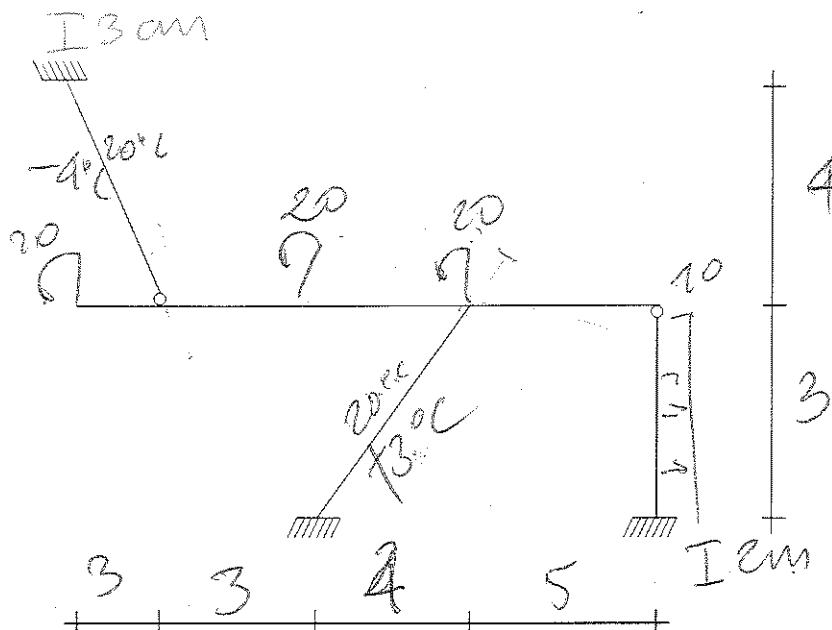


ĆWICZENIE PROJEKTOWE Z MECHANIKI BUDOWLI nr 4

Zadanie: Rozwiązać metodą przemieszczeń.

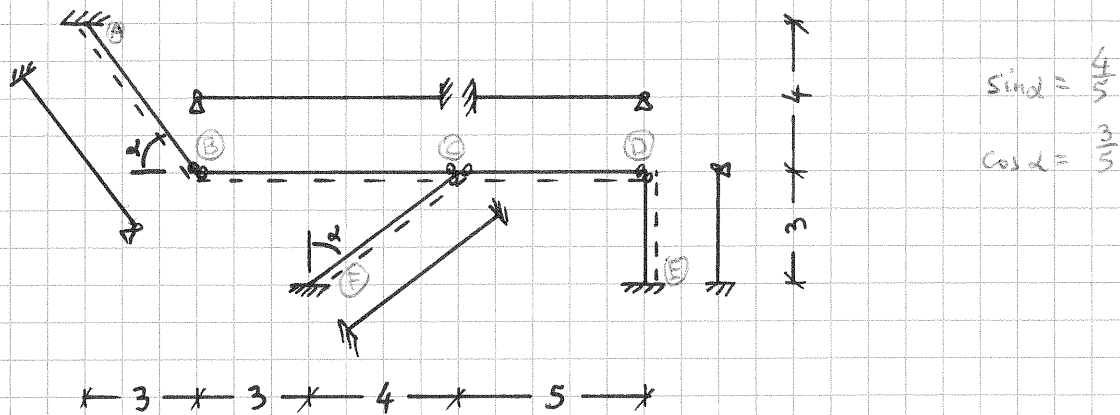


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Dane:

$$E = 210 \text{ GPa}, \quad \alpha_t = 1,2 \cdot 10^{-5} \frac{1}{\text{K}}, \quad J = 8,356 \cdot 10^{-5} \text{ m}^4$$

Stopień geometrycznej niezgodności. Podział wkładu na elementy, obliczenia kątów dane są przez transformacyjne. Przejście spójne pręty.

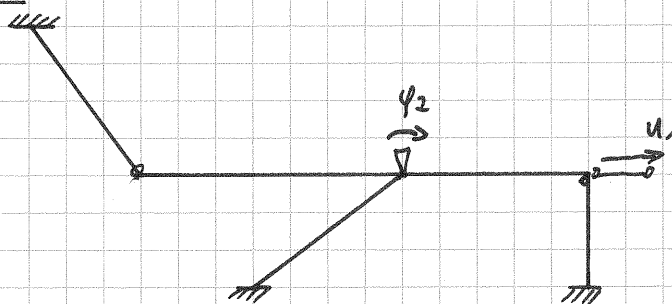


Liczba stopni swobody obrotu węzłów: $n_p = 1$

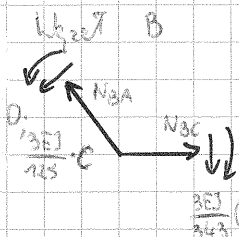
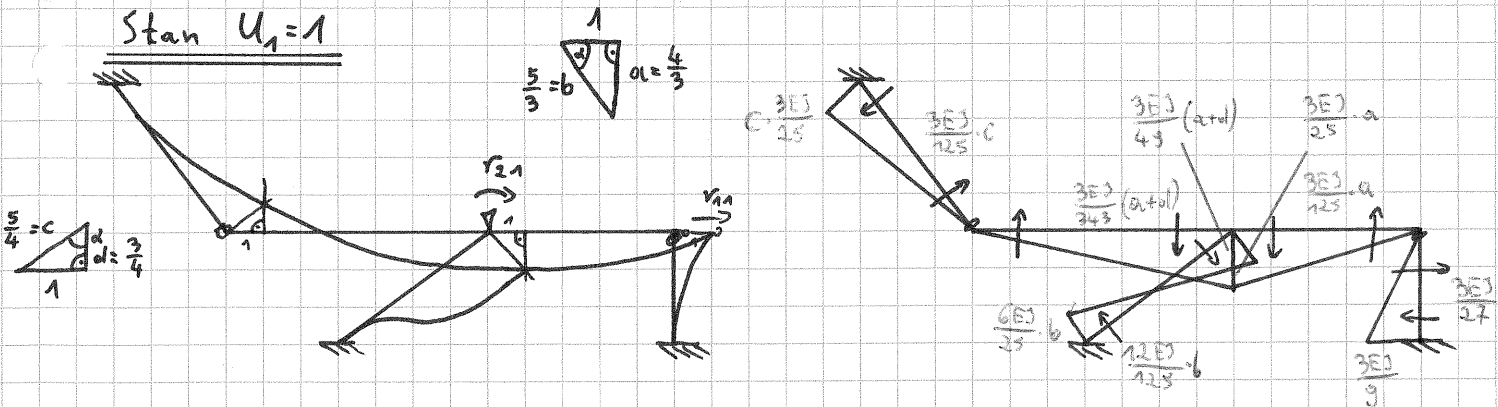
Liczba stopni swobody przesłunu węzłów: $n_g = 2 \cdot u - p - r = 2 \cdot 6 - 5 - 6 = 1$

Stopień geometrycznej niezgodności: $n_g = n_p + n_g = 1 + 1 = 2$

UPMP



Stan $u_1 = 1$

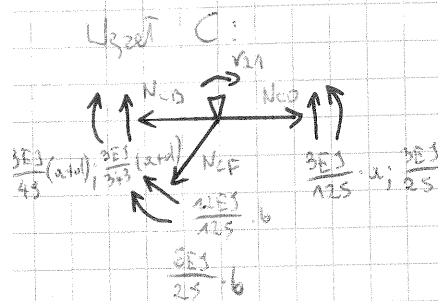


$$\sum Y = 0 \Rightarrow N_{BA} \sin \alpha - \frac{3EJ}{125} \cdot \frac{5}{4} \cdot \cos \alpha - \frac{3EJ}{343} \left(\frac{4}{3} + \frac{2}{4} \right) = 0$$

$$N_{BA} = \left(\frac{3EJ}{125} \cdot \frac{5}{4} \cdot \frac{3}{5} + \frac{3EJ}{343} \left(\frac{4}{3} + \frac{3}{4} \right) \right) \cdot \frac{5}{4} = \frac{9EJ}{400} + \frac{115EJ}{343} = \frac{9839EJ}{213500}$$

$$\sum X = 0 \Rightarrow -N_{BA} \cos \alpha - \frac{3EJ}{125} \cdot \frac{5}{4} \cdot \sin \alpha + N_{BC} = 0$$

$$N_{BC} = \frac{9839EJ}{213500} \cdot \frac{3}{5} + \frac{3EJ}{125} \cdot \frac{5}{4} \cdot \frac{4}{5} = \frac{28820EJ}{1037000} + \frac{26330EJ}{1037000} = \frac{54EJ}{1000}$$



$$\sum M_C = 0 \Rightarrow \frac{3EJ}{48} \left(\frac{4}{3} + \frac{2}{4} \right) + v_{11} - \frac{3EJ}{25} \cdot \frac{4}{3} + \frac{6EJ}{25} \cdot \frac{2}{3} = 0$$

$$v_{11} = -\frac{3EJ}{48} \cdot \frac{25}{12} + \frac{4EJ}{25} - \frac{2EJ}{5} = -\frac{488EJ}{1000}$$

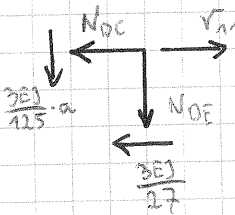
$$\sum Y = 0 \Rightarrow \frac{3EJ}{48} \left(\frac{4}{3} + \frac{2}{4} \right) + \frac{3EJ}{25} \cdot \frac{4}{3} + \frac{12EJ}{25} \cdot \frac{5}{3} \sin \alpha - N_{CF} \cos \alpha = 0$$

$$N_{CF} = \left(\frac{3EJ}{48} \cdot \frac{25}{12} + \frac{3EJ}{25} \cdot \frac{4}{3} + \frac{12EJ}{25} \cdot \frac{5}{3} \cdot \frac{4}{5} \right) \frac{5}{3} = \frac{89EJ}{300}$$

$$\sum X = 0 \Rightarrow -N_{CB} + N_{CD} - N_{CF} \sin \alpha - \frac{12EJ}{25} \cdot \frac{5}{3} \cos \alpha = 0$$

$$N_{CD} = \frac{51EJ}{1000} + \frac{89EJ}{300} \cdot \frac{4}{5} + \frac{4EJ}{20} \cdot \frac{3}{5} = \frac{408EJ}{1000}$$

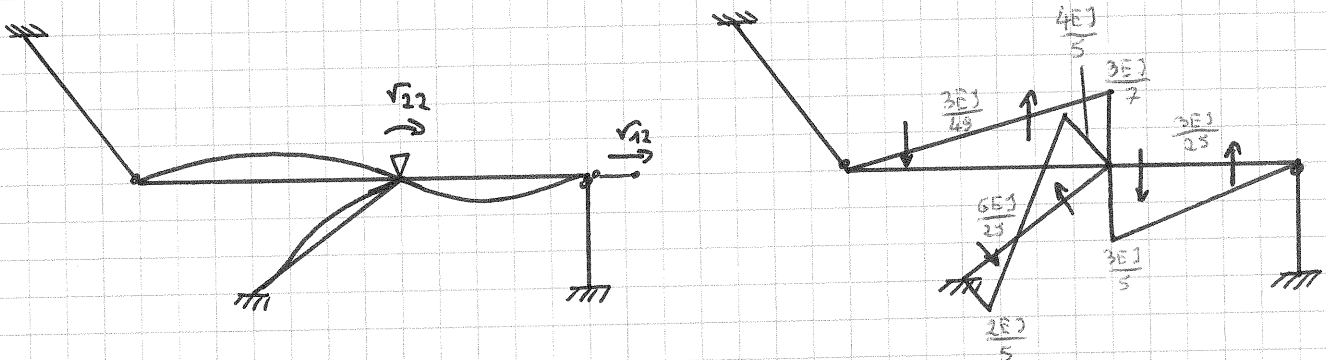
U₂₂ D:



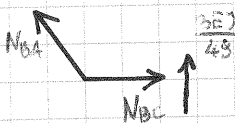
$$\sum X = 0 \Rightarrow -N_{DC} - \frac{3EJ}{27} + v_{11} = 0$$

$$v_{11} = \frac{408EJ}{1000} + \frac{3EJ}{27} = \frac{518EJ}{1000}$$

Stan $\varphi_2 = 1$



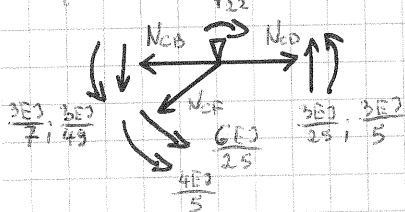
U₂₂ B:



$$\sum Y = 0 \Rightarrow \frac{3EJ}{48} + N_{BA} \sin \alpha = 0 \Rightarrow N_{BA} = -\frac{3EJ}{48} \cdot \frac{5}{4} = -\frac{15EJ}{196}$$

$$\sum X = 0 \Rightarrow -N_{BA} \cos \alpha + N_{BC} = 0 \Rightarrow N_{BC} = -\frac{15EJ}{196} \cdot \frac{3}{5} = -\frac{9EJ}{196}$$

U₂₂ C:



$$\sum M_C = 0 \Rightarrow v_{22} - \frac{3EJ}{5} - \frac{4EJ}{5} - \frac{3EJ}{7} = 0 \Rightarrow v_{22} = \frac{21EJ}{35} + \frac{28EJ}{35} + \frac{15EJ}{35} = \frac{64EJ}{35}$$

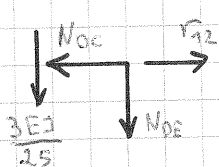
$$\sum Y = 0 \Rightarrow -\frac{3EJ}{48} + \frac{3EJ}{25} - N_{CF} \cos \alpha - \frac{6EJ}{25} \cdot \sin \alpha = 0$$

$$N_{CF} = \left(-\frac{3EJ}{48} + \frac{3EJ}{25} - \frac{6EJ}{25} \cdot \frac{4}{5} \right) \frac{5}{3} = -\frac{272EJ}{1225}$$

$$\sum X = 0 \Rightarrow -N_{CB} + N_{CD} - N_{CF} \sin \alpha + \frac{6EJ}{25} \cdot \cos \alpha = 0$$

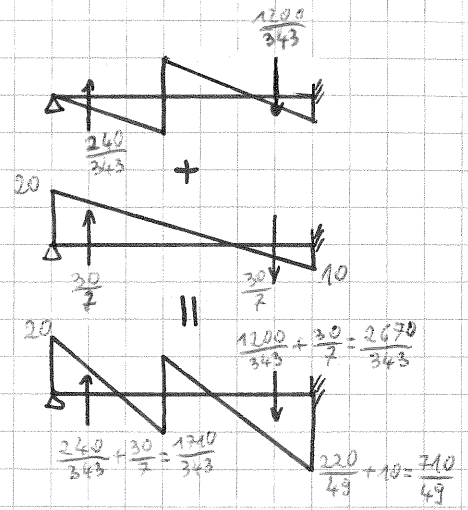
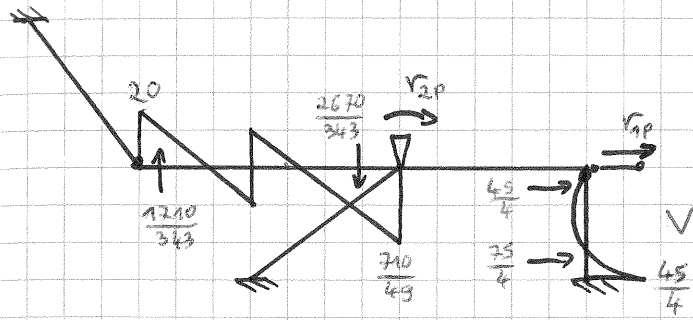
$$N_{CD} = -\frac{9EJ}{196} - \frac{272EJ \cdot 4}{1225 \cdot 5} - \frac{6EJ}{25} \cdot \frac{3}{5} = -\frac{1801EJ}{4900}$$

U₂₂ D:

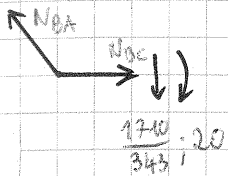


$$\sum X = 0 \Rightarrow -N_{DC} + v_{12} = 0 \Rightarrow v_{12} = -\frac{1801EJ}{4900}$$

Stan A:



Wzrost B:



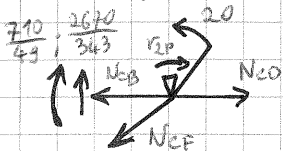
$$\sum Y = 0 \Rightarrow N_{BA} \sin \alpha - \frac{1710}{343} = 0$$

$$N_{BA} = \frac{1710}{343} \cdot \frac{5}{4} = \frac{4275}{686}$$

$$\sum X = 0 \Rightarrow -N_{BA} \cos \alpha + N_{BC} = 0$$

$$N_{BC} = \frac{4275}{686} \cdot \frac{3}{5} = \frac{2565}{686}$$

Wzrost C:

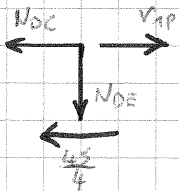


$$\sum M_C = 0 \Rightarrow v_{2P} - 20 + \frac{710}{49} = 0 \Rightarrow v_{2P} = \frac{270}{49} \checkmark$$

$$\sum Y = 0 \Rightarrow \frac{2670}{343} - N_{CF} \cos \alpha = 0 \Rightarrow N_{CF} = \frac{2670}{343} \cdot \frac{5}{3} = \frac{4450}{343}$$

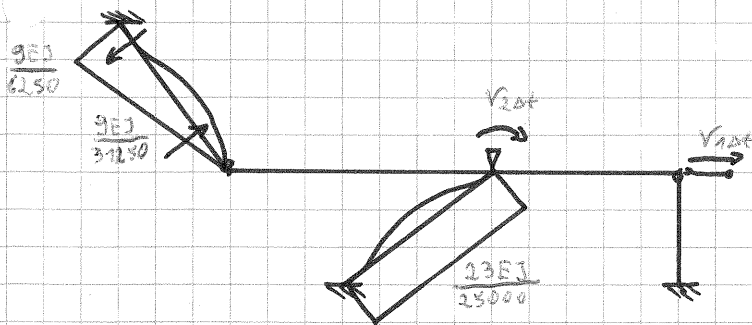
$$\sum X = 0 \Rightarrow N_{CD} - N_{CB} - N_{CF} \sin \alpha = 0 \Rightarrow N_{CD} = \frac{2565}{686} + \frac{4450}{343} \cdot \frac{4}{5} = \frac{9685}{686}$$

Wzrost D:

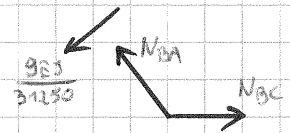


$$\sum X = 0 \Rightarrow v_{1P} - N_{DC} - \frac{45}{4} = 0 \Rightarrow v_{1P} = \frac{9685}{686} + \frac{45}{4} = \frac{34805}{1372} \checkmark$$

Stan A:



Wzrost B:



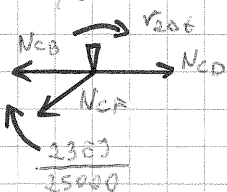
$$\sum Y = 0 \Rightarrow N_{BA} \sin \alpha - \frac{9EJ}{31250} \cdot \cos \alpha = 0$$

$$N_{BA} = \frac{9EJ}{31250} \cdot \frac{3}{5} \cdot \frac{5}{4} = \frac{27EJ}{12500}$$

$$\sum X = 0 \Rightarrow -N_{BA} \cos \alpha + N_{BC} = 0$$

$$N_{BC} = \frac{27EJ}{12500} \cdot \frac{5}{3} = \frac{9EJ}{25000}$$

Wzrost C:

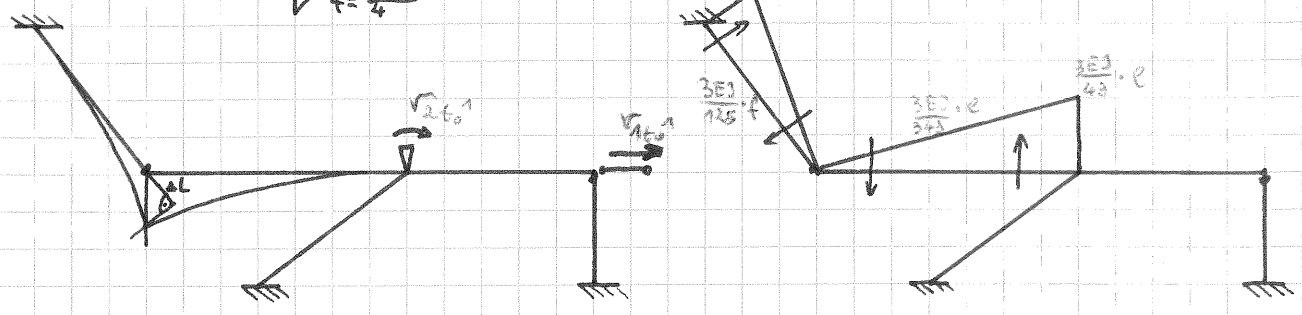


$$\sum M_C = 0 \Rightarrow v_{2At} + \frac{236J}{25000} = 0 \Rightarrow v_{2At} = -\frac{236J}{25000} \checkmark$$

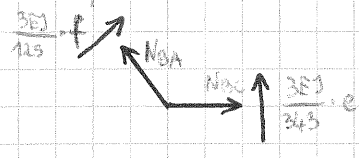
$$N_{CF} = 0$$

$$\sum X = 0 \Rightarrow -N_{CB} + N_{CD} = 0 \Rightarrow N_{CD} = \frac{9EJ}{25000} = v_{2At} \checkmark$$

Stan t_0^1 : $\frac{5\Delta l}{4} = e$ $\Delta l = \Delta t \cdot t_0 = \frac{9}{12500}$
 $f = \frac{3\Delta l}{4}$



Wzrost B:



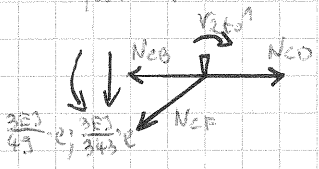
$$\sum Y = 0 \Rightarrow N_{BA} \sin \alpha + \frac{3EJ}{125} \cdot f \cdot \cos \alpha + \frac{3EJ}{343} \cdot e = 0$$

$$N_{BA} = \left(-\frac{3EJ}{125} \cdot \frac{3}{4} \cdot \frac{9}{12500} \cdot \frac{3}{5} - \frac{3EJ}{343} \cdot \frac{2}{4} \cdot \frac{9}{12500} \right) \frac{5}{4} = -\frac{3721EJ}{123306246}$$

$$\sum X = 0 \Rightarrow -N_{BA} \cos \alpha + N_{BC} + \frac{3EJ}{125} \cdot f \cdot \sin \alpha = 0$$

$$N_{BC} = -\frac{3721EJ}{123306246} \cdot \frac{3}{5} - \frac{3EJ}{125} \cdot \frac{3}{4} \cdot \frac{4}{3} = -\frac{11267EJ}{509734587}$$

Wzrost C:

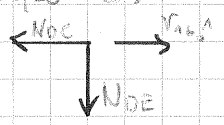


$$\sum Y = 0 \Rightarrow -\frac{3EJ}{343} \cdot e - N_{CF} \cdot \cos \alpha = 0 \Rightarrow N_{CF} = -\frac{3EJ}{343} \cdot \frac{3}{4} \cdot \frac{3}{5} \cdot \frac{9}{12500} = -\frac{9EJ}{686000}$$

$$\sum X = 0 \Rightarrow -N_{CB} - N_{CF} \sin \alpha + N_{CD} = 0 \Rightarrow N_{CD} = -\frac{11267EJ}{509734587} - \frac{9EJ}{686000} \cdot \frac{4}{5} = -\frac{13377EJ}{428250000}$$

$$\sum M_C = 0 \Rightarrow v_{2t_0^1} - \frac{3EJ}{49} \cdot e = 0 \Rightarrow v_{2t_0^1} = \frac{3EJ}{49} \cdot \frac{5}{4} \cdot \frac{9}{12500} = \frac{27EJ}{490000} \quad \checkmark$$

Wzrost D:

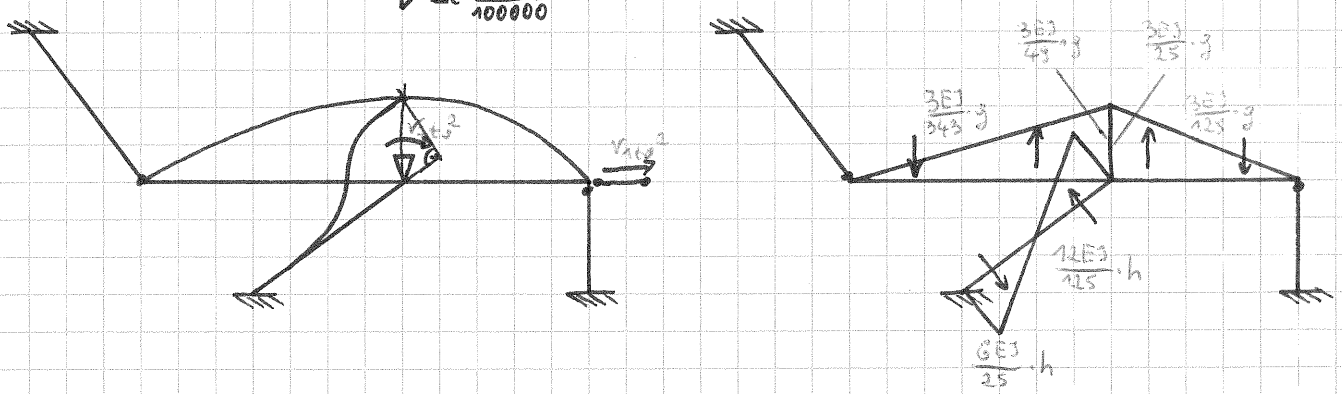


$$\sum X = 0 \Rightarrow v_{1t_0^1} = N_{DC} = -\frac{13377EJ}{428250000} \quad \checkmark$$

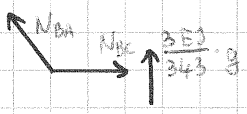
Stan t_0^2 :

$\frac{5}{3} \cdot \Delta l = g$ $h = \frac{4}{3} \cdot \Delta l$
 $\Delta l = \frac{63}{100000}$

$t_0 = ?$ $g = 9.81$ $M, SP C$
 $0.5 \Delta t \cdot 5m = 5.4 \cdot 10^{-9}$



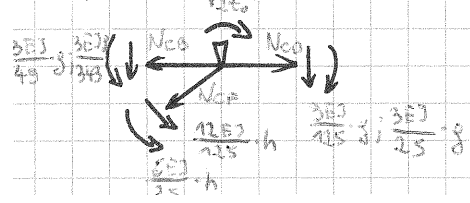
Wzrost B:



$$\sum Y = 0 \Rightarrow \frac{3EJ}{343} \cdot g + N_{BA} \sin \alpha = 0 \Rightarrow N_{BA} = -\frac{3EJ}{343} \cdot \frac{5}{3} \cdot \frac{63}{100000} \cdot \frac{5}{4} = -\frac{69EJ}{5488000}$$

$$\sum X = 0 \Rightarrow -N_{BA} \cos \alpha + N_{BC} = 0 \Rightarrow N_{BC} = -\frac{69EJ}{5488000} \cdot \frac{3}{5} = -\frac{207EJ}{27440000}$$

Wzrost C:



$$\sum M_C = 0 \Rightarrow v_{2t_0^2} + \frac{3EJ}{25} \cdot g - \frac{6EJ}{25} \cdot h - \frac{3EJ}{48} \cdot g = 0$$

$$v_{2t_0^2} = -\frac{3EJ}{25} \cdot \frac{5}{3} \cdot \frac{63}{100000} + \frac{6EJ}{25} \cdot \frac{4}{3} \cdot \frac{63}{100000} + \frac{3EJ}{48} \cdot \frac{5}{3} \cdot \frac{63}{100000} = \frac{1173EJ}{7656250} \quad \checkmark$$