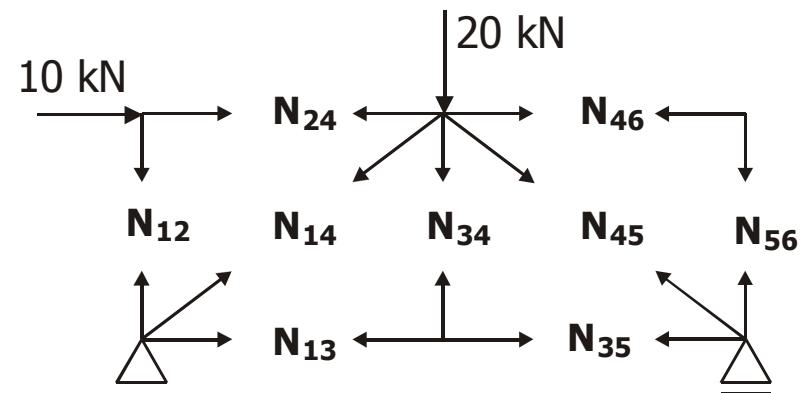
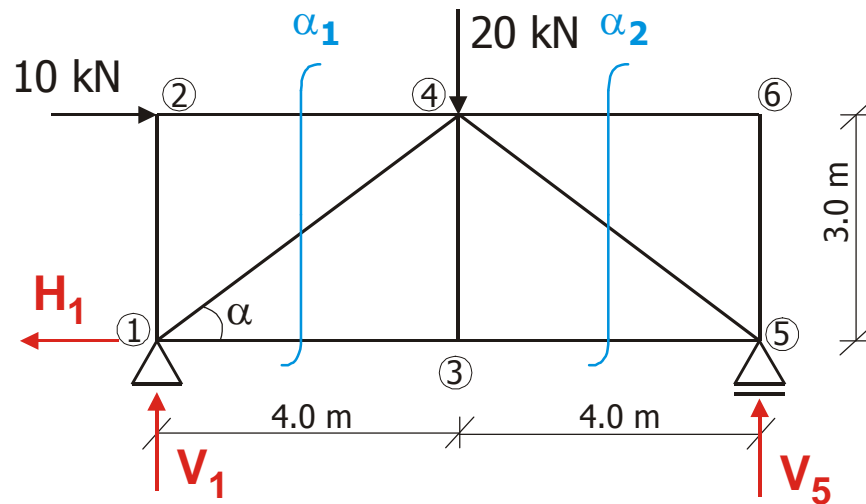


$$\left\{ \begin{array}{l} \sum X = 10 - H_1 = 0 \\ \sum Y = V_1 + V_5 - 20 = 0 \\ \sum M_1 = 10 \cdot 3 + 20 \cdot 4 - 8V_5 = 0 \end{array} \right. \quad \left\{ \begin{array}{l} H_1 = 10 \\ V_1 = 6.25 \text{ kN} \\ V_5 = 13.75 \text{ kN} \end{array} \right.$$

$$\cos \alpha = 0.6$$

$$\sin \alpha = 0.8$$



Kolejność obliczeń według metody równoważenia węzłów:

2 –  $N_{12}$ ,  $N_{24}$

1 –  $N_{14}$ ,  $N_{13}$

6 –  $N_{46}$ ,  $N_{65}$

5 –  $N_{45}$ ,  $N_{35}$

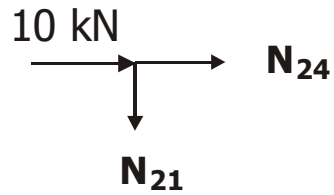
3 –  $N_{34}$ , sprawdzenie

4 – sprawdzenie

Sprawdzenie obliczeń metodą Rittera:

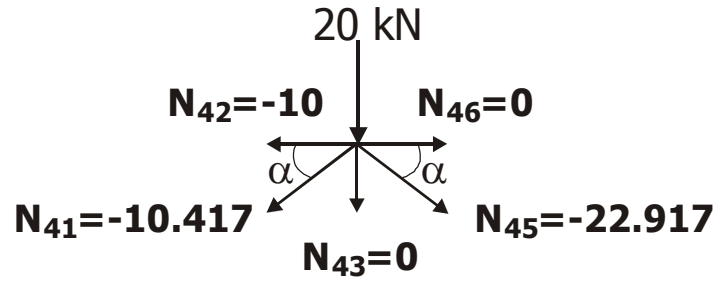
- przekrój  $\alpha_1$

- przekrój  $\alpha_2$



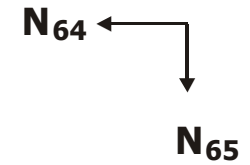
$$\begin{cases} \sum X = 10 + N_{24} = 0 \\ \sum Y = -N_{21} = 0 \end{cases}$$

$$\begin{cases} N_{24} = -10 \text{ kN} \\ N_{21} = 0 \end{cases}$$



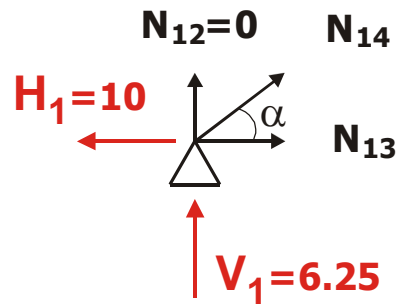
Sprawdzenie:

$$\begin{cases} \sum Y = 10 + 0 + 10.417 \cos \alpha - 22.917 \cos \alpha = 0 \\ \sum X = -20 + 10.417 \sin \alpha + 22.917 \sin \alpha = 0 \end{cases}$$



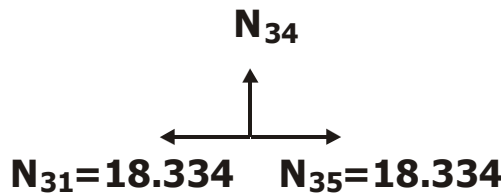
$$\begin{cases} \sum X = -N_{64} = 0 \\ \sum Y = -N_{65} = 0 \end{cases}$$

$$\begin{cases} N_{64} = 0 \\ N_{65} = 0 \end{cases}$$



$$\begin{cases} \sum Y = 6.26 + 0 + N_{14} \sin \alpha = 0 \\ \sum X = -10 + N_{13} + N_{14} \cos \alpha = 0 \end{cases}$$

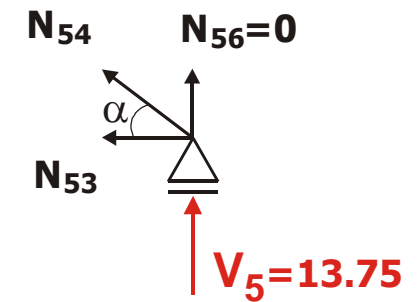
$$\begin{cases} N_{14} = -10.417 \text{ kN} \\ N_{13} = 18.334 \text{ kN} \end{cases}$$



$$\begin{cases} \sum Y = N_{34} = 0 \\ N_{34} = 0 \end{cases}$$

Sprawdzenie:

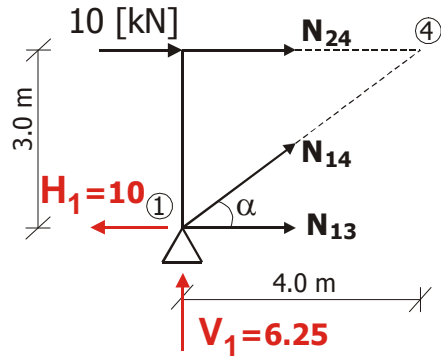
$$\sum X = -18.334 + 18.334 = 0$$



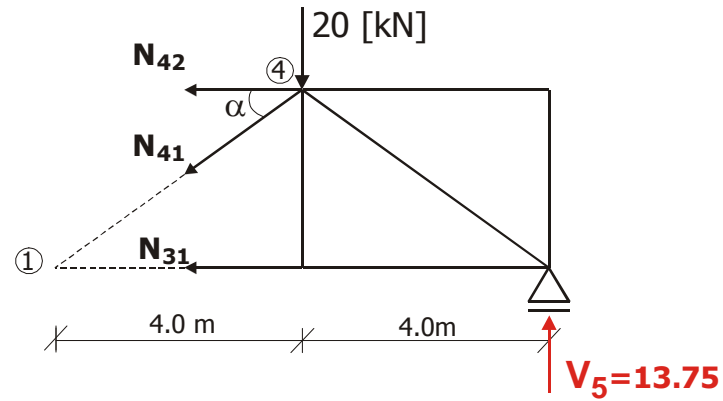
$$\begin{cases} \sum Y = 13.75 + 0 + N_{54} \sin \alpha = 0 \\ \sum X = -N_{53} - N_{54} \cos \alpha = 0 \end{cases}$$

$$\begin{cases} N_{54} = -22.917 \text{ kN} \\ N_{53} = 18.334 \text{ kN} \end{cases}$$

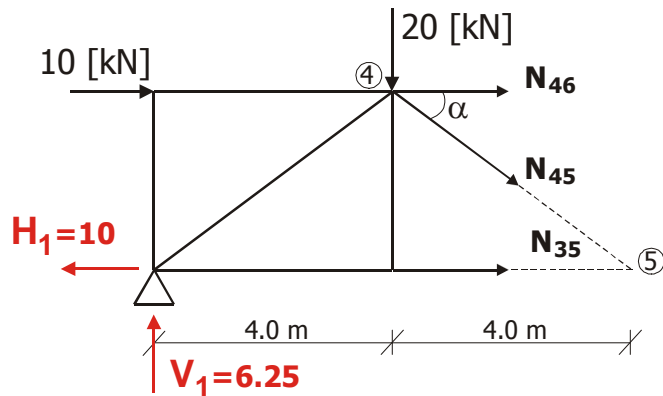
Przykład 1



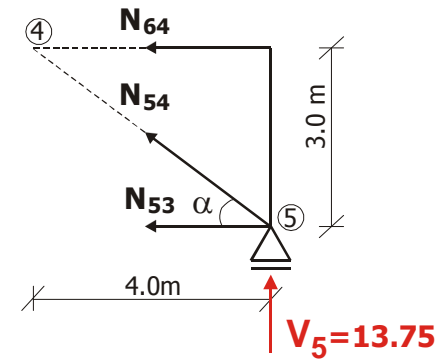
$$\begin{aligned} \sum Y^L &= 6.25 + N_{14} \sin \alpha = 0 & N_{14} &= -10.417 \text{ kN} \\ \sum M_1^L &= 10 \cdot 3 + N_{24} \cdot 3 = 0 & N_{24} &= -10 \text{ kN} \\ \sum M_4^L &= 6.25 \cdot 4 + 10 \cdot 3 - N_{13} \cdot 3 = 0 & N_{13} &= 18.33 \text{ kN} \end{aligned}$$



$$\begin{aligned} \sum Y^P &= 13.75 - N_{41} \sin \alpha = 0 & N_{41} &= -10.417 \text{ kN} \\ \sum M_1^P &= 20 \cdot 4 - N_{42} \cdot 3 - 13.75 \cdot 8 = 0 & N_{42} &= -10 \text{ kN} \\ \sum M_4^P &= -13.75 \cdot 4 + N_{31} \cdot 3 = 0 & N_{31} &= 18.33 \text{ kN} \end{aligned}$$

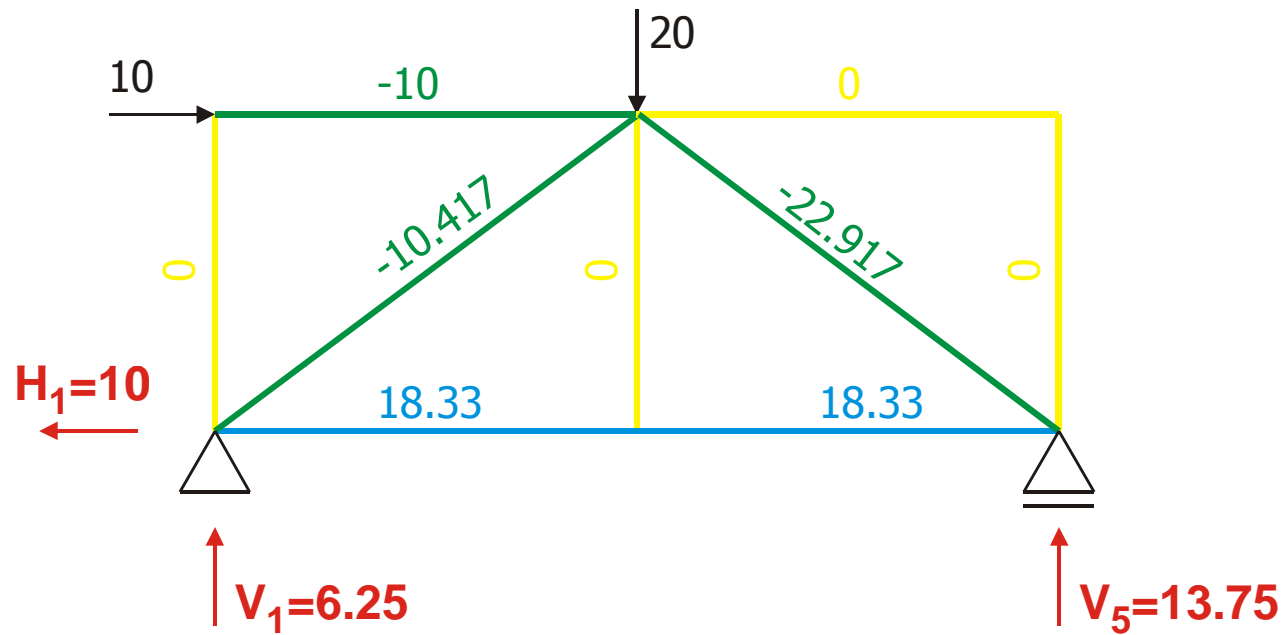


$$\begin{aligned} \sum Y^L &= 6.25 - 20 - N_{45} \sin \alpha = 0 & N_{45} &= -22.917 \text{ kN} \\ \sum M_5^L &= 10 \cdot 3 - 20 \cdot 4 + 6.25 \cdot 8 + N_{46} \cdot 3 = 0 & N_{46} &= 0 \\ \sum M_4^L &= 6.25 \cdot 4 + 10 \cdot 3 - N_{35} \cdot 3 = 0 & N_{35} &= 18.33 \text{ kN} \end{aligned}$$

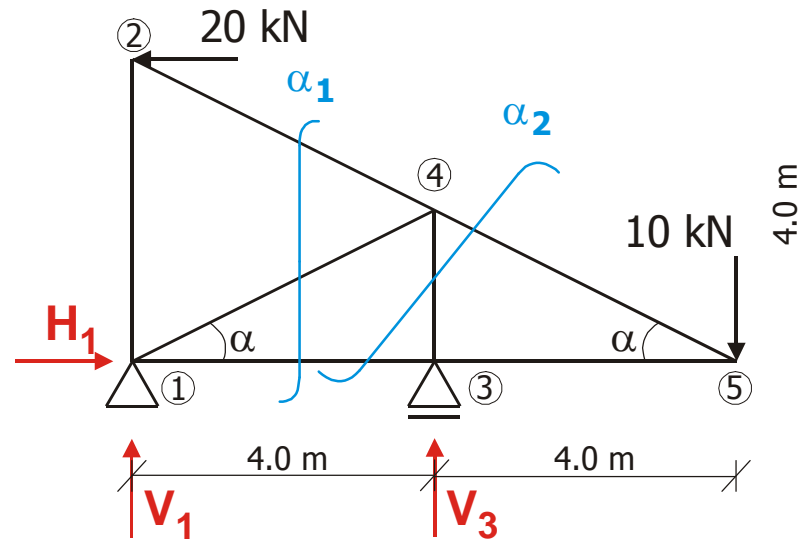


$$\begin{aligned} \sum Y^P &= 13.75 + N_{54} \sin \alpha = 0 & N_{54} &= -22.917 \text{ kN} \\ \sum M_5^P &= N_{64} \cdot 3 = 0 & N_{64} &= 0 \\ \sum M_4^P &= -13.75 \cdot 4 + N_{53} \cdot 3 = 0 & N_{53} &= 18.33 \text{ kN} \end{aligned}$$

Zestawienie sił osiowych w prętach:



pręty ściskane  
pręty rozciągane  
pręty zerowe



$$\begin{cases} \sum X = -20 + H_1 = 0 \\ \sum Y = V_1 + V_3 - 10 = 0 \\ \sum M_1 = 10 \cdot 8 - 20 \cdot 4 - 4V_3 = 0 \end{cases} \quad \begin{cases} H_1 = 20 \text{ kN} \\ V_1 = 10 \text{ kN} \\ V_3 = 0 \end{cases}$$

Kolejność obliczeń według metody równoważenia węzłów:

5 –  $N_{53}$ ,  $N_{54}$

3 –  $N_{13}$ ,  $N_{34}$

2 –  $N_{24}$ ,  $N_{21}$

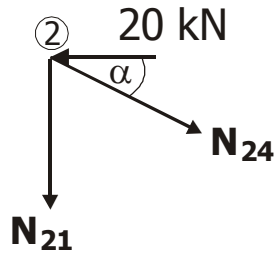
1 –  $N_{14}$ , sprawdzenie

4 – sprawdzenie

Sprawdzenie obliczeń metodą Rittera:

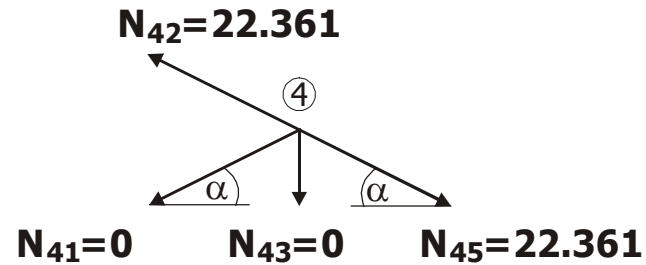
- przekrój  $\alpha_1$

- przekrój  $\alpha_2$



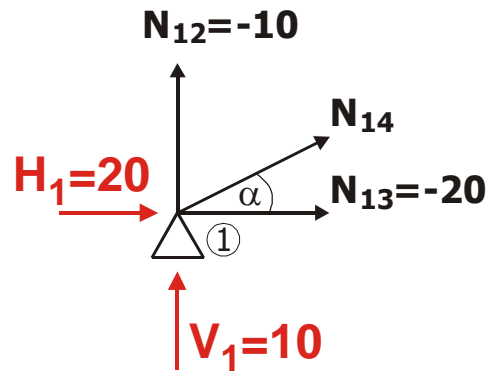
$$\begin{cases} \sum X = -20 + N_{24} \cos \alpha = 0 \\ \sum Y = -N_{21} - N_{24} \sin \alpha = 0 \end{cases}$$

$$\begin{cases} N_{24} = 22.361 \text{ kN} \\ N_{21} = -10 \text{ kN} \end{cases}$$



Sprawdzenie:

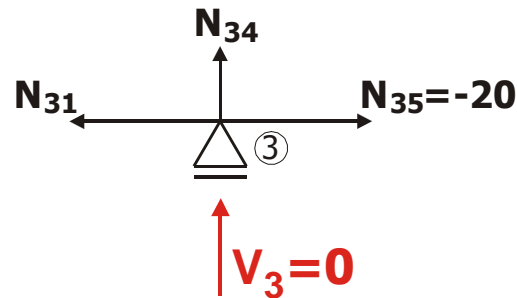
$$\begin{cases} \sum X = -22.361 \cos \alpha + 22.361 \cos \alpha = 0 \\ \sum Y = 22.361 \sin \alpha - 22.361 \sin \alpha = 0 \end{cases}$$



$$\begin{aligned} \sum Y &= -10 + 10 + N_{14} \sin \alpha = 0 \\ N_{14} &= 0 \end{aligned}$$

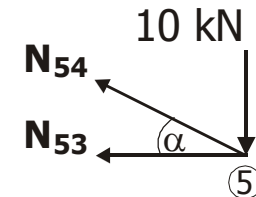
Sprawdzenie:

$$\sum X = -20 + 20 = 0$$



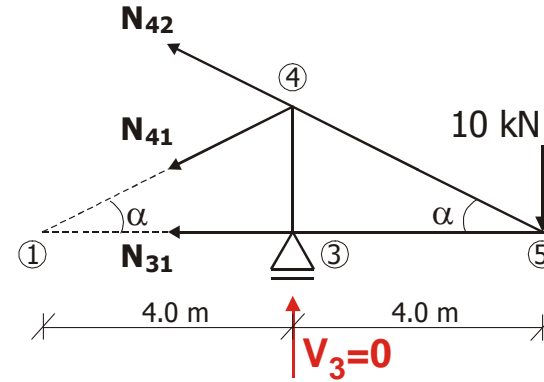
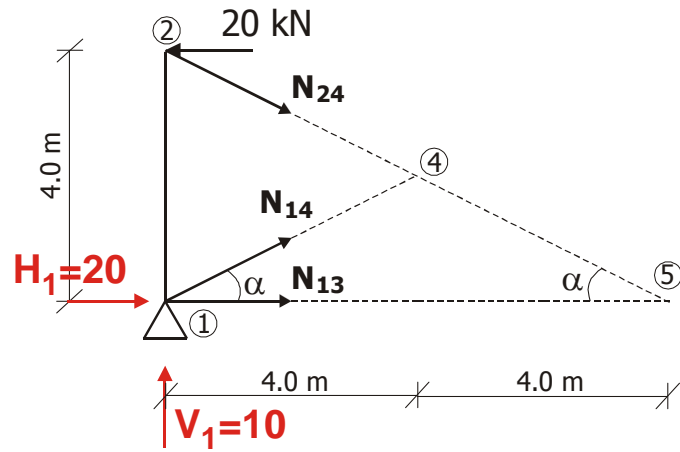
$$\begin{cases} \sum Y = 0 + N_{34} = 0 \\ \sum X = -N_{31} + 20 = 0 \end{cases}$$

$$\begin{cases} N_{34} = 0 \\ N_{31} = -20 \text{ kN} \end{cases}$$



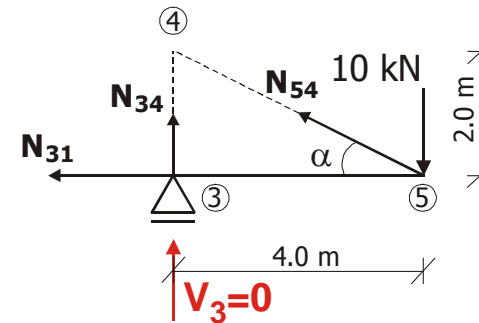
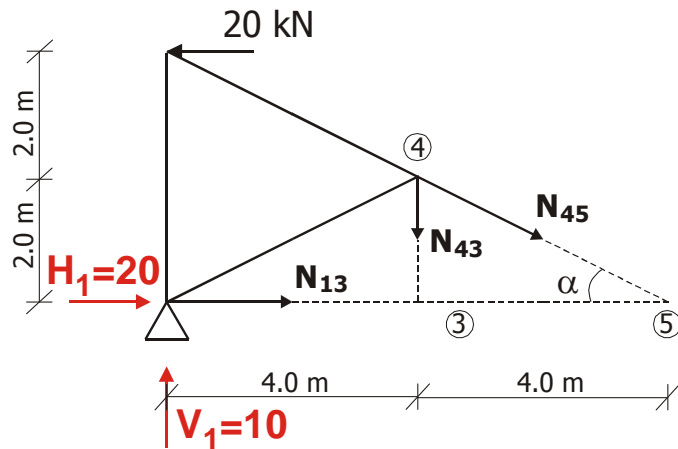
$$\begin{cases} \sum Y = -10 + N_{54} \sin \alpha = 0 \\ \sum X = -N_{53} - N_{54} \cos \alpha = 0 \end{cases}$$

$$\begin{cases} N_{24} = 22.361 \text{ kN} \\ N_{21} = -20 \text{ kN} \end{cases}$$



$$\begin{aligned} \sum M_5^L &= 10 \cdot 8 + N_{14} \sin \alpha \cdot 8 - 20 \cdot 4 = 0 & N_{14} &= 0 \\ \sum M_1^L &= -20 \cdot 4 + N_{24} \cos \alpha \cdot 4 = 0 & N_{24} &= 22.361 \text{ kN} \\ \sum M_4^L &= 10 \cdot 4 - 20 \cdot 2 - 20 \cdot 2 - N_{13} \cdot 2 = 0 & N_{13} &= -20 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_5^P &= -N_{41} \sin \alpha \cdot 4 - N_{14} \cos \alpha \cdot 2 = 0 & N_{41} &= 0 \\ \sum M_1^P &= 10 \cdot 8 + N_{42} \cos \alpha \cdot 2 = 0 & N_{42} &= 22.361 \text{ kN} \\ \sum M_4^P &= 10 \cdot 4 + N_{31} \cdot 2 = 0 & N_{31} &= -20 \text{ kN} \end{aligned}$$

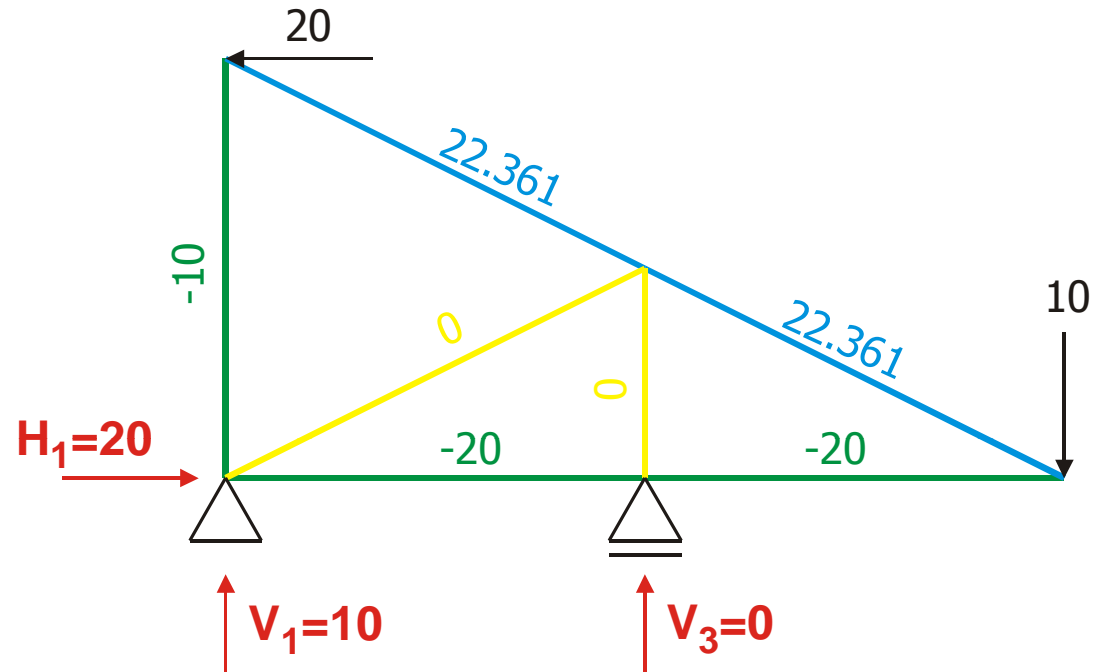


$$\begin{aligned} \sum M_5^L &= -20 \cdot 4 + 10 \cdot 8 - N_{43} \cdot 4 = 0 & N_{43} &= 0 \\ \sum M_3^L &= -20 \cdot 4 + 10 \cdot 4 + N_{45} \cos \alpha \cdot 2 = 0 & N_{45} &= 22.361 \text{ kN} \\ \sum M_4^L &= -20 \cdot 2 - 20 \cdot 2 + 10 \cdot 4 - N_{13} \cdot 2 = 0 & N_{13} &= -20 \text{ kN} \end{aligned}$$

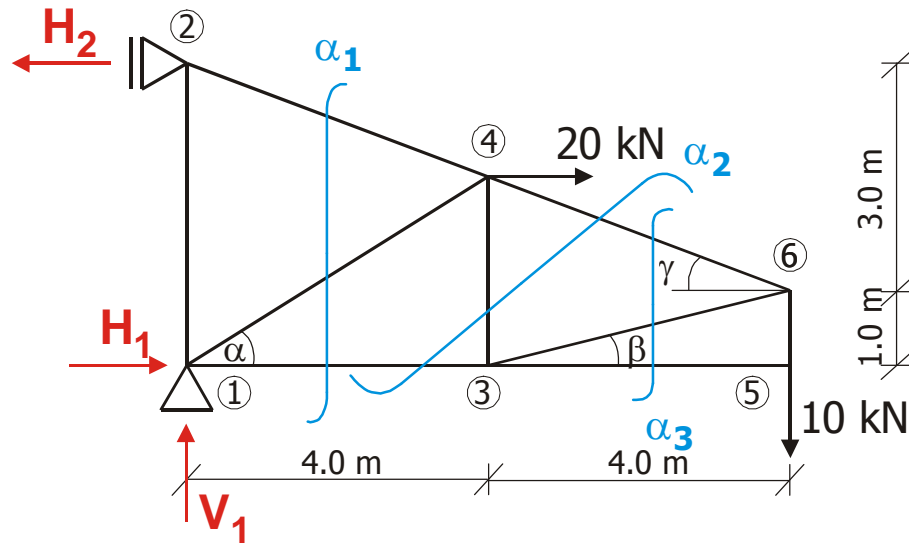
$$\begin{aligned} \sum M_5^P &= N_{34} \cdot 4 = 0 & N_{34} &= 0 \\ \sum M_3^P &= -10 \cdot 4 + N_{54} \sin \alpha \cdot 4 = 0 & N_{54} &= 22.361 \text{ kN} \\ \sum M_4^P &= 10 \cdot 4 + N_{31} \cdot 2 = 0 & N_{31} &= -20 \text{ kN} \end{aligned}$$



Zestawienie sił osiowych w prętach:



pręty ściskane  
pręty rozciągane  
pręty zerowe



$$\begin{cases} \sum X = -H_2 + H_1 + 20 = 0 \\ \sum Y = V_1 - 10 = 0 \\ \sum M_1 = 10 \cdot 8 + 20 \cdot 2.5 - 4H_2 = 0 \end{cases}$$

$$\begin{cases} H_1 = 12.5 \text{ kN} \\ V_1 = 10 \text{ kN} \\ H_2 = 32.5 \text{ kN} \end{cases}$$

Kolejność obliczeń według metody równoważenia węzłów:

5 –  $N_{53}$ ,  $N_{56}$

6 –  $N_{63}$ ,  $N_{64}$

3 –  $N_{31}$ ,  $N_{34}$

1 –  $N_{14}$ ,  $N_{12}$

2 –  $N_{24}$ , sprawdzenie

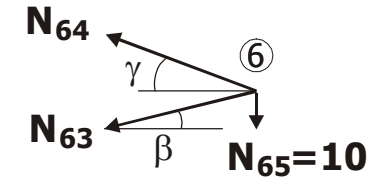
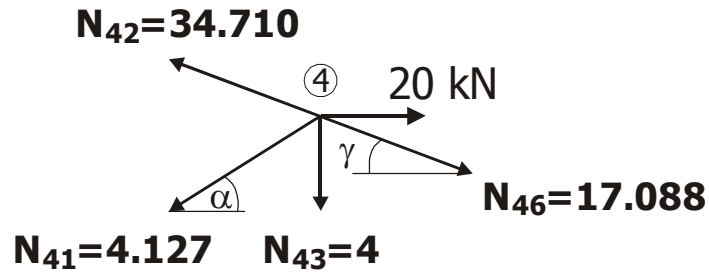
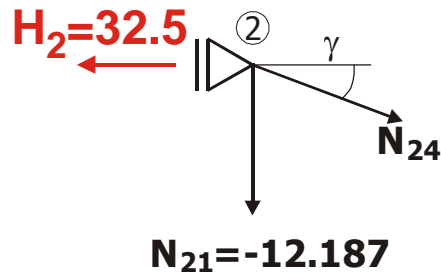
4 – sprawdzenie

Sprawdzenie obliczeń metodą Rittera:

- przekrój  $\alpha_1$

- przekrój  $\alpha_2$

- przekrój  $\alpha_3$



$$\sum X = -32.5 + N_{24} \cos \gamma = 0$$

$$N_{24} = 34.71 \text{ kN}$$

Sprawdzenie:

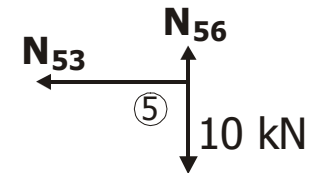
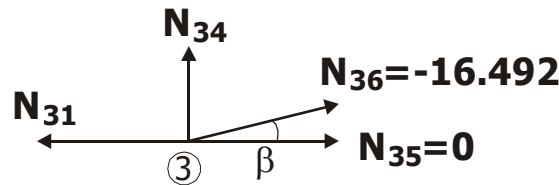
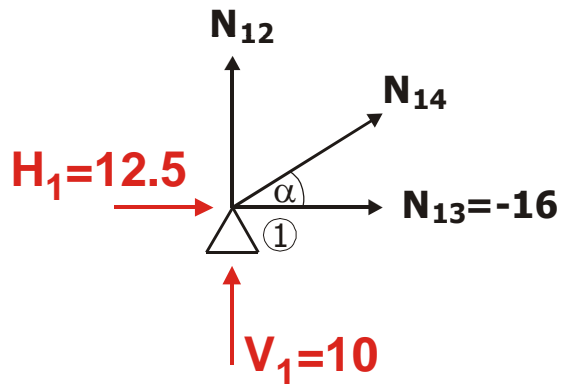
$$\sum Y = 12.187 - 34.71 \sin \gamma = 0$$

Sprawdzenie:

$$\begin{cases} \sum X = -34.710 \cos \gamma + 20 + 17.088 \cos \gamma - 4.127 \cos \alpha = 0 \\ \sum Y = 34.710 \sin \gamma - 17.088 \sin \gamma - 4.127 \sin \alpha - 4 = 0 \end{cases}$$

$$\begin{cases} \sum X = -N_{64} \cos \gamma - N_{63} \cos \beta = 0 \\ \sum Y = N_{64} \sin \gamma - N_{63} \sin \beta - 10 = 0 \end{cases}$$

$$\begin{cases} N_{64} = 17.088 \text{ kN} \\ N_{63} = -16.492 \text{ kN} \end{cases}$$



$$\begin{cases} \sum X = N_{14} \cos \alpha + 12.5 - 16 = 0 \\ \sum Y = 10 + N_{12} + N_{14} \sin \alpha = 0 \end{cases}$$

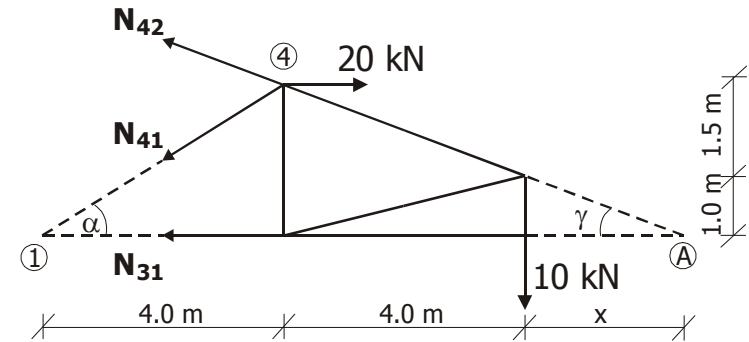
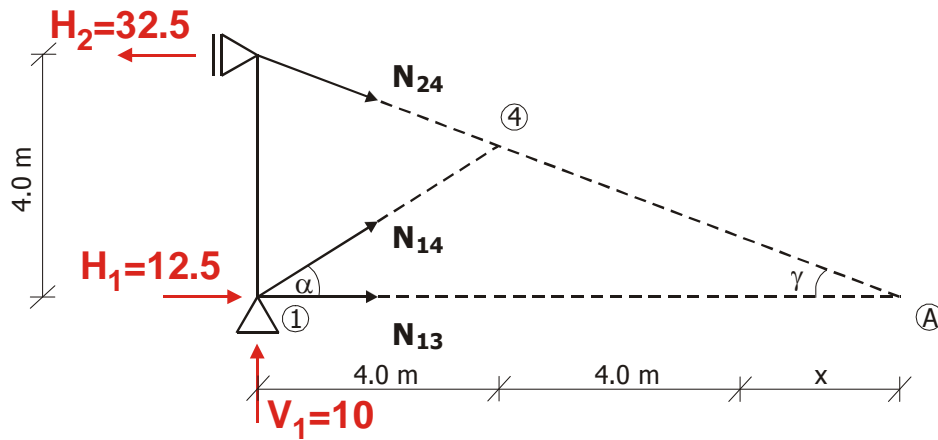
$$\begin{cases} N_{14} = 4.127 \text{ kN} \\ N_{12} = -12.187 \text{ kN} \end{cases}$$

$$\begin{cases} \sum X = -N_{31} - 16.492 \cos \beta = 0 \\ \sum Y = N_{34} - 16.492 \sin \beta = 0 \end{cases}$$

$$\begin{cases} N_{31} = -16 \text{ kN} \\ N_{34} = 4 \text{ kN} \end{cases}$$

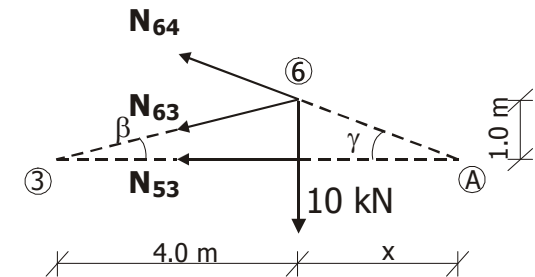
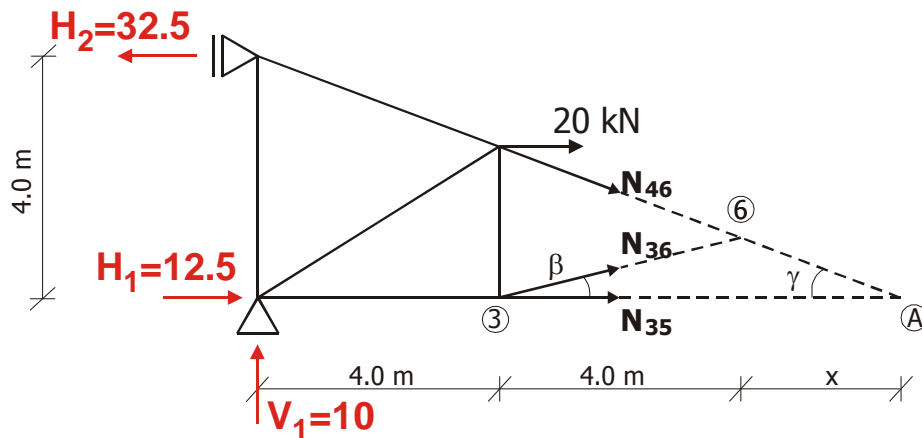
$$\begin{cases} \sum X = N_{53} = 0 \\ \sum Y = N_{56} - 10 = 0 \end{cases}$$

$$\begin{cases} N_{53} = 0 \\ N_{56} = 10 \text{ kN} \end{cases}$$



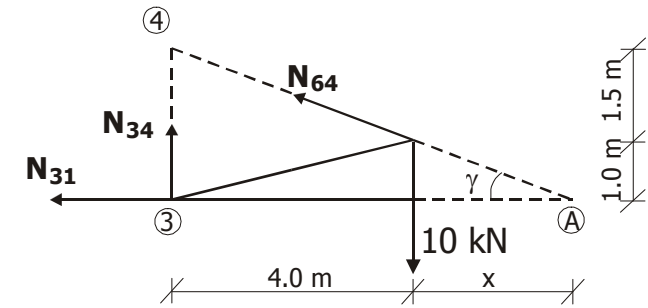
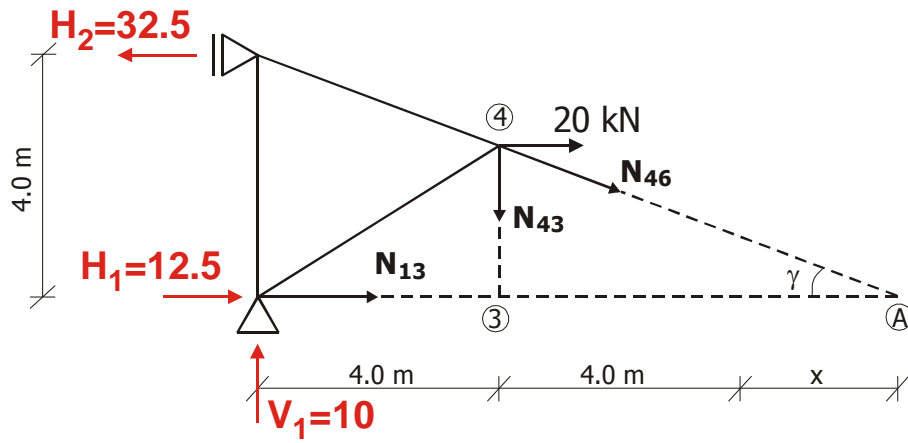
$$\begin{aligned} \sum M_1^L &= -32.5 \cdot 4 + N_{24} \cos \gamma \cdot 4 = 0 & N_{24} &= 34.71 \text{ kN} \\ \sum M_4^L &= 10 \cdot 4 - 12.5 \cdot 2.5 - 32.5 \cdot 2.5 - N_{13} \cdot 2.5 = 0 & N_{13} &= -16 \text{ kN} \\ \sum M_A^L &= 10 \cdot (8 + \frac{8}{3}) - 32.5 \cdot 4 + N_{14} \sin \alpha \cdot (8 + \frac{8}{3}) = 0 & N_{14} &= 4.127 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_1^P &= 20 \cdot 2.5 + 10 \cdot 8 - N_{42} \cos \gamma \cdot 2.5 - N_{42} \sin \gamma \cdot 4 = 0 & N_{42} &= 34.71 \text{ kN} \\ \sum M_4^P &= 10 \cdot 4 + N_{31} \cdot 2.5 = 0 & N_{31} &= -16 \text{ kN} \\ \sum M_A^P &= -10 \cdot \frac{8}{3} + 20 \cdot 2.5 - N_{41} \sin \alpha \cdot (4 + \frac{8}{3}) = 0 & N_{41} &= 4.127 \text{ kN} \end{aligned}$$



$$\begin{aligned} \sum M_3^L &= -32.5 \cdot 4 + 10 \cdot 4 + 20 \cdot 2.5 + N_{46} \cos \gamma \cdot 2.5 = 0 & N_{46} &= 17.088 \text{ kN} \\ \sum M_6^L &= 10 \cdot 8 - 12.5 \cdot 1 - 32.5 \cdot 3 + 20 \cdot 1.5 - N_{35} \cdot 1 = 0 & N_{35} &= 0 \\ \sum M_A^L &= 10 \cdot (8 + \frac{8}{3}) - 32.5 \cdot 4 + 20 \cdot 2.5 + N_{36} \sin \beta \cdot (4 + \frac{8}{3}) = 0 & N_{36} &= -16.492 \text{ kN} \end{aligned}$$

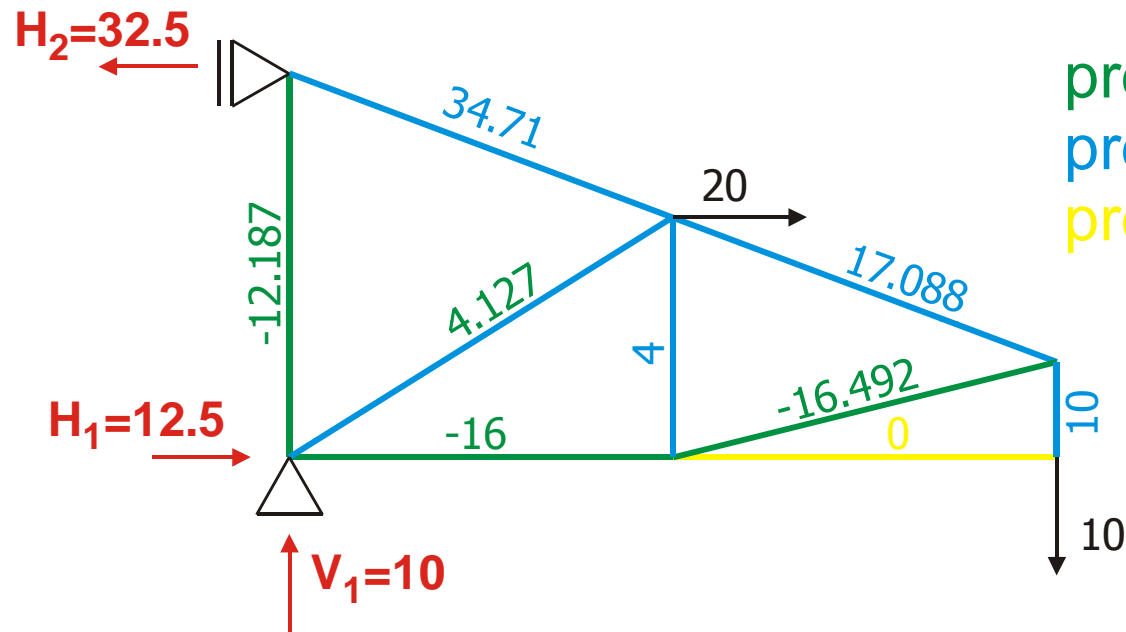
$$\begin{aligned} \sum M_3^P &= 10 \cdot 4 - N_{64} \sin \gamma \cdot 4 - N_{64} \cos \gamma \cdot 1 = 0 & N_{64} &= 17.088 \text{ kN} \\ \sum M_6^P &= N_{53} \cdot 1 = 0 & N_{53} &= 0 \\ \sum M_A^P &= -10 \cdot \frac{8}{3} - N_{63} \sin \beta \cdot \frac{8}{3} - N_{63} \cos \beta \cdot 1 = 0 & N_{63} &= -16.492 \text{ kN} \end{aligned}$$



$$\begin{aligned} \sum M_3^L &= -32.5 \cdot 4 + 10 \cdot 4 + 20 \cdot 2.5 + N_{46} \cos \gamma \cdot 2.5 = 0 & N_{46} &= 17.088 \text{ kN} \\ \sum M_4^L &= 10 \cdot 4 - 12.5 \cdot 2.5 - 32.5 \cdot 1.5 + N_{13} \cdot 2.5 = 0 & N_{13} &= -16 \text{ kN} \\ \sum M_A^L &= 10 \cdot \left(8 + \frac{8}{3}\right) - 32.5 \cdot 4 + 20 \cdot 2.5 + N_{43} \left(4 + \frac{8}{3}\right) = 0 & N_{43} &= 4.0 \text{ kN} \end{aligned}$$

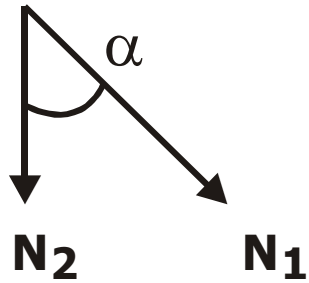
$$\begin{aligned} \sum M_3^L &= 10 \cdot 4 - N_{64} \cos \gamma \cdot 1 - N_{64} \sin \gamma \cdot 4 = 0 & N_{64} &= 17.088 \text{ kN} \\ \sum M_4^L &= 10 \cdot 4 - N_{31} \cdot 2.5 = 0 & N_{31} &= -16 \text{ kN} \\ \sum M_A^L &= -10 \cdot \frac{8}{3} + N_{34} \left(4 + \frac{8}{3}\right) = 0 & N_{34} &= 4.0 \text{ kN} \end{aligned}$$

Zestawienie sił osiowych w prętach:



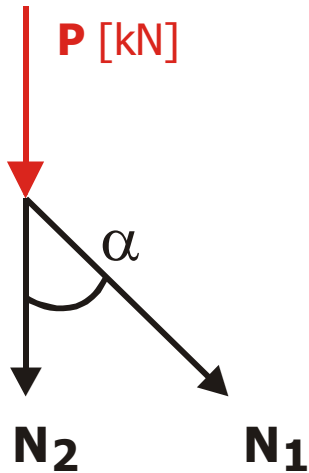
pręty ściskane  
pręty rozciągane  
pręty zerowe

## Reguły prętów zerowych:



W węźle nieobciążonym dwa pręty

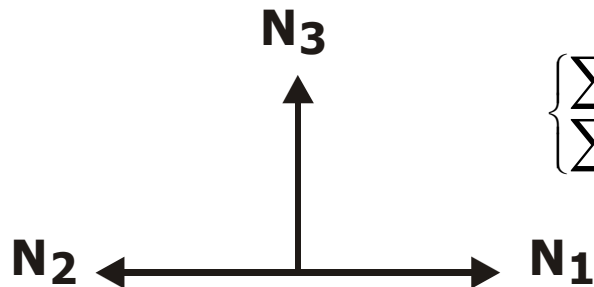
$$\begin{cases} \sum X = N_1 \sin \alpha = 0 \\ \sum Y = -N_2 - N_1 \cos \alpha = 0 \end{cases} \quad \begin{cases} N_1 = 0 \\ N_2 = 0 \end{cases}$$



W węźle dwa pręty obciążone siłą wzdłuż jednego z prętów:

$$\begin{cases} \sum X = N_1 \sin \alpha = 0 \\ \sum Y = -N_2 - N_1 \cos \alpha - P = 0 \end{cases} \quad \begin{cases} N_1 = 0 \\ N_2 = -P \end{cases}$$

W węźle nieobciążonym trzy pręty (dwa w jednej linii, trzeci pod kątem)



$$\begin{cases} \sum X = -N_2 + N_1 = 0 \\ \sum Y = N_3 = 0 \end{cases} \quad \begin{cases} N_1 = N_2 \\ N_3 = 0 \end{cases}$$