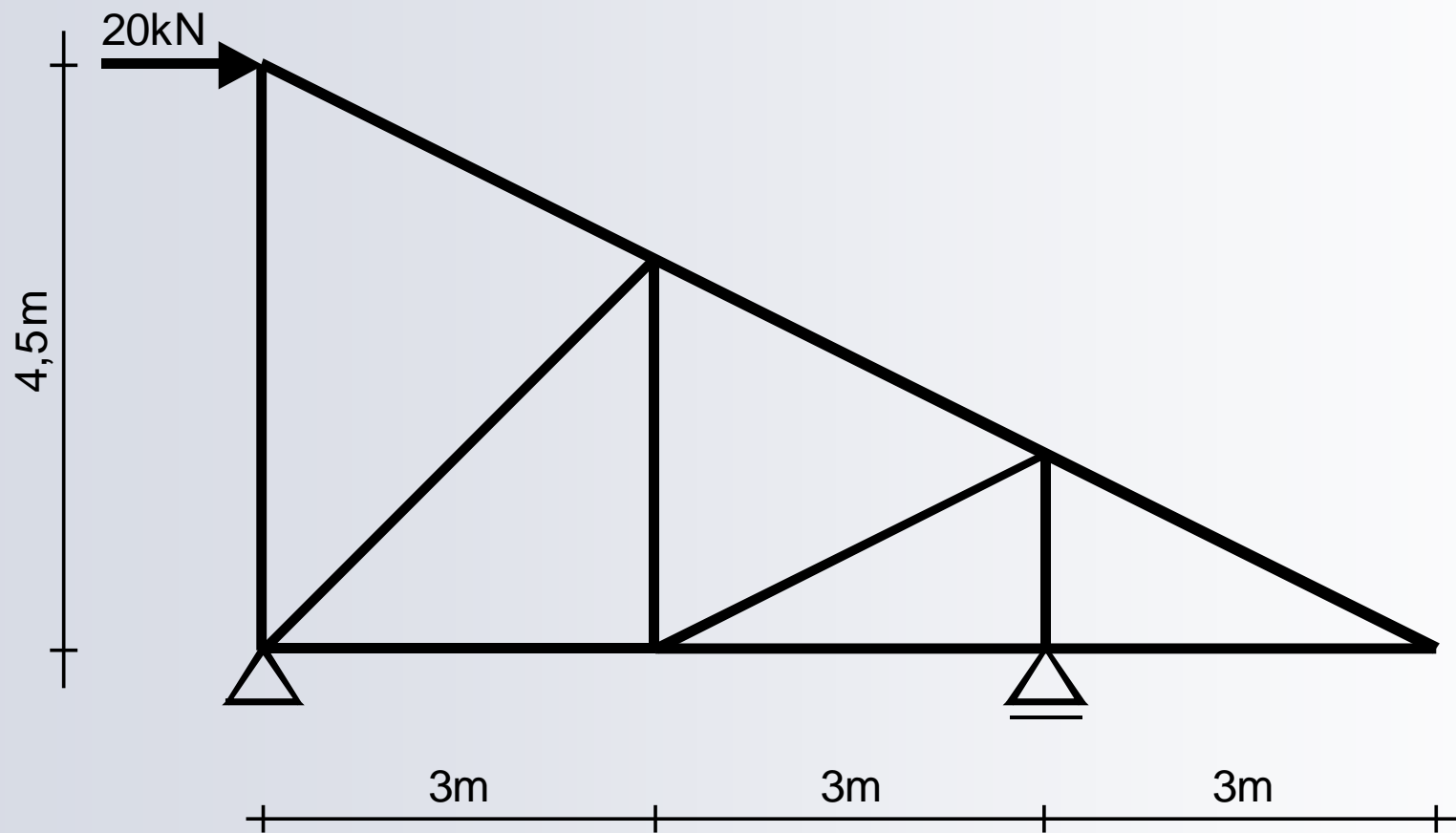


Mechanika teoretyczna

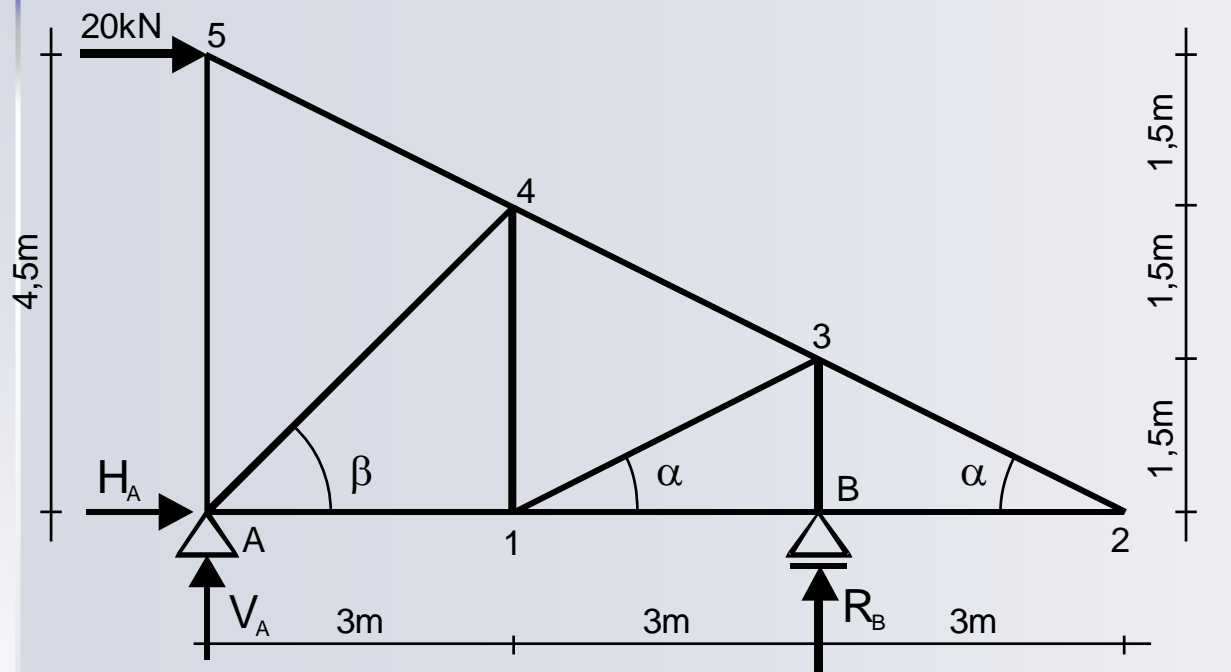
Wykład nr 3

**Obliczanie sił wewnętrznych
w układach prętowych - kratownice.**

Przykład B – kratownica trójkątna



Przykład B – reakcje



$$\sin \alpha = \frac{1,5m}{\sqrt{(1,5m)^2 + (3m)^2}} = 0,447 \quad \cos \alpha = \frac{3m}{\sqrt{(1,5m)^2 + (3m)^2}} = 0,894$$

$$\sin \beta = \cos \beta = \frac{3m}{\sqrt{(3m)^2 + (3m)^2}} = 0,707$$

$$\sum X : H_A + 20kN = 0$$

$$\sum Y : V_A + R_B = 0$$

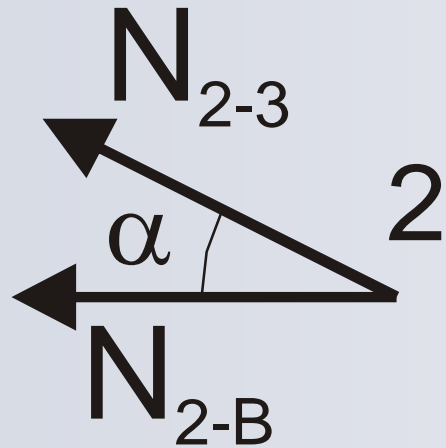
$$\sum M_A : R_B \cdot 6m - 20kN \cdot 4,5m = 0$$

$$H_A = -20kN$$

$$V_A = -15kN$$

$$R_B = 15kN$$

Węzeł 2



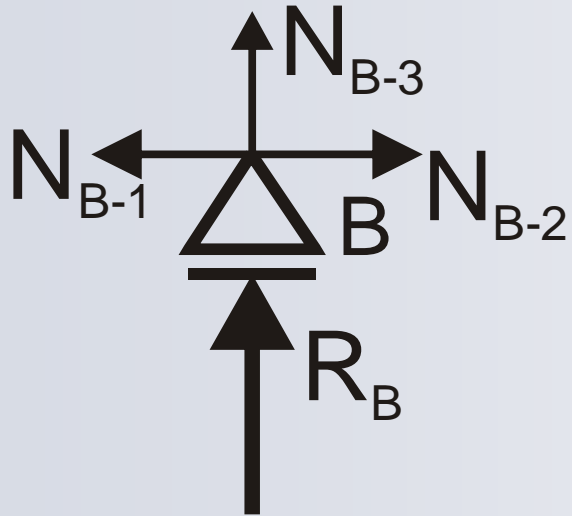
$$\sum Y : N_{2-3} \cdot \sin \alpha = 0$$

$$N_{2-3} = 0$$

$$\sum X : N_{2-B} + N_{2-3} \cdot \cos \alpha = 0$$

$$N_{2-B} = 0$$

Wezeł B



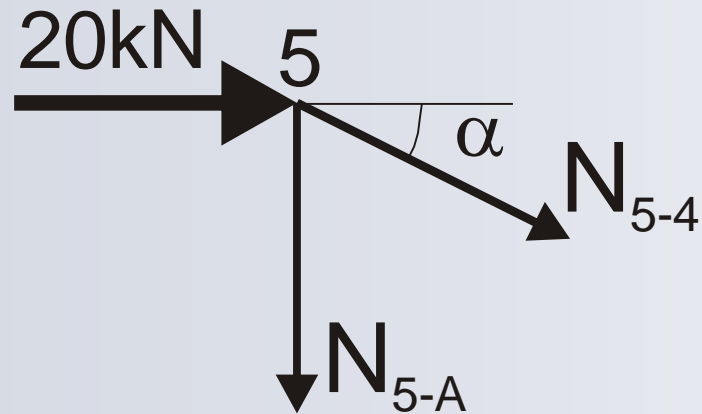
$$\sum X : N_{B-2} - N_{B-1} = 0$$

$$N_{B-1} = 0$$

$$\sum Y : R_B + N_{B-3} = 0$$

$$N_{B-3} = -15kN$$

Wezeł 5



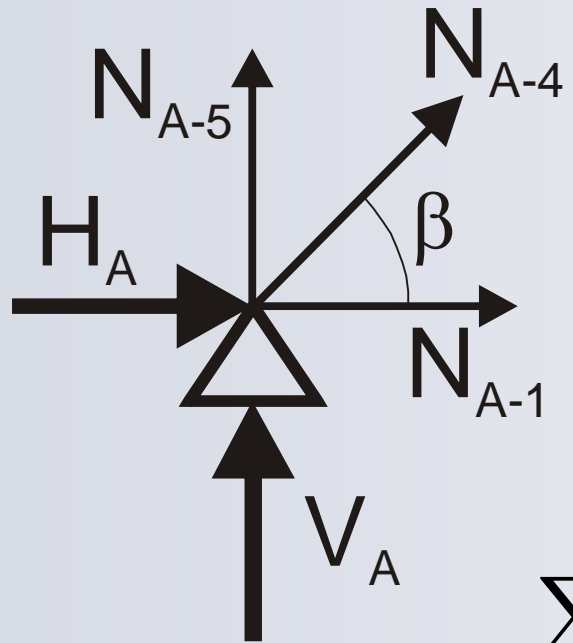
$$\sum X : 20kN + N_{5-4} \cdot \cos \alpha = 0$$

$$N_{5-4} = \frac{-20kN}{0,894} = -22,371kN$$

$$\sum Y : N_{5-A} + N_{5-4} \cdot \sin \alpha = 0$$

$$N_{5-A} = 22,371kN \cdot 0,447 = 10kN$$

Węzeł A



$$\sum Y : N_{A-5} + N_{A-4} \cdot \sin \beta + V_A = 0$$

$$N_{A-4} = \frac{-10kN + 15kN}{0,707} = 7,072kN$$

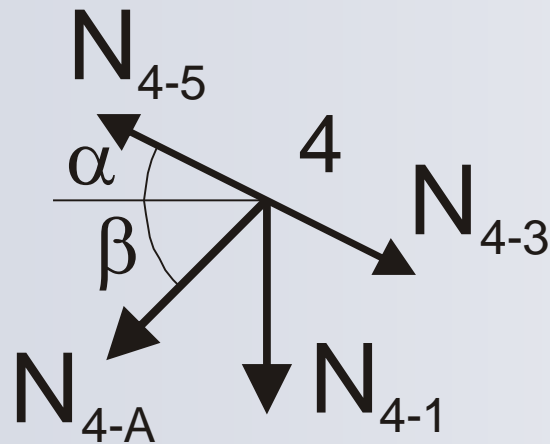
$$\sum X : N_{A-1} + N_{A-4} \cdot \cos \beta + H_A = 0$$

$$N_{A-1} = 20kN - 7,072kN \cdot 0,707 = 15kN$$

Węzeł 4

$$\sum X : N_{4-3} \cdot \cos \alpha - N_{4-A} \cdot \cos \beta - N_{4-5} \cdot \cos \alpha = 0$$

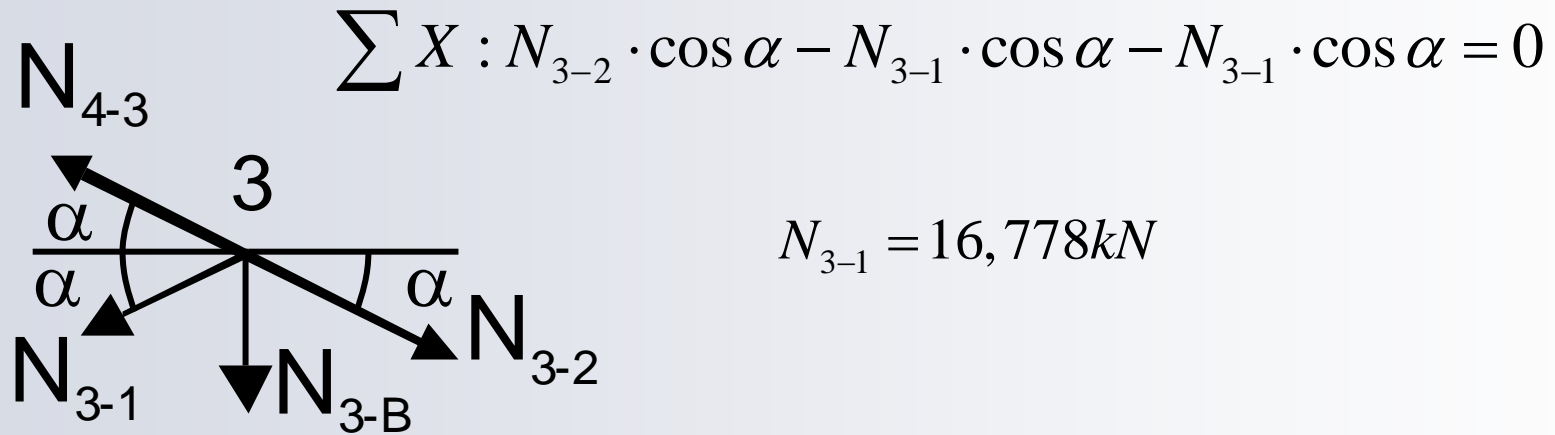
$$N_{4-3} = \frac{7,072kN \cdot 0,707 - 22,371kN \cdot 0,894}{0,894} = -16,778kN$$



$$\sum Y : N_{4-5} \cdot \sin \alpha - N_{4-A} \cdot \sin \beta + N_{4-1} - N_{4-3} \cdot \sin \alpha = 0$$

$$N_{4-1} = -22,371kN \cdot 0,447 - 7,072kN \cdot 0,707 + 16,778kN \cdot 0,447 = -7,5kN$$

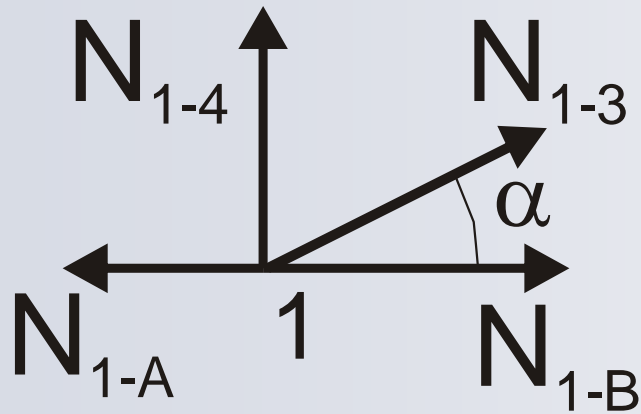
Węzeł 3



Sprawdzenie:

$$\begin{aligned} \sum Y : N_{4-3} \cdot \sin \alpha - N_{3-B} - N_{3-1} \cdot \sin \alpha - N_{3-2} \cdot \sin \alpha = \\ = -16,778kN \cdot 0,447 - 16,778kN \cdot 0,447 + 15kN = 0 \end{aligned}$$

Węzeł 1



Sprawdzenie:

$$\begin{aligned}\sum X : N_{1-3} \cdot \cos \alpha + N_{1-B} - N_{1-A} &= \\ &= 16,778kN \cdot 0,894 - 15kN = 0\end{aligned}$$

Sprawdzenie:

$$\sum Y : N_{1-4} - N_{1-3} \cdot \sin \alpha = -7,5kN + 16,778kN \cdot 0,447 = 0$$

Przykład B – metoda Rittera – przekrój 1 (z lewej)

$$\sum M_A^l : N_{5-4} \cdot \cos \alpha \cdot 4,5m + 20kN \cdot 4,5m = 0$$

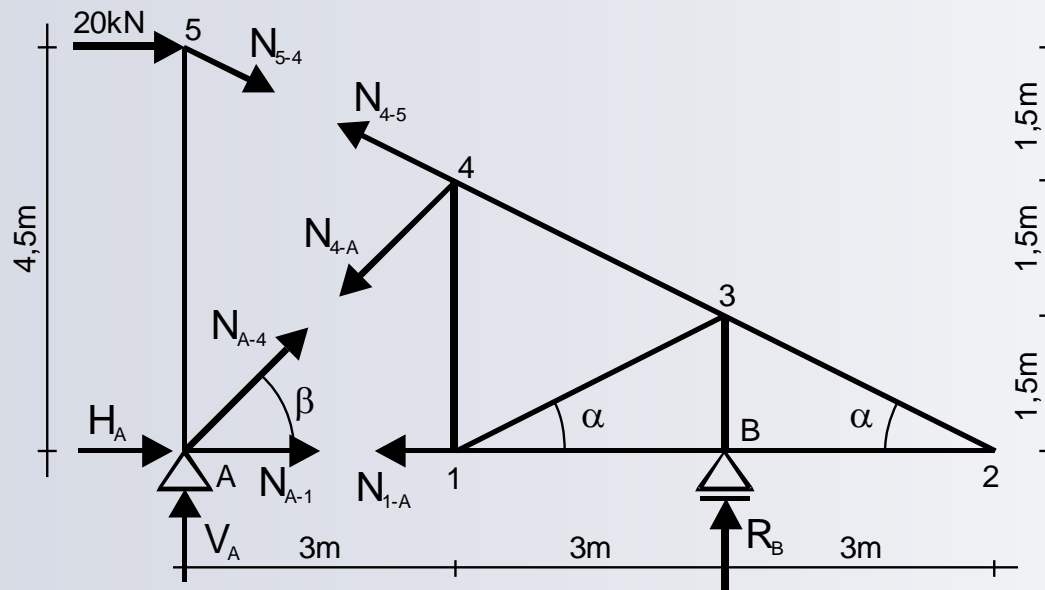
$$\sum M_4^l : V_A \cdot 3m - H_A \cdot 3m + 20kN \cdot 1,5m - N_{A-1} \cdot 3m = 0$$

$$\sum M_2^l : V_A \cdot 9m + 20kN \cdot 4,5m + N_{A-4} \cdot \sin \beta \cdot 9m = 0$$

$$N_{5-4} = \frac{-20kN}{0,894} = -22,371kN$$

$$N_{A-1} = -15kN + 20kN + 10kN = 15kN$$

$$N_{A-4} = \frac{15kN \cdot 9m - 20kN \cdot 4,5m}{0,707 \cdot 9m} = 7,072kN$$



Przykład B – metoda Rittera – przekrój 1 (z prawej)

$$\sum M_A^P : N_{4-5} \cdot \cos \alpha \cdot 3m + N_{4-5} \cdot \sin \alpha \cdot 3m + R_B \cdot 6m = 0$$

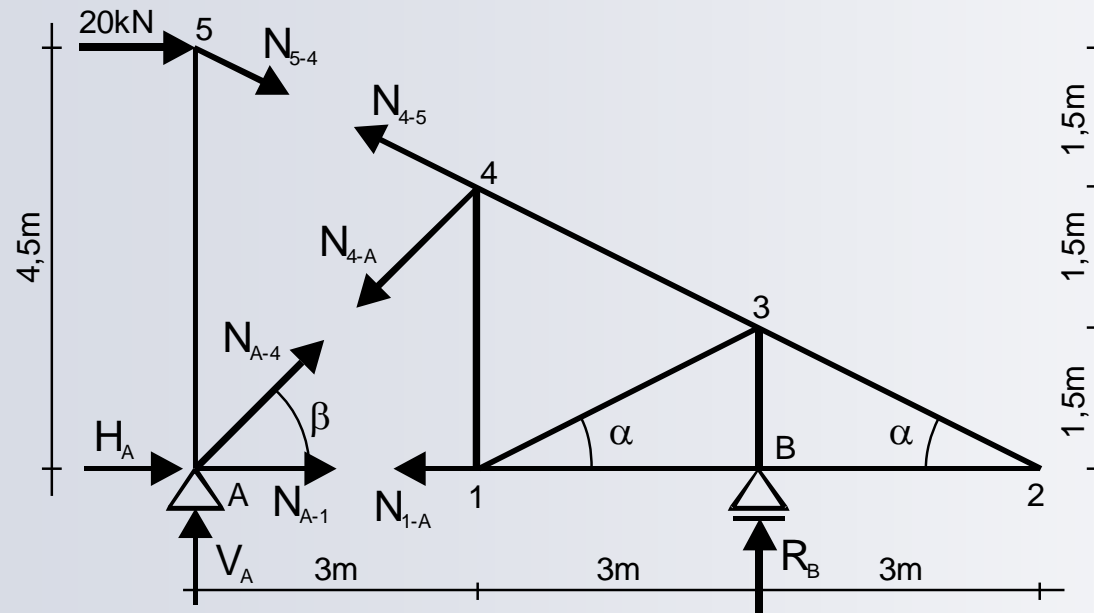
$$\sum M_4^P : R_B \cdot 3m - N_{1-A} \cdot 3m = 0$$

$$\sum M_2^P : R_B \cdot 3m - N_{4-A} \cdot \sin \beta \cdot 6m - N_{4-A} \cdot \cos \beta \cdot 3m = 0$$

$$N_{4-5} = \frac{-15kN \cdot 6m}{0,894 \cdot 3m + 0,447 \cdot 3m} = -22,371kN$$

$$N_{1-A} = 15kN$$

$$N_{4-A} = \frac{15kN \cdot 3m}{0,707 \cdot 6m + 0,707 \cdot 3m} = 7,072kN$$



Przykład B – metoda Rittera – przekrój 2

$$\sum M_1^P : N_{4-3} \cdot \cos \alpha \cdot 1,5m + N_{4-3} \cdot \sin \alpha \cdot 3m + R_B \cdot 3m = 0$$

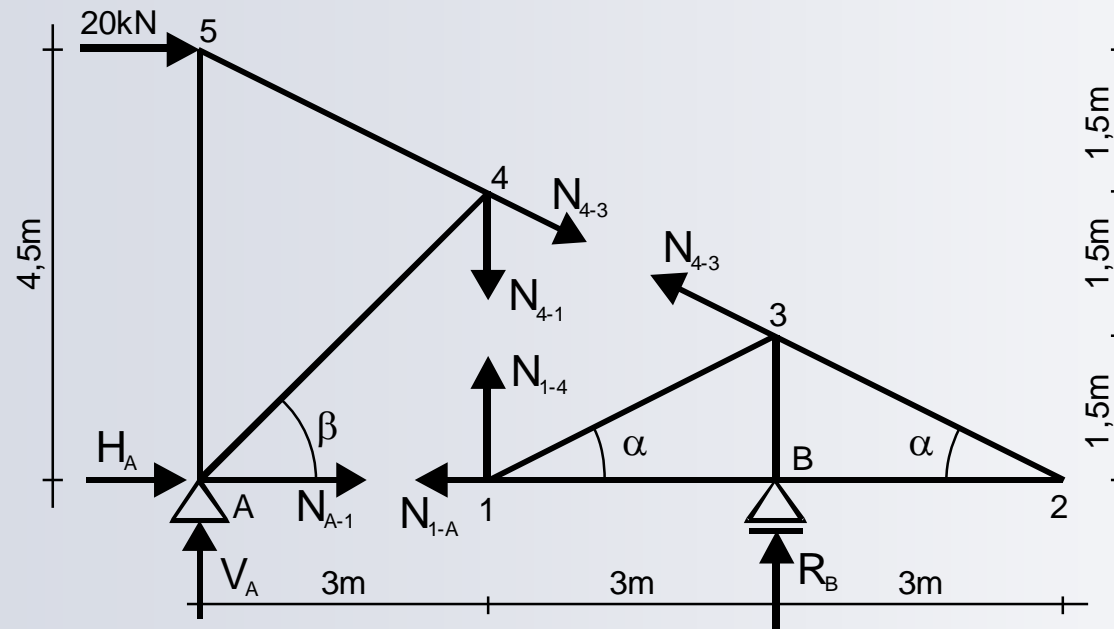
$$\sum M_4^P : R_B \cdot 3m - N_{1-A} \cdot 3m = 0$$

$$\sum M_2^P : R_B \cdot 3m - N_{1-4} \cdot 6m = 0$$

$$N_{4-3} = \frac{-15kN \cdot 3m}{0,894 \cdot 1,5m + 0,447 \cdot 3m} = -16,779kN$$

$$N_{1-A} = 15kN$$

$$N_{1-4} = \frac{15kN \cdot 3m}{6m} = 7,5kN$$



Przykład B – metoda Rittera – przekrój 3

$$\sum M_1^P : N_{4-3} \cdot \cos \alpha \cdot 1,5m + N_{4-3} \cdot \sin \alpha \cdot 3m + R_B \cdot 3m = 0$$

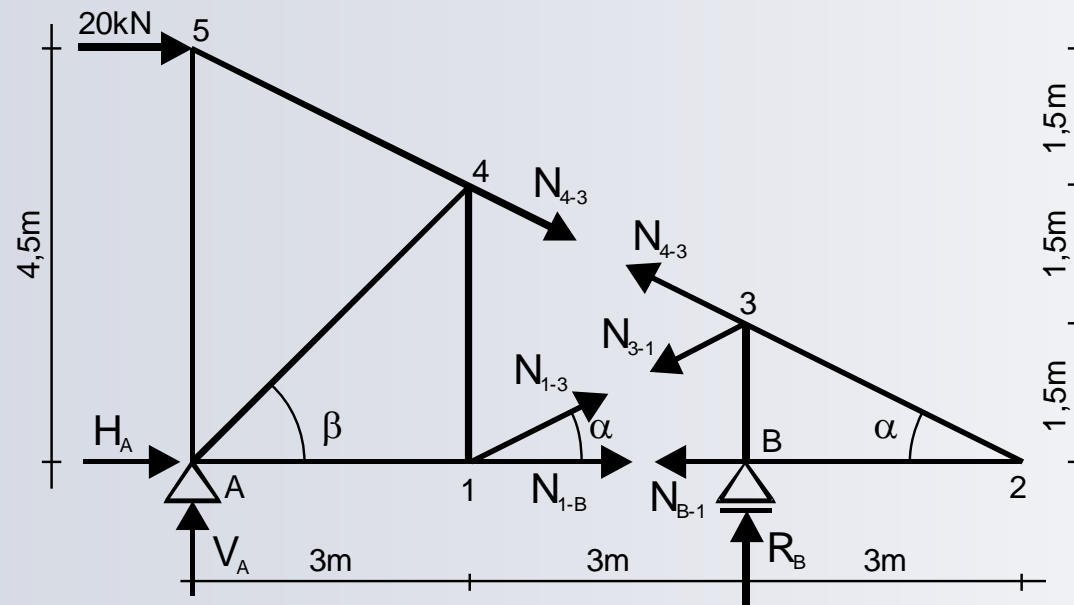
$$\sum M_3^P : N_{5-1} \cdot 1,5m = 0$$

$$\sum M_2^l : V_A \cdot 9m + 20kN \cdot 4,5m + N_{1-3} \cdot \sin \alpha \cdot 6m = 0$$

$$N_{4-3} = \frac{-15kN \cdot 3m}{0,894 \cdot 1,5m + 0,447 \cdot 3m} = -16,779kN$$

$$N_{5-1} = 0$$

$$N_{1-3} = \frac{15kN \cdot 9m - 20kN \cdot 4,5m}{0,447 \cdot 6m} = 16,779kN$$



Przykład B – metoda Rittera – przekrój 4

$$\sum M_3^P : N_{B-1} \cdot 1,5m = 0$$

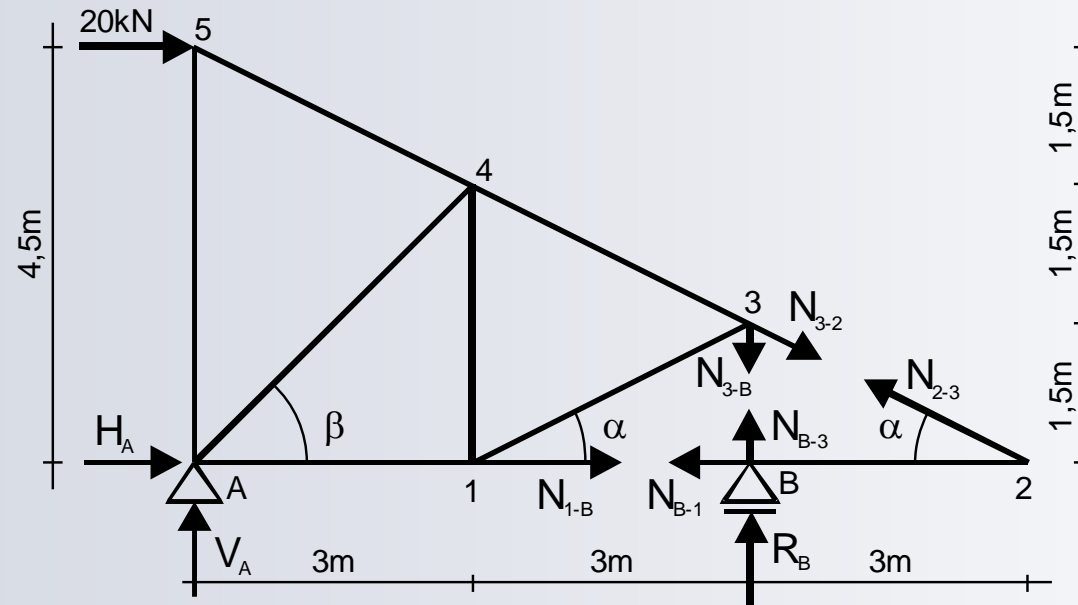
$$N_{B-1} = 0$$

$$\sum M_B^P : N_{2-3} \cdot \sin \alpha \cdot 3m = 0$$

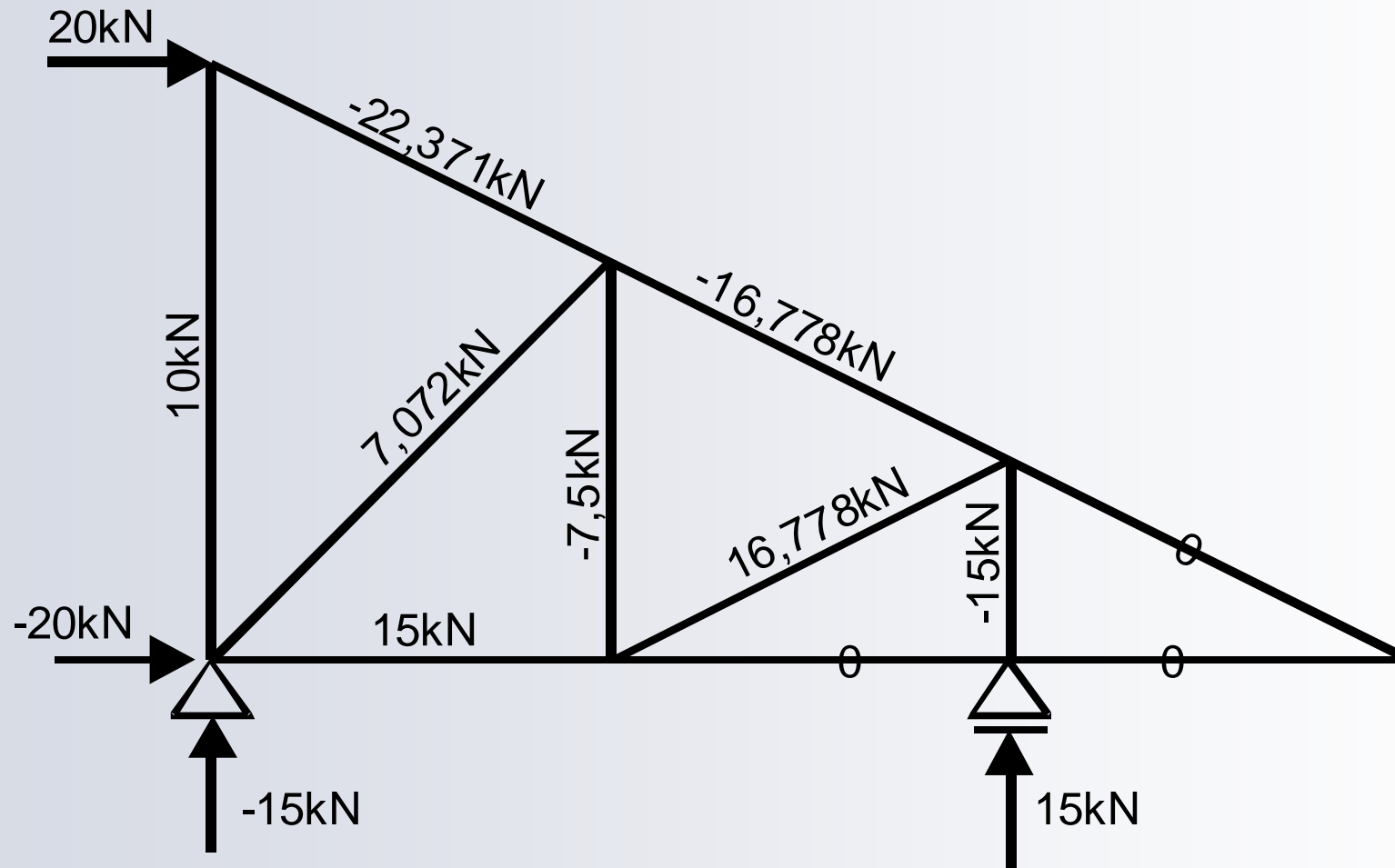
$$N_{2-3} = 0$$

$$\sum M_2^P : R_B \cdot 3m + N_{B-3} \cdot 3m = 0$$

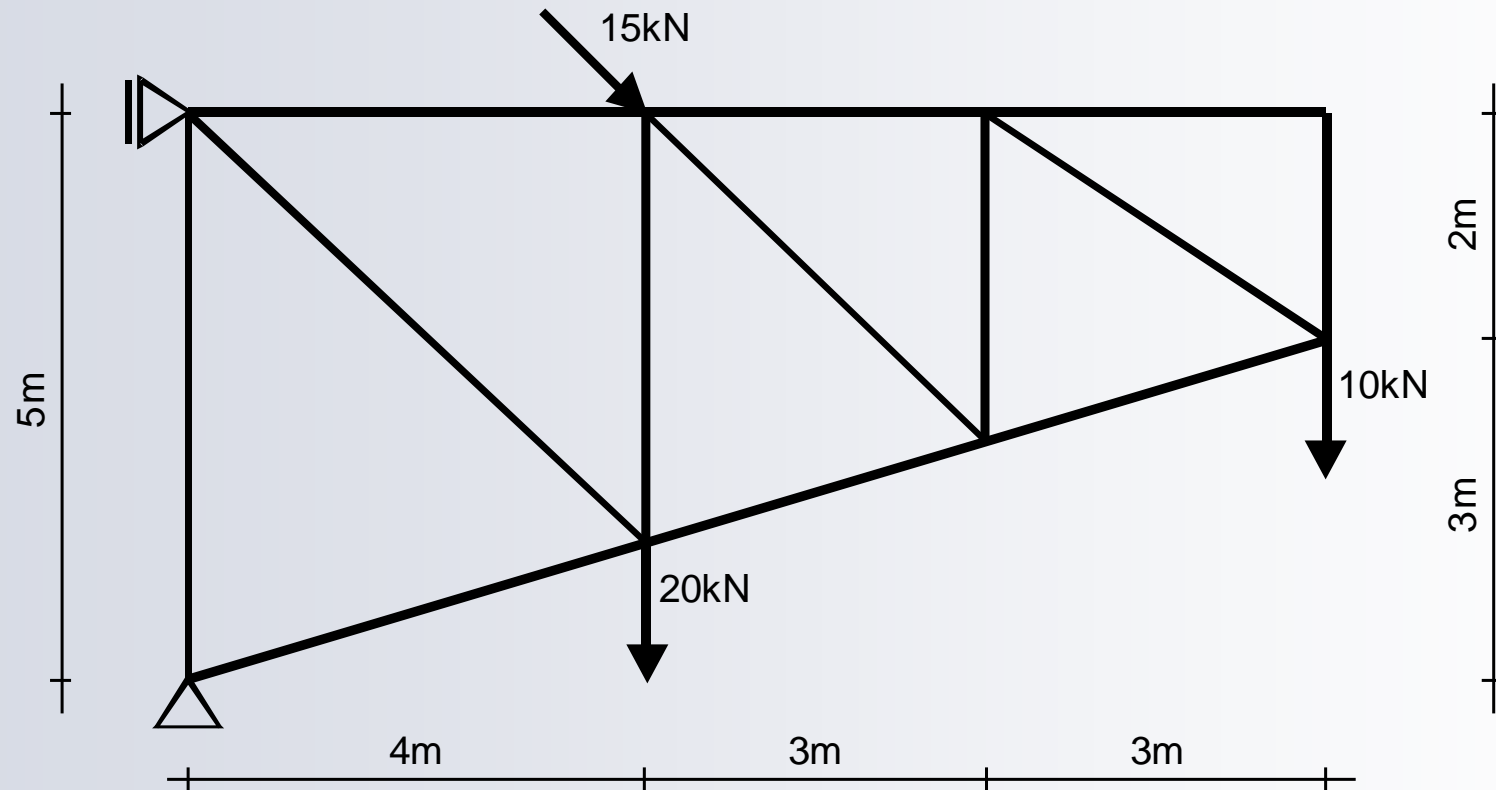
$$N_{B-3} = -15kN$$



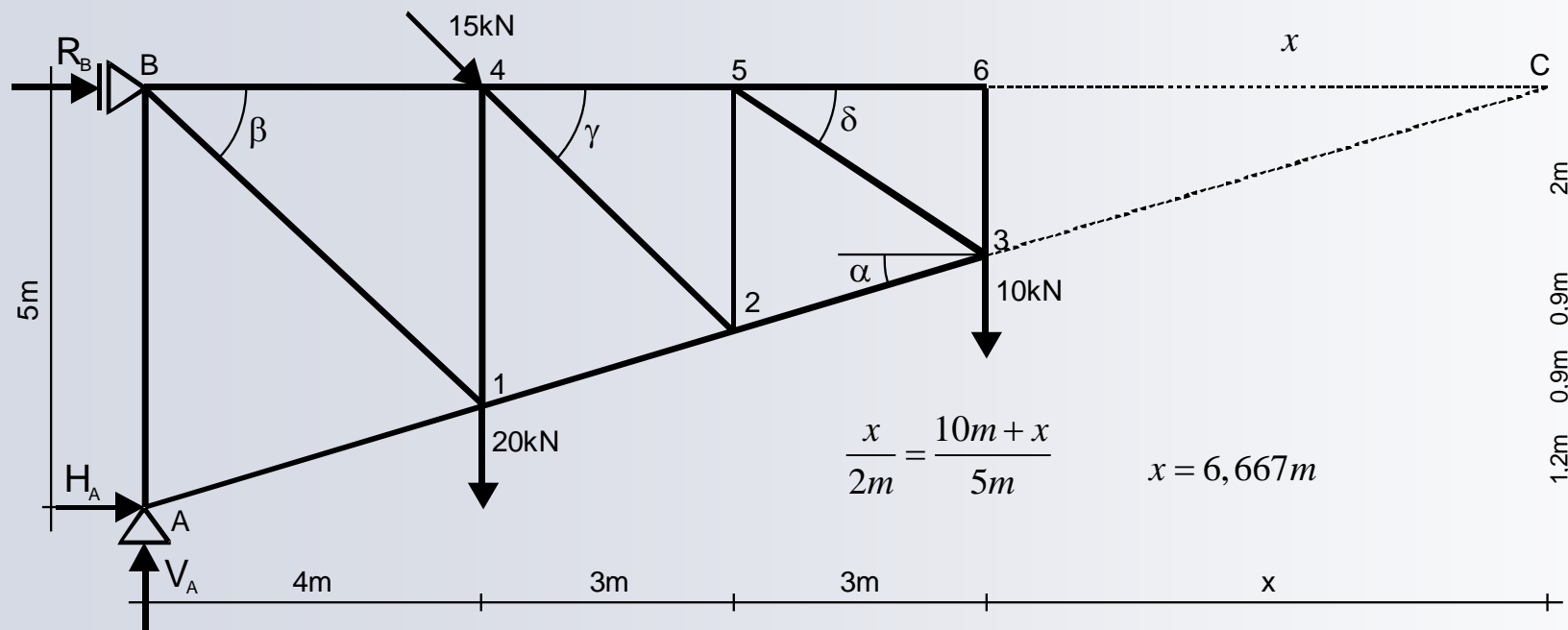
Przykład B – wyniki: zestawienie sił



Przykład C – kratownica z pasami zbieżnymi



Przykład C – wymiary



$$\sin \delta = \frac{2m}{\sqrt{(2m)^2 + (3m)^2}} = 0,555$$

$$\cos \delta = \frac{3m}{\sqrt{(2m)^2 + (3m)^2}} = 0,832$$

$$\frac{x}{2m} = \frac{10m + x}{5m} \quad x = 6,667m$$

$$\sin \beta = \frac{3,8m}{\sqrt{(3,8m)^2 + (4m)^2}} = 0,689$$

$$\cos \beta = \frac{4m}{\sqrt{(3,8m)^2 + (4m)^2}} = 0,725$$

$$\sin \gamma = \frac{2,9m}{\sqrt{(2,9m)^2 + (3m)^2}} = 0,695$$

$$\cos \gamma = \frac{3m}{\sqrt{(2,9m)^2 + (3m)^2}} = 0,719$$

$$\sin \alpha = \frac{3m}{\sqrt{(10m)^2 + (3m)^2}} = 0,287$$

$$\cos \alpha = \frac{10m}{\sqrt{(10m)^2 + (3m)^2}} = 0,958$$

Przykład C - reakcje

$$\sum X : H_A + R_B + 15kN \cdot \cos \gamma = 0$$

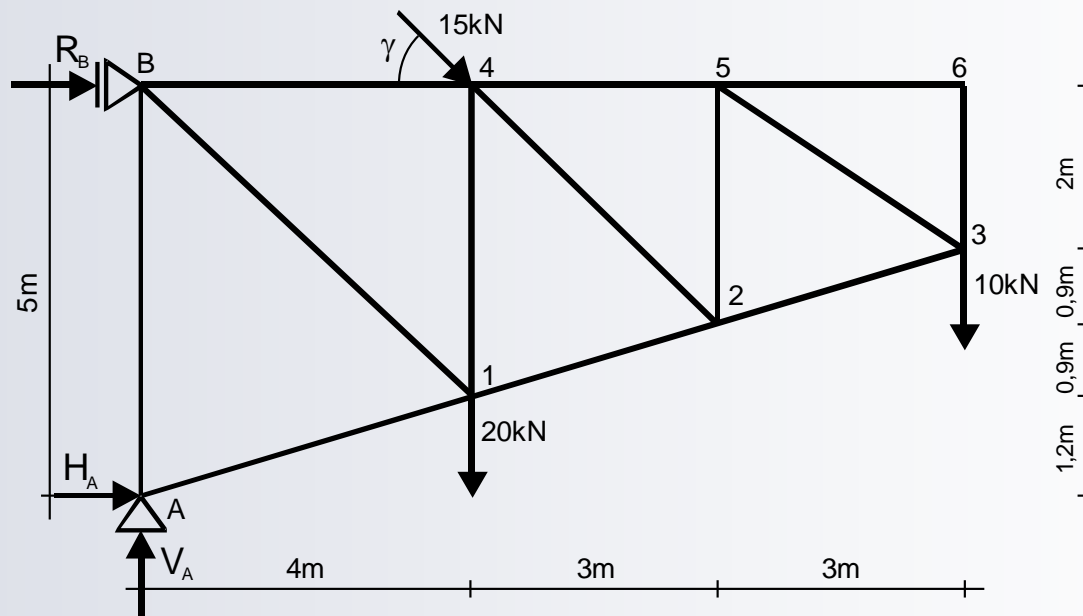
$$H_A = 44,340kN$$

$$\sum Y : V_A - 20kN - 10kN - 15kN \cdot \sin \gamma = 0$$

$$V_A = 40,425kN$$

$$\sum M_A : R_B \cdot 5m + 20kN \cdot 4m + 10kN \cdot 10m + \\ + 15kN \cdot \sin \gamma \cdot 4m + 15kN \cdot \cos \gamma \cdot 5m = 0$$

$$R_B = -55,125kN$$



Przykład C – metoda Rittera – przekrój 1

$$\sum M_1^P : N_{4-B} \cdot 3,8m - 15kN \cdot \cos \gamma \cdot 3,8m - 10kN \cdot 6m = 0$$

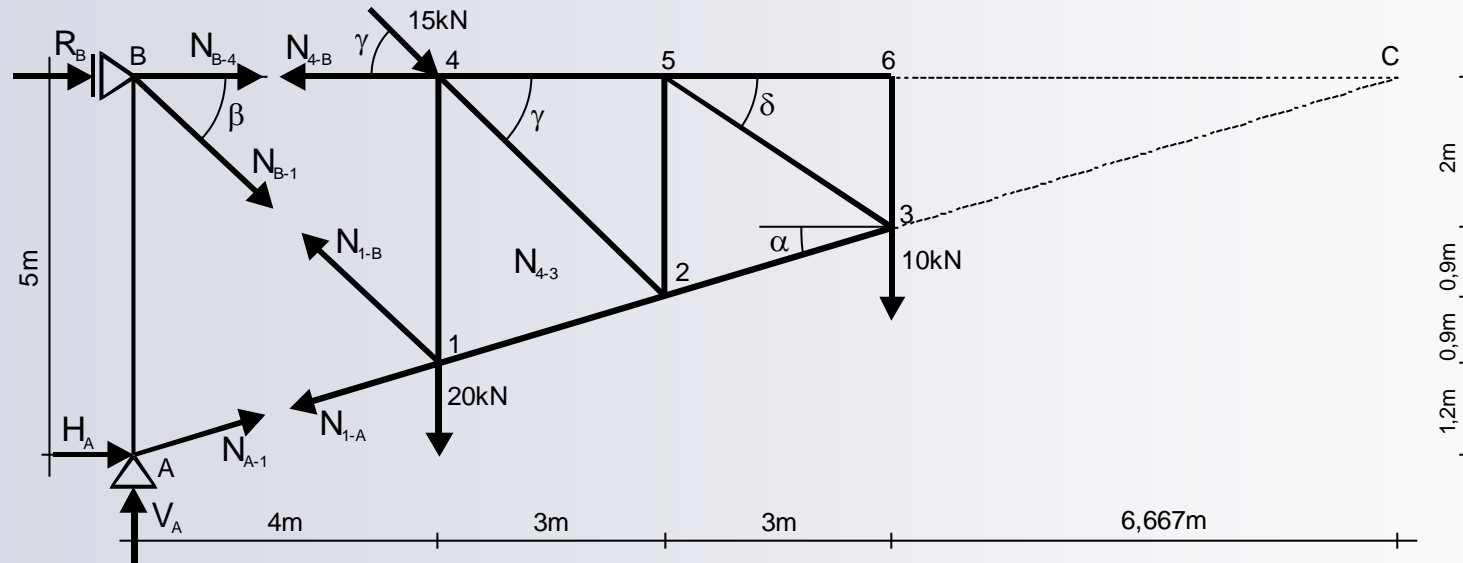
$$N_{4-B} = 26,574kN$$

$$\sum M_B^l : N_{A-1} \cdot \cos \alpha \cdot 5m + H_A \cdot 5m = 0$$

$$N_{A-1} = -46,284kN$$

$$\sum M_C^l : V_A \cdot 16,667m - H_A \cdot 5m - N_{B-1} \cdot \sin \beta \cdot 16,667m = 0$$

$$N_{4-B} = 39,366kN$$



Przykład C – metoda Rittera – przekrój 2

$$\sum M_2^P : N_{5-4} \cdot 2,9m - 10kN \cdot 3m = 0$$

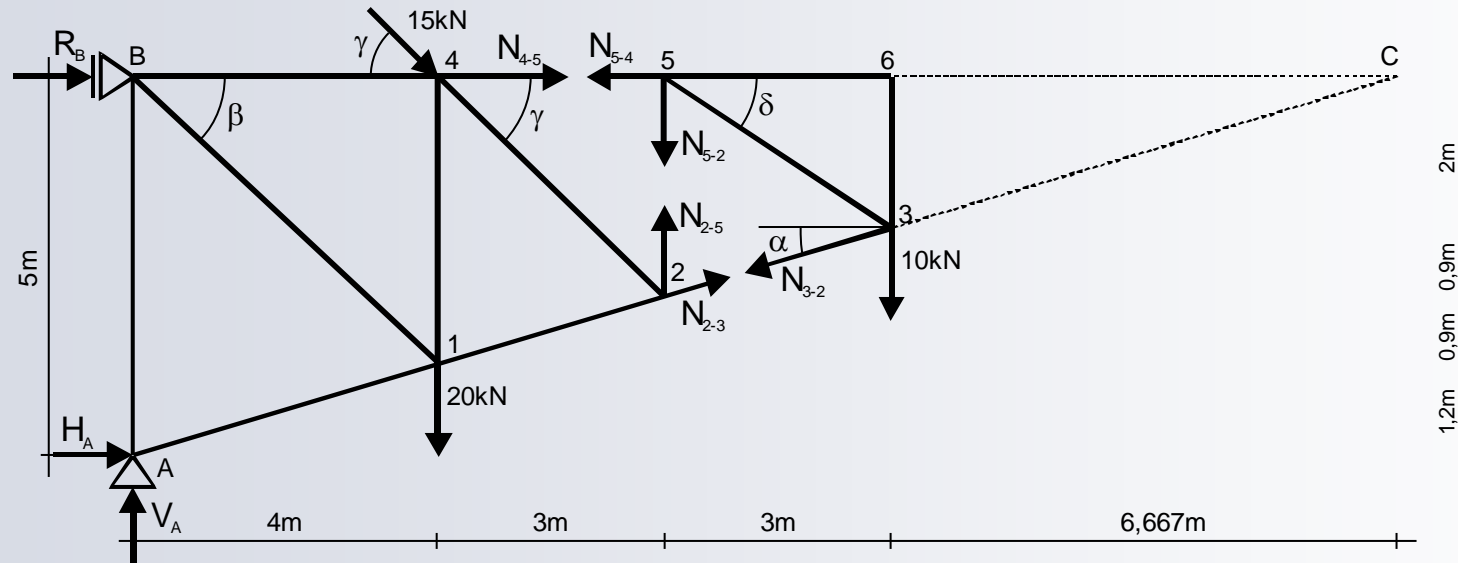
$$N_{5-4} = 10,345kN$$

$$\sum M_5^P : N_{3-2} \cdot \cos \alpha \cdot 2m + N_{3-2} \cdot \sin \alpha \cdot 3m + 10kN \cdot 3m = 0$$

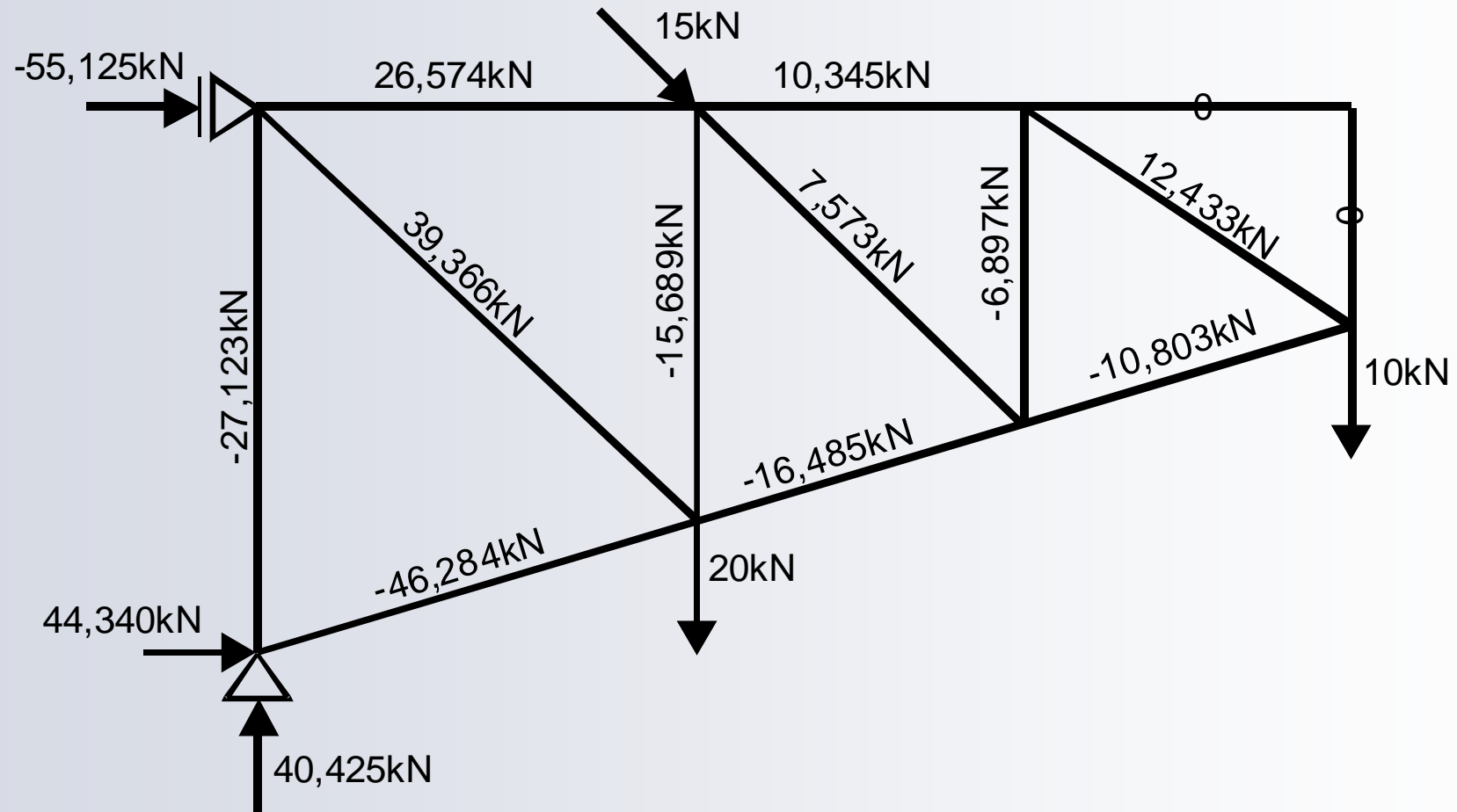
$$N_{3-2} = -10,803kN$$

$$\sum M_C^P : N_{5-2} \cdot 9,667m + 10kN \cdot 6,667m = 0$$

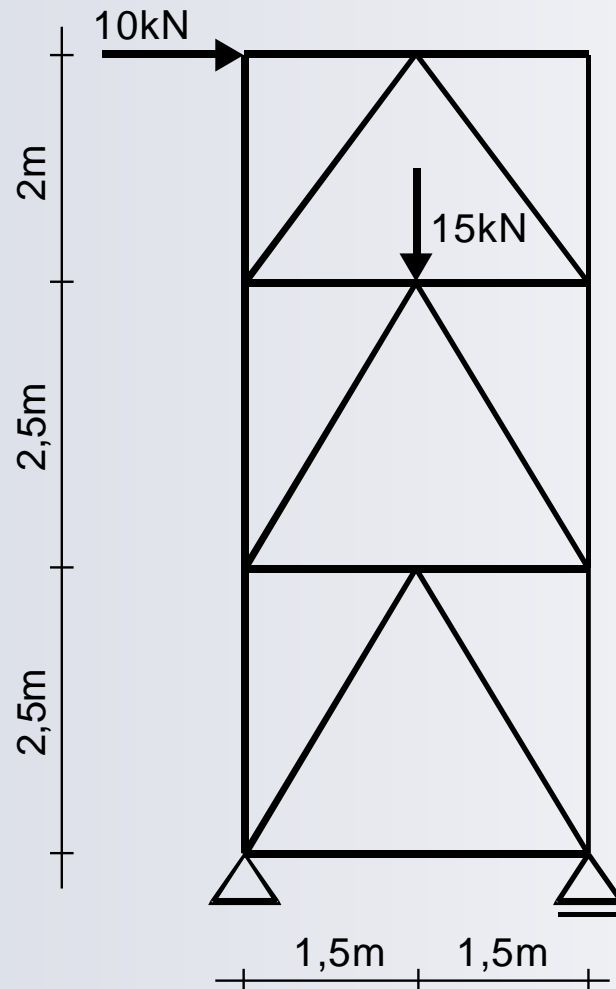
$$N_{5-2} = -6,897kN$$



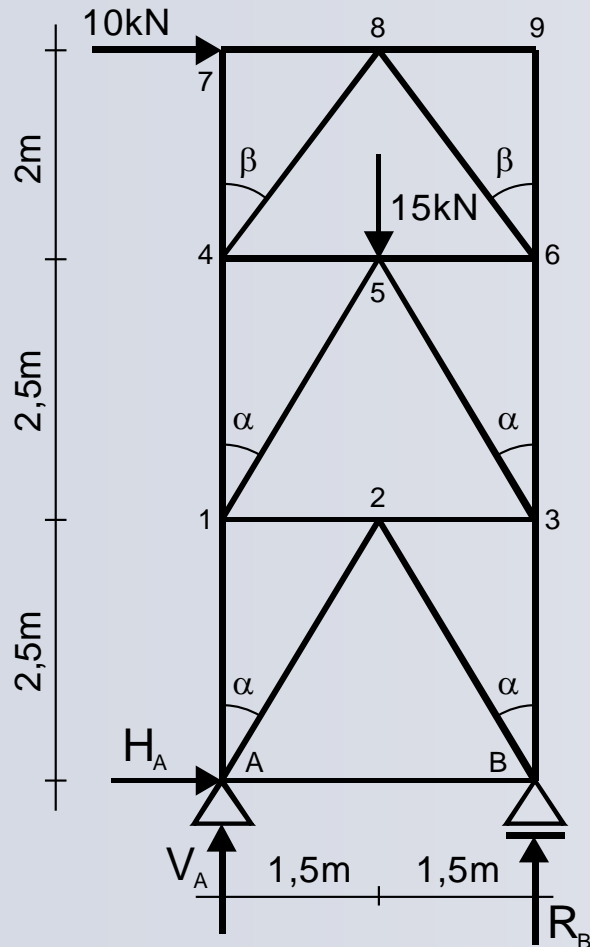
Przykład C – wyniki: zestawienie sił



Przykład D – kratownica typu „K”



Przykład D – reakcje



$$\sum X : H_A + 10kN = 0$$

$$H_A = -20kN$$

$$\sum Y : V_A + R_B - 15kN = 0$$

$$V_A = -15,833kN$$

$$\sum M_A : R_B \cdot 3m - 15kN \cdot 1,5m - 10kN \cdot 7m = 0$$

$$R_B = 30,833kN$$

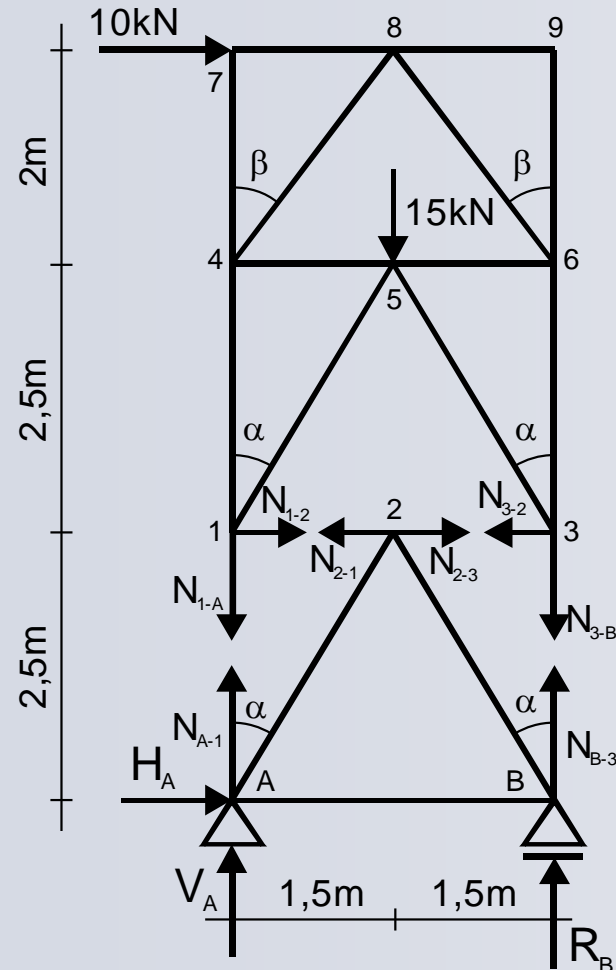
$$\sin \alpha = \frac{1,5m}{\sqrt{(1,5m)^2 + (2,5m)^2}} = 0,514$$

$$\cos \alpha = \frac{2,5m}{\sqrt{(1,5m)^2 + (2,5m)^2}} = 0,857$$

$$\sin \beta = \frac{1,5m}{\sqrt{(1,5m)^2 + (2m)^2}} = 0,6$$

$$\cos \beta = \frac{2m}{\sqrt{(1,5m)^2 + (2m)^2}} = 0,8$$

Przykład D – metoda Rittera – przekrój 1



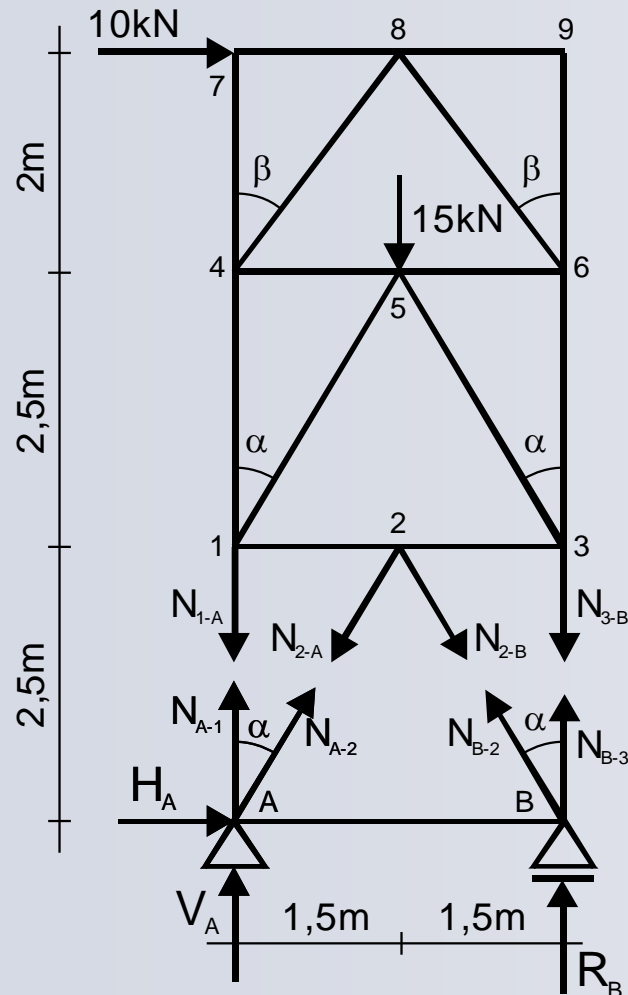
$$\sum M_1^g : N_{3-B} \cdot 3m + 15kN \cdot 1,5m + 10kN \cdot 4,5m = 0$$

$$N_{3-B} = -22,500kN$$

$$\sum M_3^g : N_{1-A} \cdot 3m + 15kN \cdot 1,5m - 10kN \cdot 4,5m = 0$$

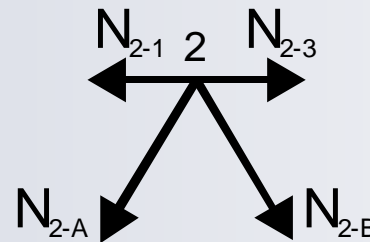
$$N_{1-A} = 7,500kN$$

Przykład D – metoda Rittera – przekrój 2



$$\sum X^g : N_{2-A} \cdot \sin \alpha - N_{2-B} \cdot \sin \alpha - 10kN = 0$$

$$N_{2-A} - N_{2-B} = \frac{10kN}{\sin \alpha}$$



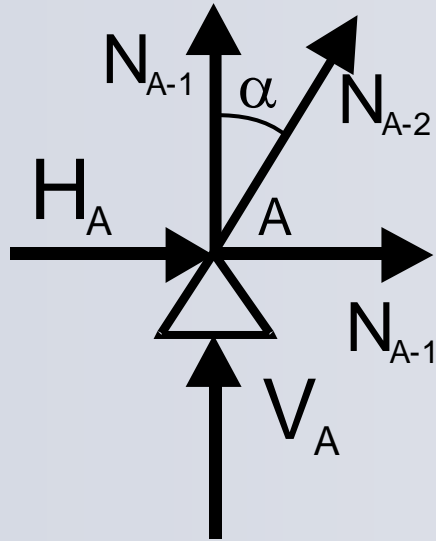
$$\sum Y : N_{2-A} \cdot \cos \alpha + N_{2-B} \cdot \cos \alpha = 0$$

$$N_{2-B} = -N_{2-A} \quad 2N_{2-A} = \frac{10kN}{\sin \alpha}$$

$$N_{2-A} = \frac{10kN}{2 \sin \alpha} = \frac{10kN}{2 \cdot 0,514} = 9,728kN$$

$$N_{2-B} = -9,728kN$$

Przykład D – metoda równoważenia węzłów



$$\sum X : N_{A-2} \cdot \sin \alpha + N_{A-1} + H_A = 0$$

$$N_{A-1} = 10kN - 9,728kN \cdot 0,514 = 5kN$$

Przykład D – wyniki: zestawienie sił

