$\mathrm{E}:=210 \cdot 10^{3} \cdot \mathrm{MPa} \quad \nu:=0.3$
$\mathrm{G}:=\frac{\mathrm{E}}{2(1+\nu)}=8.077 \times 10^{4} \cdot \mathrm{MPa}$
$\mathrm{Jz}:=11280 \cdot \mathrm{~cm}^{4} \quad \mathrm{Jy}:=3920 \cdot \mathrm{~cm}^{4} \quad \mathrm{JX}:=103 \cdot \mathrm{~cm}^{4}$
1240
$\mathrm{E}=210 \mathrm{GPa}$ $\mathrm{Jz}=11260 \mathrm{~cm} 4$ $\mathrm{Jy}=3920 \mathrm{~cm} 4$ $J x=103 \mathrm{~cm} 4$


## UPMS



$$
\begin{aligned}
& \delta_{11}:=\left(\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4+\frac{1}{2} \cdot 3 \cdot 3 \cdot \frac{2}{3} \cdot 3+3 \cdot 5 \cdot 3\right) \cdot \frac{\mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jz}}+(4 \cdot 5 \cdot 4) \cdot \frac{\mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jy}}+(4 \cdot 3 \cdot 4) \cdot \frac{\mathrm{m}^{3}}{\mathrm{G} \cdot \mathrm{JX}}=0.59 \mathrm{~m} \cdot \frac{1}{\mathrm{kN}} \\
& \delta_{12}:=(0) \cdot \frac{\mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jz}}+\left(\frac{1}{2} \cdot 5 \cdot 5 \cdot 4\right) \cdot \frac{\mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jy}}+0 \cdot \frac{\mathrm{~m}^{3}}{\mathrm{G} \cdot \mathrm{JX}}=6.074 \times 10^{-3} \mathrm{~m} \cdot \frac{1}{\mathrm{kN}} \\
& \delta_{22}:=(0) \cdot \frac{\mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{JZ}}+\left(\frac{1}{2} \cdot 3 \cdot 3 \cdot \frac{2}{3} 3+\frac{1}{2} \cdot 5 \cdot 5 \cdot \frac{2}{3} 5\right) \cdot \frac{\mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jy}}+(3 \cdot 5 \cdot 3) \cdot \frac{\mathrm{m}^{3}}{\mathrm{G} \cdot \mathrm{Jx}}=0.547 \mathrm{~m} \cdot \frac{1}{\mathrm{kN}}
\end{aligned}
$$

$$
\Delta_{\mathrm{lp}}:=\left[\frac{-1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 80+\frac{2}{3} \cdot 4 \cdot \frac{10 \cdot 4^{2}}{8} \frac{1}{2} 4+\frac{1}{2} 3 \cdot 3 \cdot\left(\frac{1}{3} 20-\frac{2}{3} 100\right)-3 \cdot 5 \cdot 75\right] \cdot \frac{\mathrm{kN} \cdot \mathrm{~m}^{3}}{\mathrm{E} \cdot \mathrm{Jz}} \ldots
$$

$$
+\left[(-4 \cdot 5 \cdot 80) \cdot \frac{\mathrm{kN} \cdot \mathrm{~m}^{3}}{\mathrm{E} \cdot \mathrm{Jy}}+(-4 \cdot 3 \cdot 80) \cdot \frac{\mathrm{kN} \cdot \mathrm{~m}^{3}}{\mathrm{G} \cdot \mathrm{JX}}\right]
$$

$\Delta_{1 p}=-11.806 \mathrm{~m}$
$\Delta_{2 \mathrm{p}}:=(0) \cdot \frac{\mathrm{kN} \cdot \mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jz}}+\left(\frac{-1}{2} \cdot 3 \cdot 3 \cdot 40-\frac{1}{2} \cdot 5 \cdot 5 \cdot 80\right) \cdot \frac{\mathrm{kN} \cdot \mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jy}}+(-3 \cdot 5 \cdot 40) \cdot \frac{\mathrm{kN} \cdot \mathrm{m}^{3}}{\mathrm{G} \cdot \mathrm{JX}}$
$\Delta_{2 p}=-7.356 \mathrm{~m}$
Rozewiązanie układu równań

$$
\begin{array}{ll}
\mathrm{D}:=\left(\begin{array}{cc}
\delta_{11} & \delta_{12} \\
\delta_{12} & \delta_{22}
\end{array}\right) & \mathrm{P}:=\binom{-\Delta_{1 \mathrm{p}}}{-\Delta_{2 \mathrm{p}}} \\
\mathrm{X}:=\mathrm{D}^{-1} \cdot \mathrm{P} & \mathrm{X}=\binom{19.87874}{13.22464} \cdot \mathrm{kN}
\end{array}
$$

| Momenty |  |  |  | $\square$ |  |  |  |  |  |  | Most |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Stan X1=1 |  |  | Stan X2=1 |  |  |  | Stan P |  |  |  |  |
|  | M x | My | Mz | Mx | My | Mz | Mx | My | Mz | Mx | My | Mz |
| A-B | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B-A | 0 | 0 | 4 | 0 | 0 | 0 | 0 | -40 | -80 | 0 | -40 | -0.484 |
| B-C | 4 | 0 | 0 | 0 | 0 | 0 | -80 | -40 | 20 | -0.484 | -40 | 20 |
| C-B | 4 | 0 | 3 | 0 | -3 | 0 | -80 | 40 | -100 | -0.484 | 0.325 | -40.363 |
| C-D | 0 | -4 | -3 | -3 | 0 | 0 | 40 | 80 | 100 | 0.325 | 0.484 | 40.363 |
| D-C | 0 | -4 | -3 | -3 | -5 | 0 | 40 | 80 | 50 | 0.325 | -65.641 | -9.637 |
| TNACE |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Stan X1=1 |  |  | $S \tan \mathrm{X}=1$ |  |  |  | Stan P |  |  | Sily ost |  |
|  | Nx | Ty | Tz | Nx | Ty | Tz | Nx | Ty | Tz | Nx | Ty | Tz |
| A-B | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 10 | 0 | 19.879 | 10 |
| B-A | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 40 | -10 | 0 | 20.121 | -10 |
| B-C | 0 | 1 | 0 | 0 | 0 | -1 | -10 | -40 | 0 | -10 | -20.121 | -13.225 |
| C-B | 0 | -1 | 0 | 0 | 0 | 1 | -10 | 40 | 0 | -10 | 20.121 | 13.225 |
| C-D | 1 | 0 | 0 | 0 | 0 | -1 | -40 | -10 | 0 | -20.121 | -10 | -13.225 |
| D-C | 1 | 0 | 0 | 0 | 0 | 1 | -40 | 10 | 0 | -20.121 | 10 | 13.225 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| REAKCJE |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Stan X1=1 | Stan X2=1 | Stan P | Rost |  |  |  |  |  |  |  |  |
| RDX | 0 | 1 | 0 | 13.225 |  |  |  |  |  |  |  |  |
| RDY | 0 | 0 | -10 | -10 |  |  |  |  |  |  |  |  |
| RDZ | -1 | 0 | 40 | 20.121 |  |  |  |  |  |  |  |  |
| MDX | 3 | 0 | -50 | 9.637 |  |  |  |  |  |  |  |  |
| MDY | 4 | 5 | -80 | 65.641 |  |  |  |  |  |  |  |  |
| MDZ | 0 | 3 | -40 | -0.325 |  |  |  |  |  |  |  |  |

Sprawdzenie kinematyczne

$$
\begin{aligned}
\mathrm{u}_{\mathrm{Az}}= & \int \frac{\text { M1y } \cdot \text { Mosty }}{\mathrm{E} \cdot \mathrm{Jy}} \mathrm{ds}+\int \frac{\mathrm{M} 1 \mathrm{z} \cdot \mathrm{Mostz}}{\mathrm{E} \cdot \mathrm{Jz}} \mathrm{ds}+\int \frac{\mathrm{M} 1 \mathrm{x} \cdot \mathrm{Mostx}}{\mathrm{G} \cdot \mathrm{Jx}} \mathrm{ds} \\
\mathrm{u}_{\mathrm{Az}}:= & {\left[\frac{-1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 0.484+\frac{2}{3} \cdot 4 \cdot \frac{10 \cdot 4^{2}}{8} \frac{1}{2} 4+\frac{1}{2} 3 \cdot 3 \cdot\left(\frac{1}{3} 20-\frac{2}{3} 40.363\right)+3 \cdot 5 \cdot \frac{1}{2}(9.627-40.363)\right] \cdot \frac{\mathrm{kN} \cdot \mathrm{~m}^{3}}{\mathrm{E} \cdot \mathrm{Jz}} \ldots } \\
& +\left[\left[4 \cdot 5 \cdot \frac{1}{2}(65.641-0.484)\right] \cdot \frac{\mathrm{kN} \cdot \mathrm{~m}^{3}}{\mathrm{E} \cdot \mathrm{Jy}}+(-4 \cdot 3 \cdot 0.484) \cdot \frac{\mathrm{kN} \cdot \mathrm{~m}^{3}}{\mathrm{G} \cdot \mathrm{Jx}}\right] \\
\mathrm{u}_{\mathrm{Az}}= & 1.539 \times 10^{-4} \mathrm{~m}
\end{aligned}
$$

błąd względny
blad1 $:=\frac{\left|\mathrm{u}_{\mathrm{Az}}\right|}{\left|\Delta_{\mathrm{lp}}\right|}=1.303 \times 10^{-3} . \%$
$u_{B x}=\int \frac{M 2 y \cdot \text { Mosty }}{E \cdot J y} d s+\int \frac{\text { M2z•Mostz }}{E \cdot J z} d s+\int \frac{\text { M2x•Mostx }}{G \cdot J x} d s$
$u_{B x}:=(0) \cdot \frac{\mathrm{kN} \cdot \mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jz}}+\left[\frac{-1}{2} \cdot 3 \cdot 3 \cdot \frac{1}{2}(40+0.325)+\frac{1}{2} \cdot 5 \cdot 5 \cdot \frac{1}{2}(65.641-0.484)\right] \cdot \frac{\mathrm{kN} \cdot \mathrm{m}^{3}}{\mathrm{E} \cdot \mathrm{Jy}}+(-3 \cdot 5 \cdot 0.325) \cdot \frac{\mathrm{kN} \cdot \mathrm{m}^{3}}{\mathrm{G} \cdot \mathrm{Jx}}$

$$
\begin{aligned}
& \mathrm{u}_{\mathrm{Bx}}=-0.02 \mathrm{~m} \\
& \text { blad2 }:=\frac{\left|\mathrm{u}_{\mathrm{Bx}}\right|}{\left|\Delta_{2 \mathrm{p}}\right|}=0.274 . \%
\end{aligned}
$$

Sprawdzenie statyczne

$$
\begin{aligned}
& \mathrm{RDx}:=-13.225+13.225=0 \\
& \mathrm{RDy}:=10-10=0 \\
& \mathrm{RDz}:=-40+19.879+20.121=0
\end{aligned}
$$

$$
\text { MDx }:=-10 \cdot 5-20+40 \cdot 3+9.637-19.879 \cdot 3=0
$$

$$
\text { MDy }:=-19.879 \cdot 4+10 \cdot 4 \cdot 2-13.225 \cdot 5+65.641=0
$$

$$
\mathrm{MDz}:=-0.325+10 \cdot 4-13.225 \cdot 3=0
$$


JEŻELI UKŁAD LOKALNY PRĘTA C-D WYGLĄDA TAK, TOZ ZNAKI WYKRESÓW MAJĄ POSTAĆ




