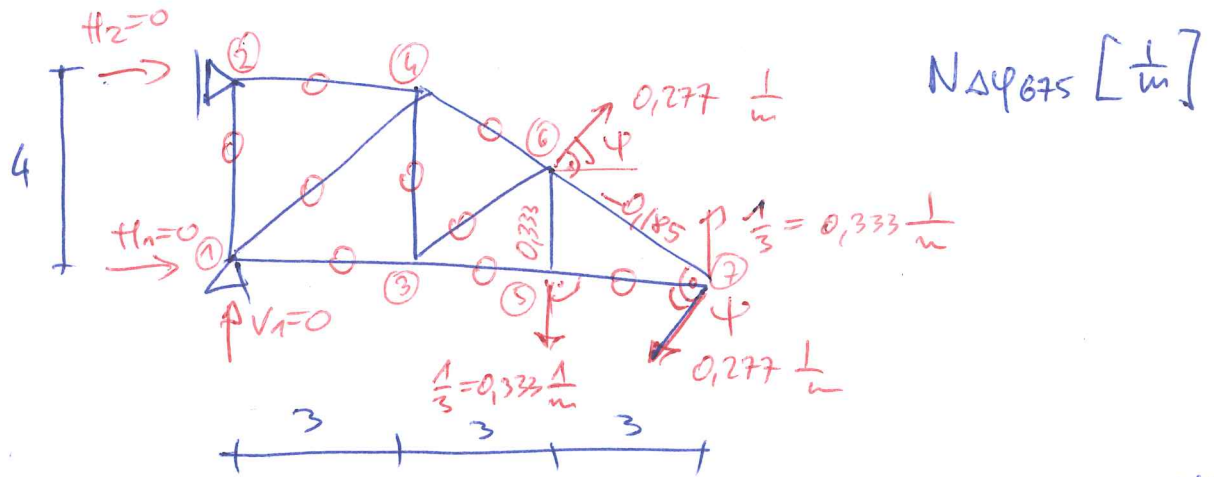


$$\begin{cases} \sum X = 0,499 - 0,277 \cos \psi - N_{76} \cos \gamma = 0 \\ \sum X = 0,415 \cos \gamma + 0,277 \cos \psi - 0,300 \cos \gamma - 0,300 \cos \beta = 0 \\ \sum Y = -0,415 \sin \gamma + 0,277 \sin \psi + 0,300 \sin \gamma - 0,300 \sin \beta = 0 \end{cases}$$

Siły N wyznaczone poprawnie.



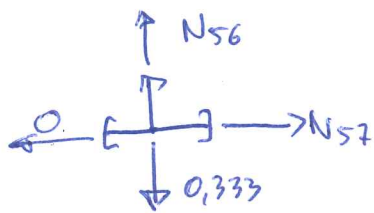
5) Wyznaczenie zmiany kąta między prętami 6-7 i 5-7 -  $\Delta\varphi_{675}$



$$\sum M_n = 0,277 \cdot \cos \varphi \cdot 2 - 0,277 \cdot \sin \varphi \cdot 6 + 0,277 \sin \varphi \cdot 9 - \frac{1}{3} \cdot 9 + \frac{1}{3} \cdot 6 \stackrel{+H_2 \cdot 4 = 0}{=} 0$$

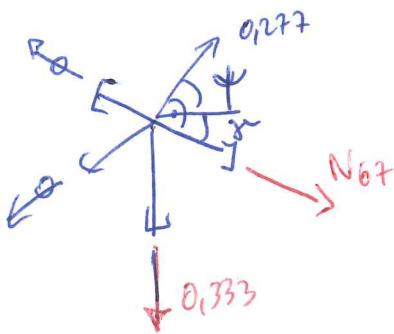
$$\Rightarrow H_2 = 0$$

przy odpowiednim założeniu siły 0,277 jako  $\frac{1}{13}$  i dobrać wyznaczone  $\cos \varphi$  i  $\sin \varphi$   $[H_2 = 0]$ , a więc proste równanie różnic



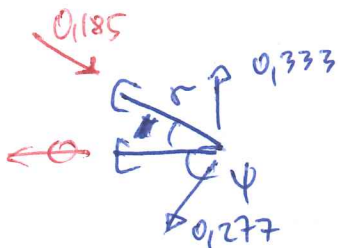
$$\sum X = N_{57} = 0$$

$$\sum Y = -0,333 + N_{56} = 0 \Rightarrow N_{56} = 0,333 \frac{1}{m}$$



$$\sum X = N_{67} \cos \gamma + 0,277 \cos \varphi = 0 \Rightarrow N_{67} = -0,185 \frac{1}{m}$$

$$\sum Y = 0,277 \sin \varphi - N_{67} \sin \gamma - 0,333 = 0$$



$$\sum X = 0,185 \cos \gamma - 0,277 \cos \varphi = 0$$

$$\sum Y = 0,333 - 0,185 \sin \gamma - 0,277 \sin \varphi = 0$$

siły wyznaczone poprawnie

Tabelki proponuję wykonać w exelu

**Wyznaczenie  $y_5$**

nr pręta	L [m]	$N_p$ [kN]	$N_{y7}$ [-]	$N_p * N_{y7} * L$ [kNm]	$t_0$ [C]	$N_{y7} * L * t_0$ [mC]
1-3	3	-3.286	-1.5	14.787		0.000
3-5	3	-15	-1.5	67.500		0.000
5-7	3	-15	-1.5	67.500		0.000
2-4	3	33.394	2.25	225.410		0.000
4-6	3.606	22.834	1.803	148.458	20	130.032
6-7	3.606	18.028	1.803	117.211	20	130.032
1-2	4	0	0	0.000		0.000
3-4	4	6.527	0	0.000	30	0.000
5-6	2	0	0	0.000		0.000
1-4	5	-23.991	-1.25	149.944		0.000
3-6	3.606	4.809	0	0.000	15	0.000
<b>suma</b>				<b>790.809</b>		<b>260.065</b>

**Wyznaczenie  $\Delta I_{25}$**

nr pręta	L [m]	$N_p$ [kN]	$N_{\Delta 2-5}$ [-]	$N_p * N_{\Delta 2-5} * L$ [kNm]	$t_0$ [C]	$N_{\Delta 2-5} * L * t_0$ [mC]
1-3	3	-3.286	-0.416	4.101		0.000
3-5	3	-15	-0.832	37.440		0.000
5-7	3	-15	0	0.000		0.000
2-4	3	33.394	-0.832	-83.351		0.000
4-6	3.606	22.834	-0.5	-41.170	20	-36.060
6-7	3.606	18.028	0	0.000	20	0.000
1-2	4	0	-0.555	0.000		0.000
3-4	4	6.527	-0.277	-7.232	30	-33.240
5-6	2	0	-0.555	0.000		0.000
1-4	5	-23.991	0.694	-83.249		0.000
3-6	3.606	4.809	0.5	8.671	15	27.045
<b>suma</b>				<b>-164.790</b>		<b>-42.255</b>

**Wyznaczenie  $\phi_{67}$**

nr pręta	L [m]	$N_p$ [kN]	$N_{\bar{n}67}$ [1/m]	$N_p * N_{\bar{n}67} * L$ [kN]	$t_0$ [C]	$N_{\bar{n}67} * L * t_0$ [C]
1-3	3	-3.286	-0.25	2.465		0.000
3-5	3	-15	-0.5	22.500		0.000
5-7	3	-15	-0.5	22.500		0.000
2-4	3	33.394	0.25	25.046		0.000
4-6	3.606	22.834	0.3	24.702	20	21.636
6-7	3.606	18.028	0.415	26.979	20	29.930
1-2	4	0	0	0.000		0.000
3-4	4	6.527	-0.166	-4.334	30	-19.920
5-6	2	0	0	0.000		0.000
1-4	5	-23.991	0	0.000		0.000
3-6	3.606	4.809	0.3	5.202	15	16.227
<b>suma</b>				<b>125.059</b>		<b>47.873</b>

**Wyznaczenie  $\Delta \phi_{576}$**

nr pręta	L [m]	$N_p$ [kN]	$N_{\bar{n}576}$ [1/m]	$N_p * N_{\bar{n}576} * L$ [kN]	$t_0$ [C]	$N_{\bar{n}576} * L * t_0$ [C]
1-3	3	-3.286	0	0.000		0.000
3-5	3	-15	0	0.000		0.000
5-7	3	-15	0	0.000		0.000
2-4	3	33.394	0	0.000		0.000
4-6	3.606	22.834	0	0.000	20	0.000
6-7	3.606	18.028	-0.185	-12.027	20	-13.342
1-2	4	0	0	0.000		0.000
3-4	4	6.527	0	0.000	30	0.000
5-6	2	0	0.333	0.000		0.000
1-4	5	-23.991	0	0.000		0.000
3-6	3.606	4.809	0	0.000	15	0.000
<b>suma</b>				<b>-12.027</b>		<b>-13.342</b>

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$$1. \sigma = \sum_{i=1}^n \frac{N_p \cdot N_i}{EA} L + \sum_{i=1}^m N_i L t_{0i} \alpha_i - \sum_{i=1}^k R_i \cdot \Delta p$$

n - liczba prętów obciążonych w krańce (liczba prętów w krawędzie)

m - liczba prętów obciążonych temperaturą

k - liczba obciążeń geometrycznych.

$$\begin{aligned} \bar{1} \cdot y_7 &= \frac{780,809 \text{ kNm}}{210 \cdot 10^6 \frac{\text{kN}}{\text{m}^2} \cdot 5,7 \cdot 10^{-4} \text{ m}^2} + 1,2 \cdot 10^{-5} \frac{1}{^\circ\text{C}} \cdot 260,065 \text{ m}^\circ\text{C} = \\ &= 0,006607 \text{ m} + 0,003121 \text{ m} = 0,009727 \text{ m} \quad (\text{SOLDIS} - 0,0097) \end{aligned}$$

$$\begin{aligned} \bar{1} \cdot \Delta l_{25} &= \frac{-164,790 \text{ kNm}}{210 \cdot 10^6 \frac{\text{kN}}{\text{m}^2} \cdot 5,7 \cdot 10^{-4} \text{ m}^2} + 1,2 \cdot 10^{-5} \frac{1}{^\circ\text{C}} \cdot (-42,255) = \\ &= -0,001377 \text{ m} - 0,000507 \text{ m} = -0,001884 \text{ m} \quad (\text{SOLDIS} - 0,00186) \end{aligned}$$

(te wartości proszę wyznaczyć - nie odczytać jej się bezpośrednio z SOLDISA lub RM-WINA)

$$\begin{aligned} \bar{1} \cdot \varphi_{67} &= \frac{125,059 \text{ kN}}{210 \cdot 10^6 \frac{\text{kN}}{\text{m}^2} \cdot 5,7 \cdot 10^{-4} \text{ m}^2} + 1,2 \cdot 10^{-5} \frac{1}{^\circ\text{C}} \cdot 47,873^\circ\text{C} = \\ &= 0,0010448 + 0,000574 = 0,001619 \approx 0,09278^\circ \quad (0,0928^\circ - \text{SOLDIS}) \end{aligned}$$

$$\begin{aligned} 1 \cdot \Delta \varphi_{675} &= \frac{-12,027 \text{ kN}}{210 \cdot 10^6 \frac{\text{kN}}{\text{m}^2} \cdot 5,7 \cdot 10^{-4} \text{ m}^2} + 1,2 \cdot 10^{-5} \frac{1}{^\circ\text{C}} \cdot (-13,342^\circ\text{C}) = \\ &= -0,0001 - 0,00016 = -0,00026 \approx -0,01493^\circ \quad (-0,015^\circ - \text{SOLDIS}) \end{aligned}$$