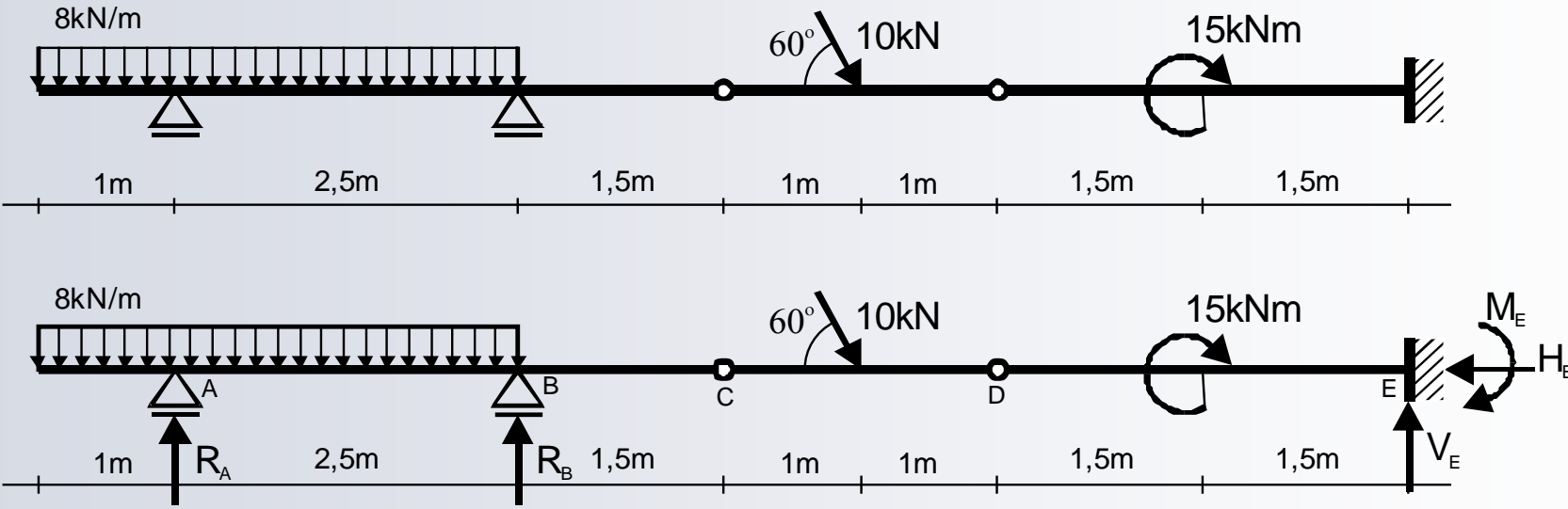


# **Mechanika teoretyczna**

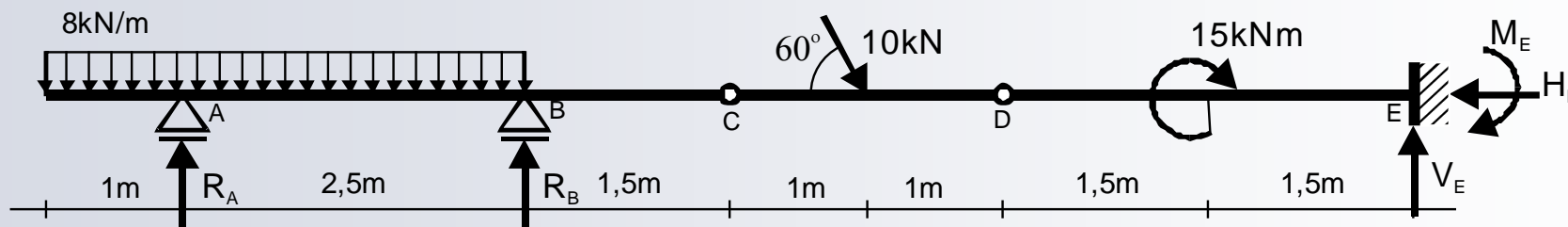
Wykład nr 5

**Siły wewnętrzne – przykłady**

# Przykład – belka przegubowa



# Reakcije



$$\sum X : H_E - 10 \cos 60^\circ = 0$$

$$\sum Y : R_A + R_B + V_E - 8 \text{ kN/m} \cdot 3,5 \text{ m} - 10 \text{ kN} \sin 60^\circ = 0$$

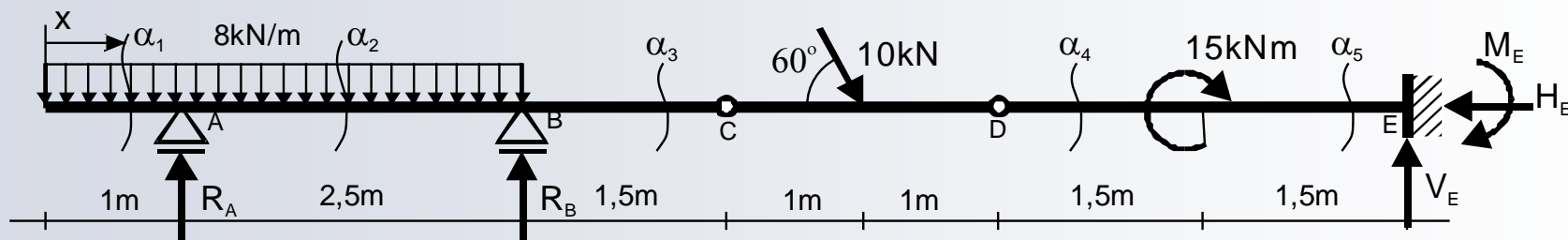
$$\sum M_E : R_A \cdot 9 \text{ m} + R_B \cdot 6,5 \text{ m} + M_E +$$

$$+ 15 \text{ kNm} - 10 \text{ kN} \sin 60^\circ \cdot 4 \text{ m} - 8 \cdot 3,5 \text{ m} \cdot \left( \frac{1}{2} 3,5 \text{ m} + 6,5 \text{ m} \right) = 0$$

$$\sum M_C^l : R_A \cdot 4 \text{ m} + R_B \cdot 1,5 \text{ m} - 8 \text{ kN/m} \cdot 3,5 \text{ m} \cdot \left( \frac{1}{2} 3,5 \text{ m} + 1,5 \text{ m} \right) = 0$$

$$\sum M_D^l : R_A \cdot 6 \text{ m} + R_B \cdot 3,5 \text{ m} - 8 \text{ kN/m} \cdot 3,5 \text{ m} \cdot \left( \frac{1}{2} 3,5 \text{ m} + 3,5 \text{ m} \right) - 10 \text{ kN} \sin 60^\circ \cdot 1 \text{ m} = 0$$

# Przyjęcie przekrojów, przedziały



$$R_A = 17,002kN$$

$$R_B = 15,328kN$$

$$H_E = 5kN$$

$$V_E = 4,330kN$$

$$M_E = -2,010kN$$

$$\alpha_1 - \alpha_1 \quad x \in \langle 0; 1m \rangle$$

$$\alpha_2 - \alpha_2 \quad x \in \langle 1m; 3,5m \rangle$$

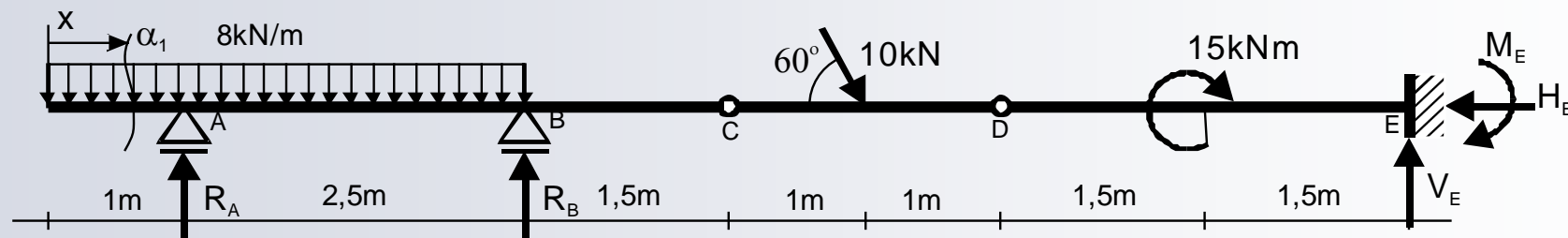
$$\alpha_3 - \alpha_3 \quad x \in \langle 3,5m; 6m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 6m; 8,5m \rangle$$

$$\alpha_5 - \alpha_5 \quad x \in \langle 8,5m; 10m \rangle$$

# Przekrój

$$\alpha_1 - \alpha_1 \quad x \in \langle 0; 1m \rangle$$



$$N_{\alpha_1} = 0$$

$$T_{\alpha_1} = -8\text{ kN} / \text{m} \cdot x$$

$$M_{\alpha_1} = -8\text{ kN} / \text{m} \cdot x \cdot \frac{x}{2} =$$
$$= -4\text{ kN} / \text{m} \cdot x^2$$

$$x = 0 \quad T_{\alpha_1} = 0$$

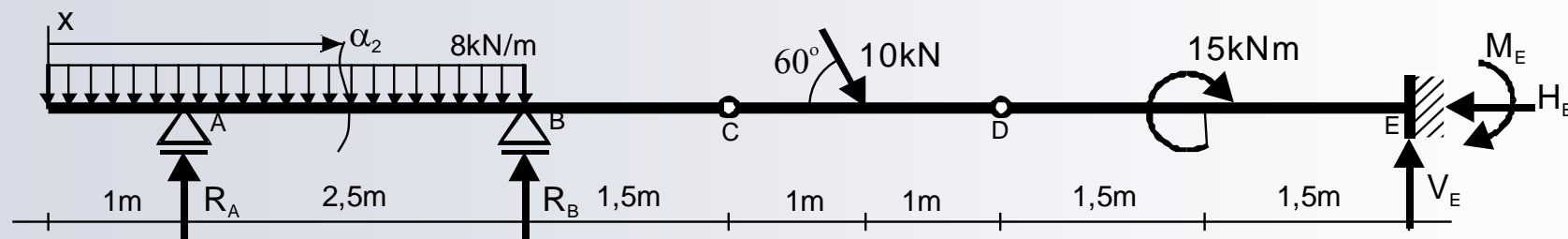
$$x = 1\text{ m} \quad T_{\alpha_1} = -8\text{ kN}$$

$$x = 0 \quad M_{\alpha_1} = 0$$

$$x = 1\text{ m} \quad M_{\alpha_1} = -4\text{ kNm}$$

# Przekrój

$$\alpha_2 - \alpha_2 \quad x \in \langle 1m; 3,5m \rangle$$



$$N_{\alpha_2} = 0$$

$$T_{\alpha_2} = -8kN/m \cdot x + 17,002kN$$

$$\begin{aligned} M_{\alpha_2} &= -8kN/m \cdot x \cdot \frac{x}{2} + 17,002kN \cdot (x - 1m) = \\ &= -4kN/m \cdot x^2 + 17,002kN \cdot x - 17,002kNm \end{aligned}$$

$$x = 1m \quad T_{\alpha_2} = 9,002kN$$

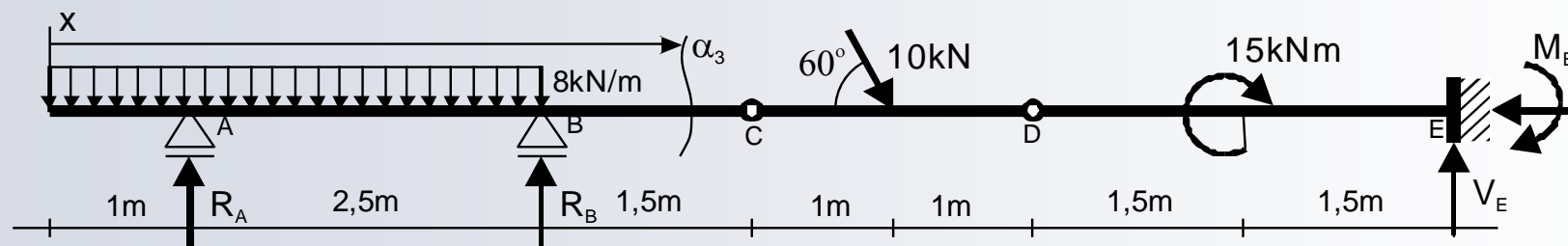
$$x = 3,5m \quad T_{\alpha_2} = -10,998kN$$

$$x = 1m \quad M_{\alpha_2} = -4kNm$$

$$x = 3,5m \quad M_{\alpha_2} = -6,495kNm$$

# Przekrój

$$\alpha_3 - \alpha_3 \quad x \in \langle 3,5m; 6m \rangle$$



$$N_{\alpha_3} = 0$$

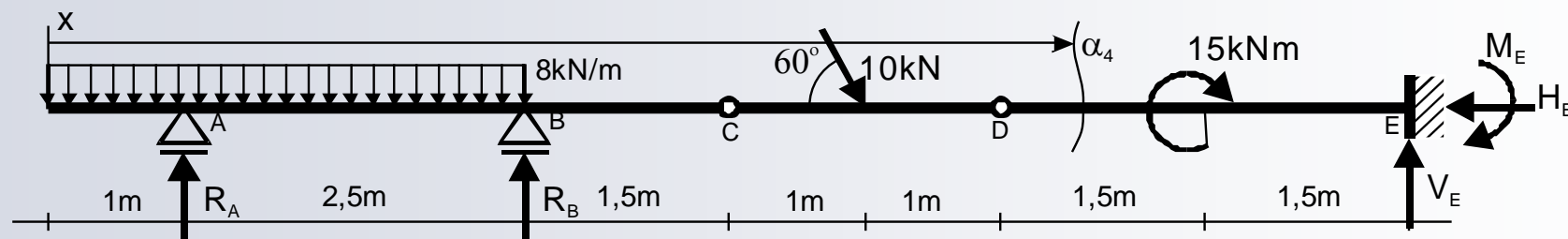
$$\begin{aligned} T_{\alpha_3} &= -8kN/m \cdot 3,5m + 17,002kN + 15,328kN \\ &= 4,33kN \end{aligned}$$

$$\begin{aligned} M_{\alpha_3} &= -8kN/m \cdot 3,5m \cdot \left(x - \frac{3,5m}{2}\right) + \\ &+ 17,002kN \cdot (x - 1m) + 15,328kN \cdot (x - 3,5m) = \\ &= 4,33kN \cdot x - 21,65kNm \end{aligned}$$

$$\left| \begin{array}{ll} x = 3,5m & M_{\alpha_3} = -6,495kNm \\ x = 5m & M_{\alpha_3} = 0 \\ x = 6m & M_{\alpha_3} = 4,330kNm \end{array} \right.$$

# Przekrój

$$\alpha_4 - \alpha_4 \quad x \in \langle 6m; 8,5m \rangle$$



$$N_{\alpha_4} = -10kN \cos 60^\circ = -5kN$$

$$T_{\alpha_4} = 4,33kN - 10kN \sin 60^\circ = -4,33kN$$

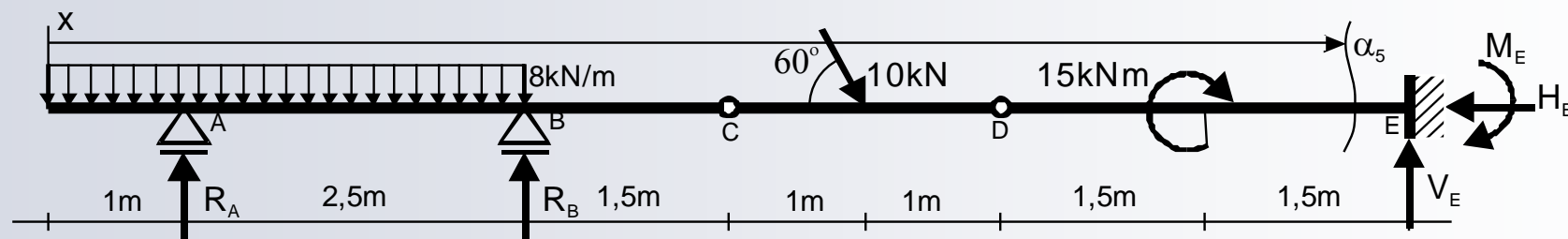
$$\begin{aligned} M_{\alpha_4} &= 4,33kN \cdot x - 21,65kNm - 10kN \sin 60^\circ \cdot (x - 6m) = \\ &= -4,33kN \cdot x + 30,31kNm \end{aligned}$$

$$\left| \begin{array}{ll} x = 6m & M_{\alpha_4} = 4,33m \\ x = 7m & M_{\alpha_4} = 0 \\ x = 8,5m & M_{\alpha_4} = -6,495kNm \end{array} \right.$$



# Przekrój

$$\alpha_5 - \alpha_5 \quad x \in \langle 8,5m; 10m \rangle$$



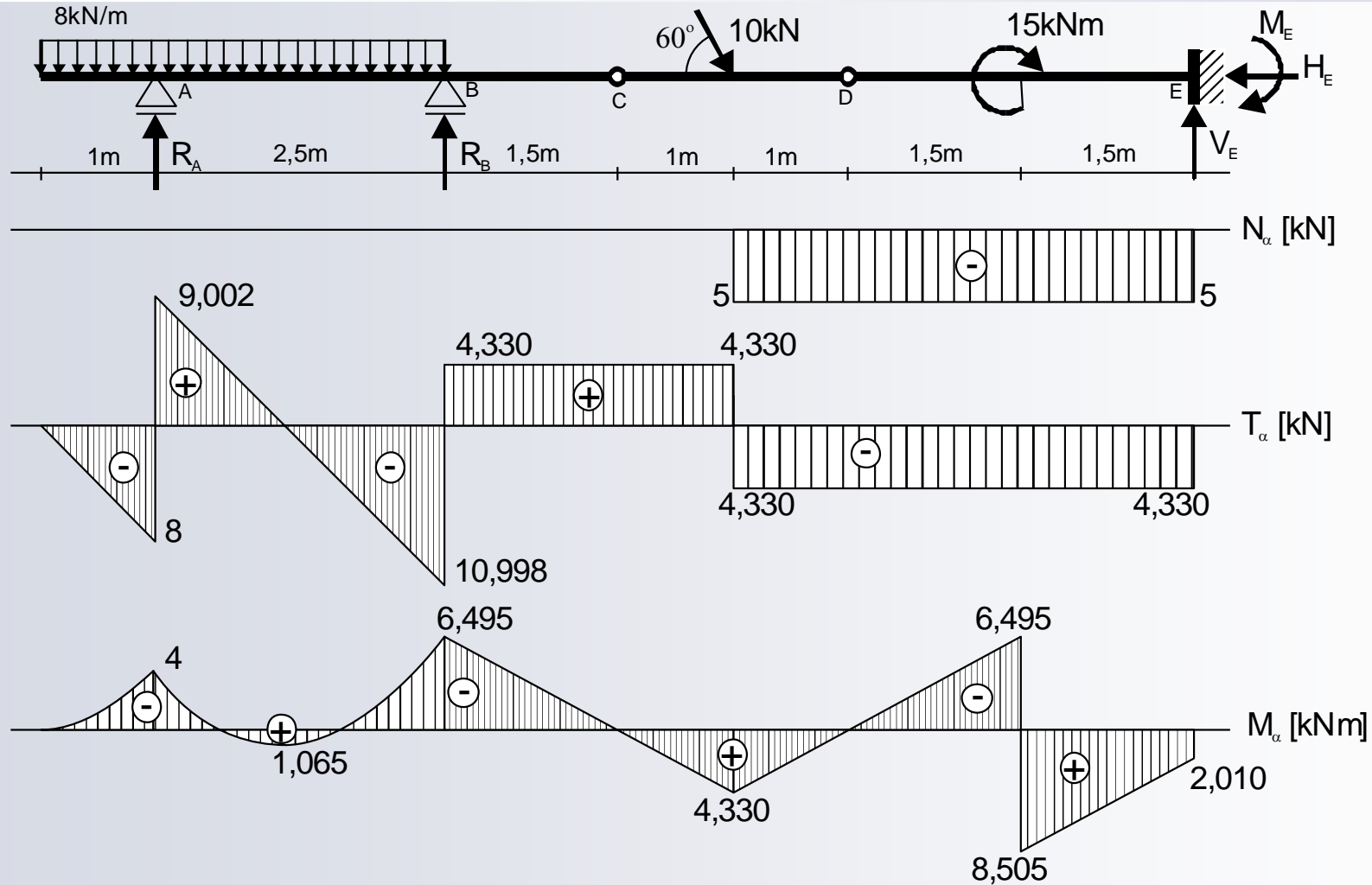
$$N_{\alpha_5} = -5kN$$

$$T_{\alpha_5} = -4,33kN$$

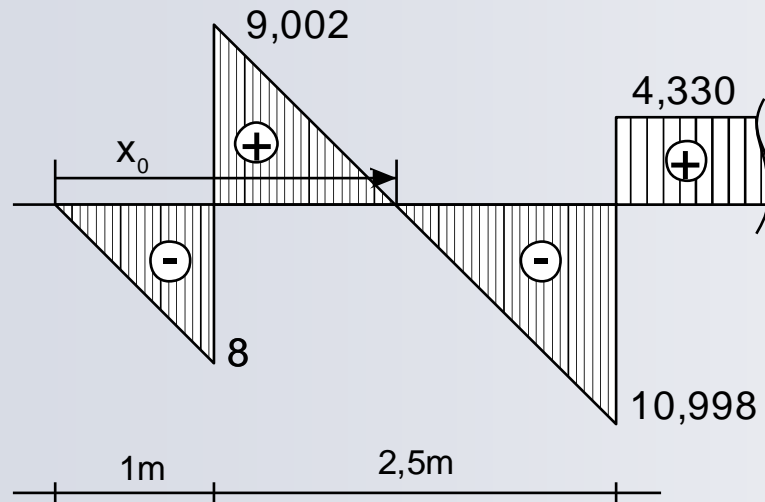
$$\begin{aligned} M_{\alpha_5} &= -4,33kN \cdot x + 30,31kNm + 15kNm = \\ &= -4,33kN \cdot x + 45,31kNm \end{aligned}$$

$$\left| \begin{array}{ll} x = 8,5m & M_{\alpha_5} = 8,505kNm \\ x = 10m & M_{\alpha_5} = 2,01kNm \end{array} \right.$$

# Wykresy



# Ekstremum



$$\frac{9,002 \text{ kNm} + 10,998 \text{ kNm}}{2,5 \text{ m}} = \frac{9,002 \text{ kNm}}{x_0 - 1 \text{ m}}$$

$T_\alpha$  [kN]

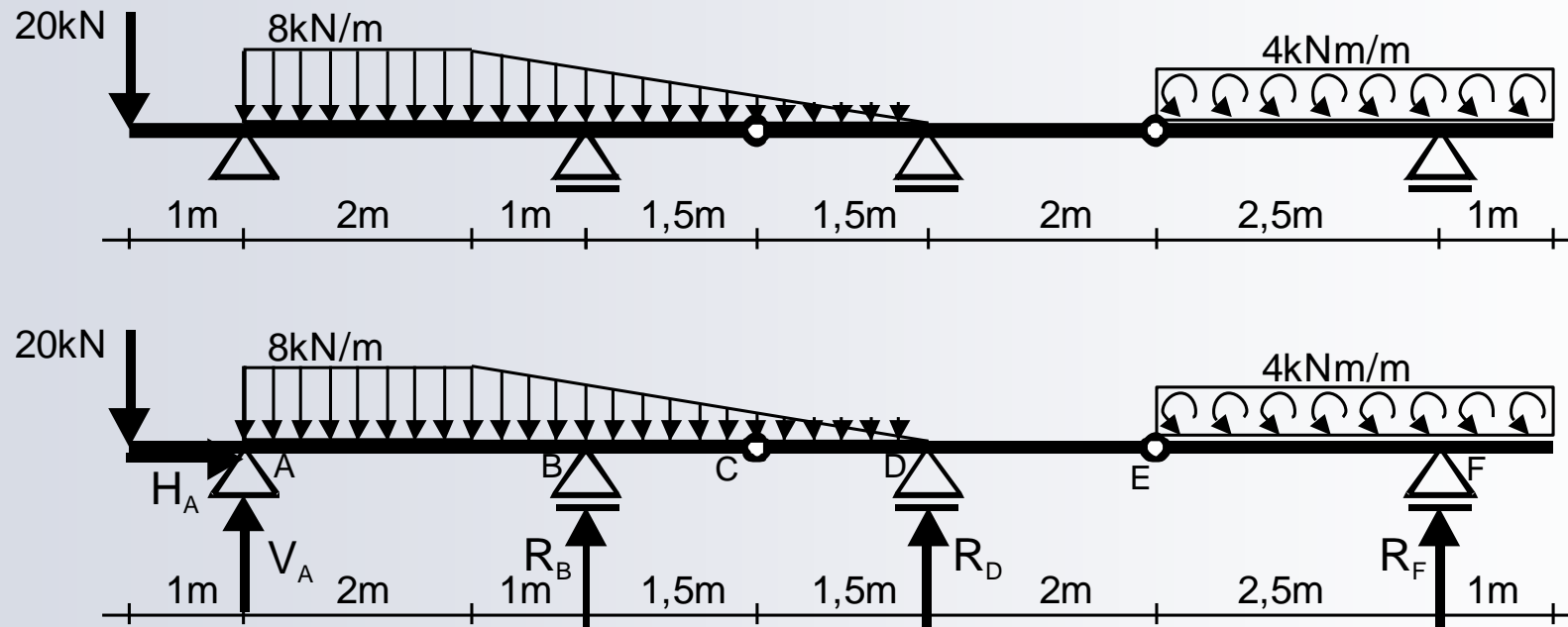
$$T_{\alpha 2} = -8 \text{ kN} / \text{m} \cdot x + 17,002 \text{ kN} = 0$$

$$x_0 = 2,125 \text{ m}$$

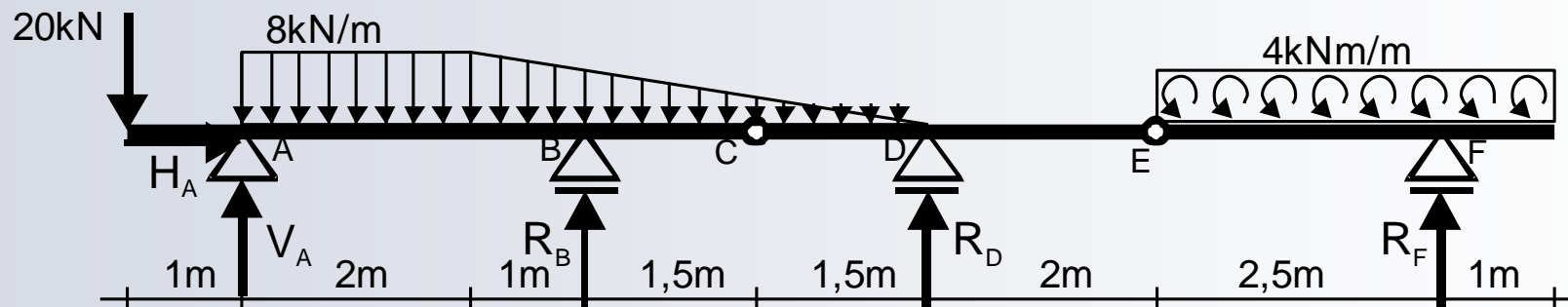
$$M_{\alpha 2} = -4 \text{ kN} / \text{m} \cdot x^2 + 17,002 \text{ kN} \cdot x - 17,002 \text{ kNm}$$

$$M_{\alpha 2}(x_0 = 2,125 \text{ m}) = 1,065 \text{ kNm}$$

# Przykład – belka przegubowa



# Reakcije



$$\sum X : H_A = 0$$

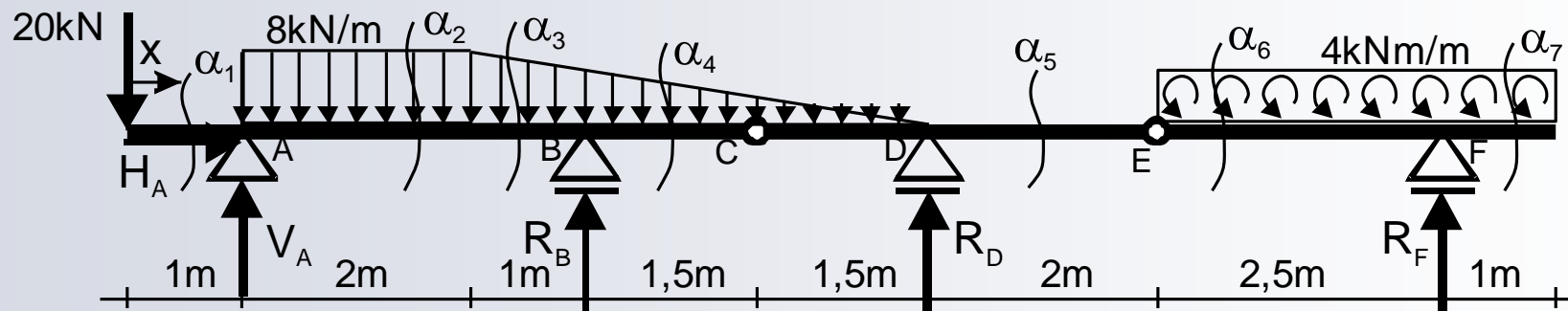
$$\sum Y : V_A + R_B + R_D + R_F - 20\text{kN} - 8\text{kN/m} \cdot 2\text{m} - \frac{1}{2} 8\text{kN/m} \cdot 4\text{m} = 0$$

$$\sum M_A : R_B \cdot 3\text{m} + R_D \cdot 6\text{m} + R_F \cdot 10,5\text{m} + 20\text{kN} \cdot 1\text{m} - 8\text{kN/m} \cdot 2\text{m} \cdot 1\text{m} + \\ - \frac{1}{2} 8\text{kN/m} \cdot 4\text{m} \cdot \left( 2\text{m} + \frac{1}{3} 4\text{m} \right) + 4\text{kNm/m} \cdot 3,5\text{m} = 0$$

$$\sum M_E^P : R_F \cdot 2,5\text{m} + 4\text{kNm/m} \cdot 3,5\text{m} = 0$$

$$\sum M_C^P : R_D \cdot 1,5\text{m} + R_F \cdot 6\text{m} - \frac{1}{2} 3\text{kN/m} \cdot 1,5\text{m} \cdot \frac{1}{3} 1,5\text{m} + 4\text{kNm/m} \cdot 3,5\text{m} = 0$$

# Przyjęcie przekrojów, przedziały



$$H_A = 0$$

$$R_F = -5,6kN$$

$$R_D = 13,817kN$$

$$R_B = 3,744kN$$

$$V_A = 40,039kN$$

$$\alpha_1 - \alpha_1 \quad x \in \langle 0; 1m \rangle$$

$$\alpha_2 - \alpha_2 \quad x \in \langle 1m; 3m \rangle$$

$$\alpha_3 - \alpha_3 \quad x \in \langle 3m; 4m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 4m; 7m \rangle$$

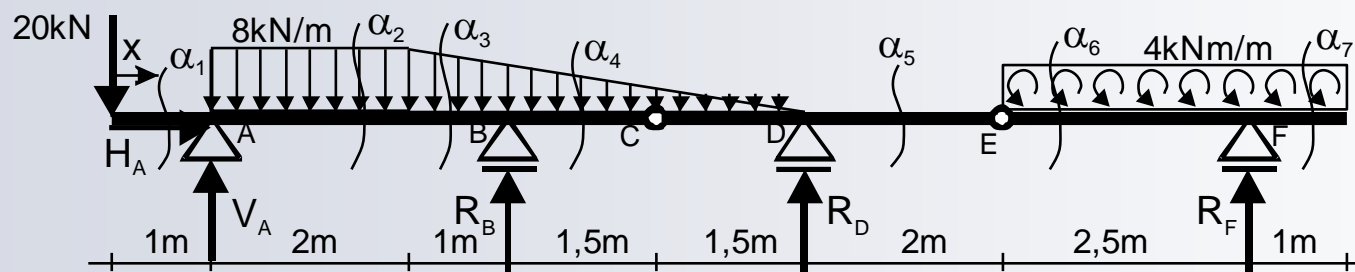
$$\alpha_5 - \alpha_5 \quad x \in \langle 7m; 9m \rangle$$

$$\alpha_6 - \alpha_6 \quad x \in \langle 9m; 11,5m \rangle$$

$$\alpha_7 - \alpha_7 \quad x \in \langle 11,5m; 12,5m \rangle$$

# Przekrój

$$\alpha_1 - \alpha_1 \quad x \in \langle 0; 1m \rangle$$



$$N_{\alpha_1} = 0$$

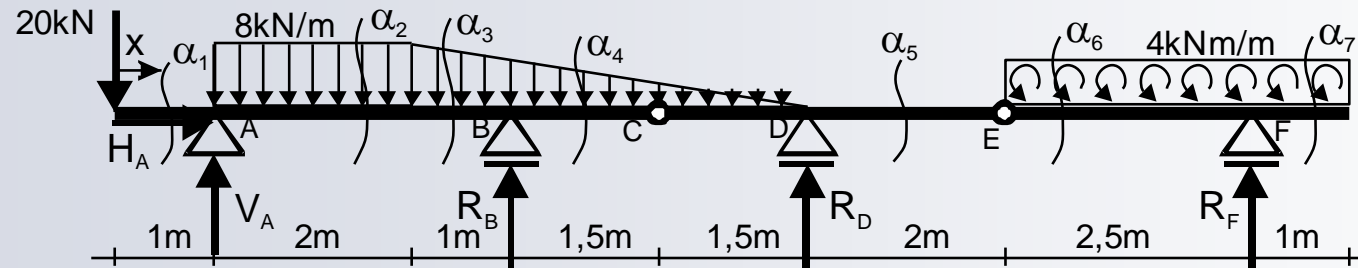
$$T_{\alpha_1} = -20kN$$

$$M_{\alpha_1} = -20kN \cdot x$$

$$\left| \begin{array}{l} x = 0 \quad M_{\alpha_1} = 0 \\ x = 1m \quad M_{\alpha_1} = -20kNm \end{array} \right.$$

# Przekrój

$$\alpha_2 - \alpha_2 \quad x \in \langle 1m; 3m \rangle$$



$$N_{\alpha_2} = H_A = 0$$

$$T_{\alpha_2} = -20kN + V_A - 8kN/m \cdot (x - 1m) = -20kN + 40,039kN - 8kN/m \cdot (x - 1m) =$$

$$= 28,039kN - 8kN/m \cdot x$$

$$\left| \begin{array}{l} x = 1m \quad T_{\alpha_2} = 20,039kN \\ x = 3m \quad T_{\alpha_2} = 4,039kN \end{array} \right.$$

$$M_{\alpha_2} = -20kN \cdot x + V_A \cdot (x - 1m) - 8kN/m \cdot (x - 1m) \cdot \frac{x - 1m}{2} =$$

$$= -20kN \cdot x + 40,039kN \cdot (x - 1m) - 8kN/m \cdot (x - 1m) \cdot \frac{x - 1m}{2} =$$

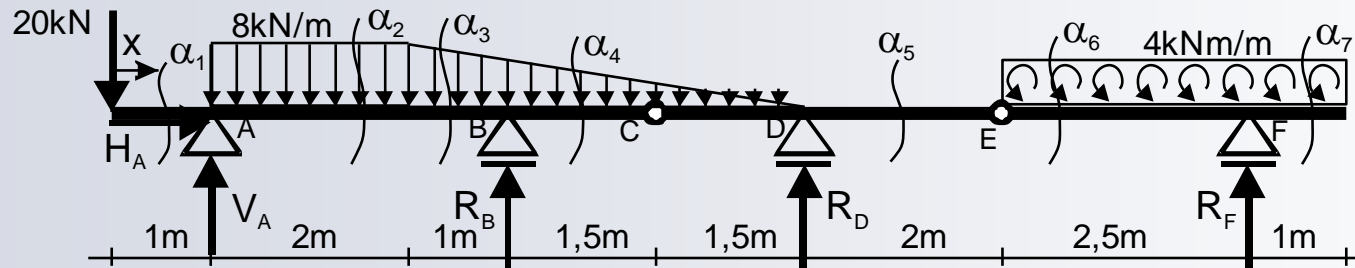
$$= -4kN/m \cdot x^2 + 28,039kN \cdot x - 44,039kNm$$

$$\left| \begin{array}{l} x = 1m \quad M_{\alpha_2} = -20kNm \\ x = 3m \quad M_{\alpha_2} = 4,078kNm \end{array} \right.$$



# Przekrój $\alpha_3 - \alpha_3 \quad x \in \langle 3m; 4m \rangle$

## – siły normalne i tnące



$$\frac{q_3(x)}{7m-x} = \frac{8kN/m}{4m}$$

$$q_3(x) = 2kN/m^2 \cdot (7m-x)$$

$$N_{\alpha_3} = H_A = 0$$

$$T_{\alpha_3} = -20kN + V_A - 8kN/m \cdot 2m - q_3(x) \cdot (x-3m) - \frac{1}{2} (8kN/m - q_3(x)) \cdot (x-3m) =$$

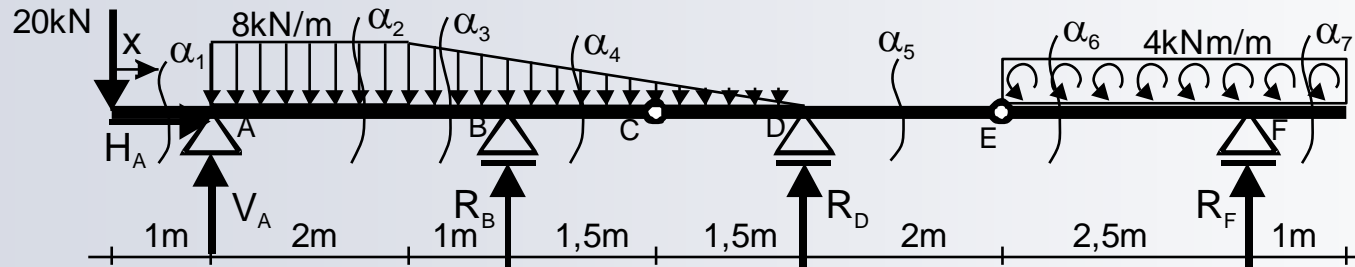
$$= -20kN + 40,039kN - 16kN - 2kN/m^2 \cdot (7m-x) \cdot (x-3m) +$$

$$- \frac{1}{2} (8kN/m - 2kN/m^2 \cdot (7m-x)) \cdot (x-3m) =$$

$$= 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 37,039kN$$

$$\left| \begin{array}{l} x = 3m \quad T_{\alpha_3} = 4,039kN \\ x = 4m \quad T_{\alpha_3} = -2,961kN \end{array} \right.$$

# Przekrój $\alpha_3 - \alpha_3 \quad x \in \langle 3m; 4m \rangle$ – momenty zginające



$$M_{\alpha_3} = -20kN \cdot x + V_A \cdot (x - 1m) - 8kN/m \cdot 2m \cdot (x - 2m) +$$

$$- q_3(x) \cdot (x - 3m) \frac{(x - 3m)}{2} - \frac{1}{2} (8kN/m - q_3(x)) \cdot (x - 3m) \cdot \frac{2}{3} (x - 3m) =$$

$$= -20kN \cdot x + 40,039kN \cdot (x - 1m) - 16kN \cdot (x - 2m) +$$

$$- 2kN/m^2 \cdot (7m - x) \cdot (x - 3m) \frac{(x - 3m)}{2} +$$

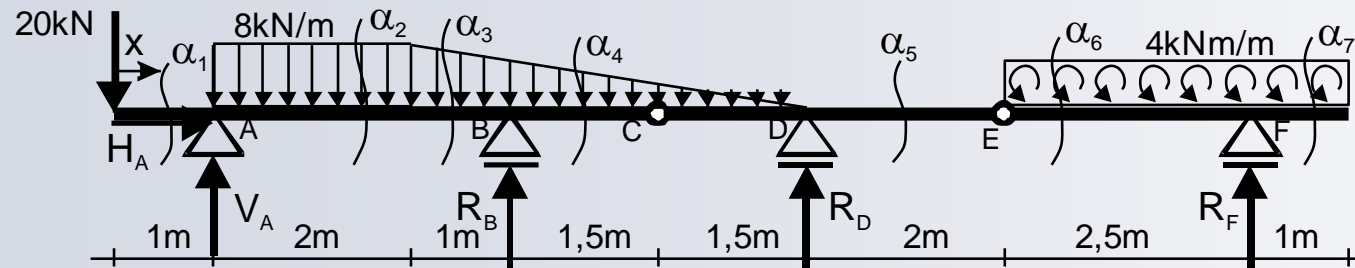
$$- \frac{1}{3} (8kN/m - 2kN/m^2 \cdot (7m - x)) \cdot (x - 3m) =$$

$$= \frac{1}{3} kN/m^2 \cdot x^3 - 7kN/m \cdot x^2 + 37,039kN \cdot x - 53,039kNm$$

$$\left| \begin{array}{l} x = 3m \quad M_{\alpha_3} = 4,078kNm \\ x = 4m \quad M_{\alpha_3} = 4,450kNm \end{array} \right.$$

# Przekrój

$$\alpha_4 - \alpha_4 \quad x \in \langle 4m; 7m \rangle$$



$$N_{\alpha_4} = H_A = 0$$

$$T_{\alpha_4} = -20kN + V_A - 8kN/m \cdot 2m - q_3(x) \cdot (x-3m) - \frac{1}{2}(8kN/m - q_3(x)) \cdot (x-3m) + R_B =$$

$$= 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 37,039kN + 3,744kN =$$

$$= 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 40,783kN$$

$$\left| \begin{array}{l} x = 4m \quad T_{\alpha_4} = 0,783kN \\ x = 7m \quad T_{\alpha_4} = -8,217kN \end{array} \right.$$

$$M_{\alpha_4} = -20kN \cdot x + V_A \cdot (x-1m) - 8kN/m \cdot 2m \cdot (x-2m) - q_3(x) \cdot (x-3m) \frac{(x-3m)}{2} +$$

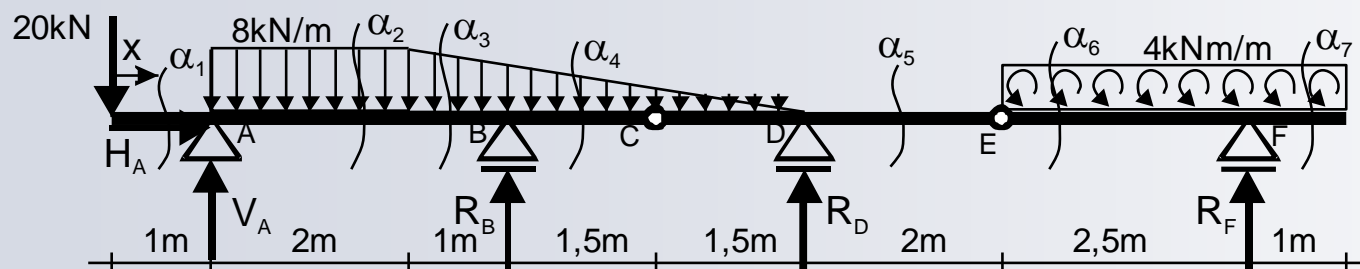
$$- \frac{1}{2}(8kN/m - q_3(x)) \cdot (x-3m) \cdot \frac{2}{3}(x-3m) + R_B \cdot (x-4m) =$$

$$= \frac{1}{3} kN/m^2 \cdot x^3 - 7kN/m \cdot x^2 + 40,783kN \cdot x - 68,015kNm$$

$$\left| \begin{array}{l} x = 4m \quad M_{\alpha_4} = 4,450kNm \\ x = 5,5m \quad M_{\alpha_4} = 0 \\ x = 7m \quad M_{\alpha_4} = -11,201kNm \end{array} \right.$$

# Przekrój

$$\alpha_5 - \alpha_5 \quad x \in \langle 7m; 9m \rangle$$



$$N_{\alpha_5} = H_A = 0$$

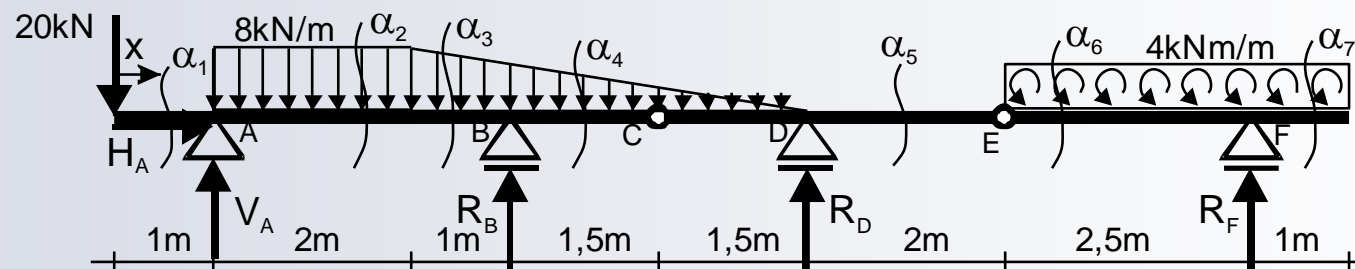
$$\begin{aligned} T_{\alpha_5} &= -20kN + V_A - 8kN/m \cdot 2m - \frac{1}{2} 8kN/m \cdot 4m + R_B + R_D = \\ &= -20kN + 40,039kN - 16kN - 16kN + 3,744kN + 13,817kN = \\ &= 5,6kN \end{aligned}$$

$$\begin{aligned} M_{\alpha_5} &= -20kN \cdot x + V_A \cdot (x - 1m) - 8kN/m \cdot 2m \cdot (x - 2m) + \\ &\quad - \frac{1}{2} 8kN/m \cdot 4m \cdot \left( x - \left( 3m + \frac{1}{3} 4m \right) \right) + R_B \cdot (x - 4m) + R_D \cdot (x - 7m) = \\ &= 5,6kN \cdot x - 50,401kNm \end{aligned}$$

$$\left| \begin{array}{l} x = 7m \quad M_{\alpha_5} = -11,201kNm \\ x = 9m \quad M_{\alpha_5} = -0,001kNm \approx 0 \end{array} \right.$$

# Przekrój

$$\alpha_6 - \alpha_6 \quad x \in \langle 9m; 11,5m \rangle$$



$$N_{\alpha_6} = H_A = 0$$

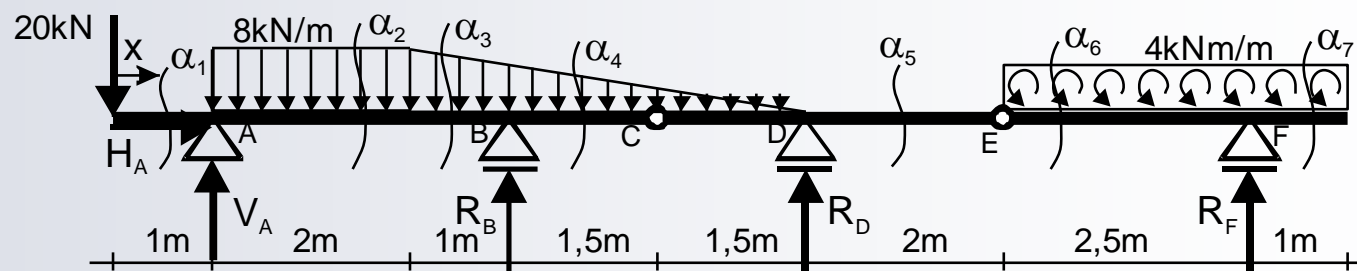
$$T_{\alpha_6} = 5,6kN$$

$$\begin{aligned} M_{\alpha_6} &= 5,6kN \cdot x - 50,401kNm - 4kNm/m \cdot (x - 9m) = \\ &= 1,6kN \cdot x - 14,401kNm \end{aligned}$$

$$\left| \begin{array}{l} x = 9m \quad M_{\alpha_6} = -0,001kNm \approx 0 \\ x = 11,5m \quad M_{\alpha_6} = 3,999kNm \end{array} \right.$$

# Przekrój

$$\alpha_7 - \alpha_7 \quad x \in \langle 11,5m; 12,5m \rangle$$



$$N_{\alpha_7} = H_A = 0$$

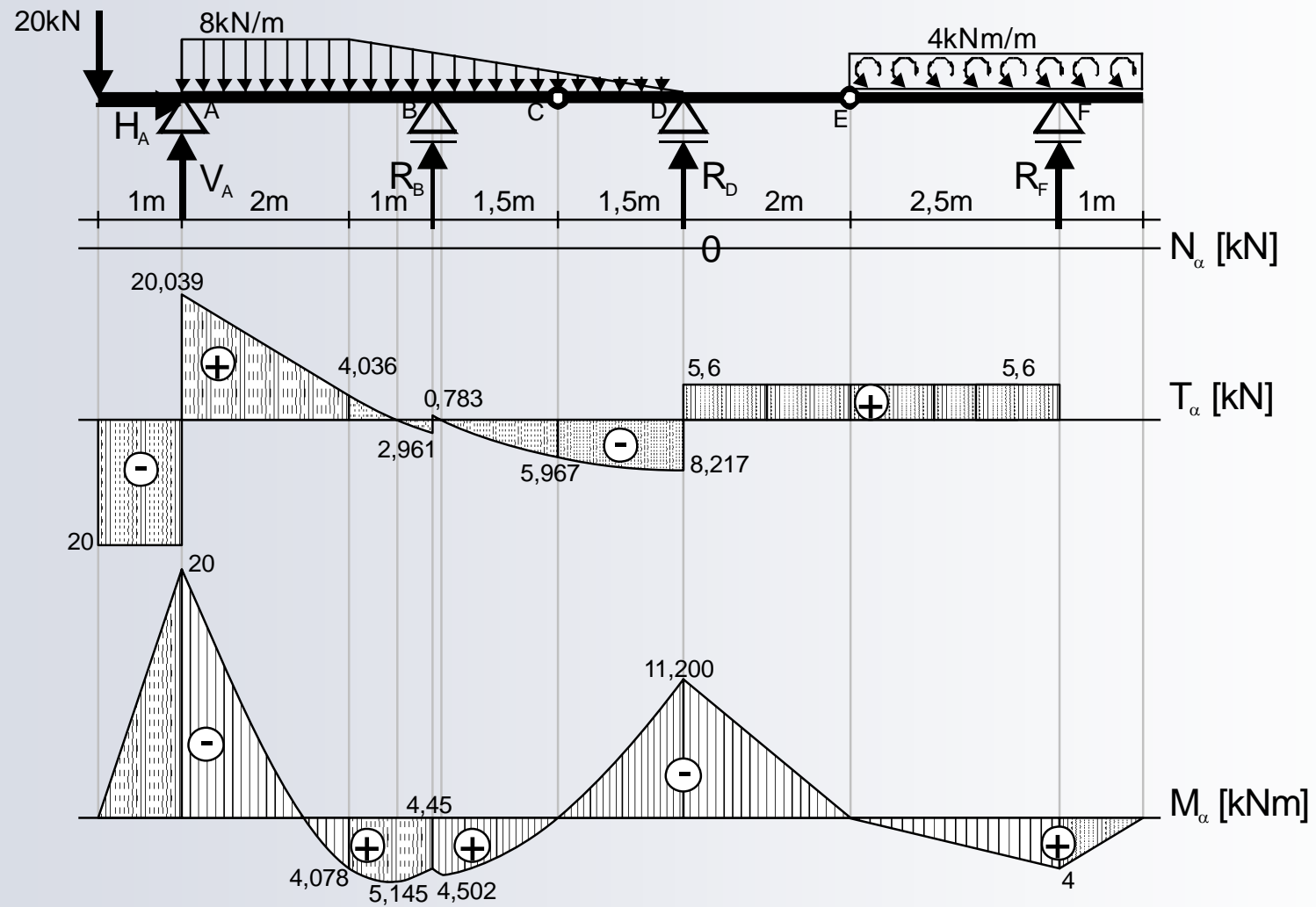
$$T_{\alpha_7} = 5,6kN + R_F = 0$$

$$M_{\alpha_7} = 1,6kN \cdot x - 14,401kNm - 5,6kN \cdot (x - 11,5m) =$$

$$= -4kN \cdot x + 49,999kNm$$

$$\left| \begin{array}{ll} x = 11,5m & M_{\alpha_7} = 3,999kNm \\ x = 12,5m & M_{\alpha_7} = -0,001kNm \approx 0 \end{array} \right.$$

# Wykresy



# Ekstrema

$$T_{\alpha 3} = 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 37,039 kN = 0 \quad x_0 = 3,542 m$$

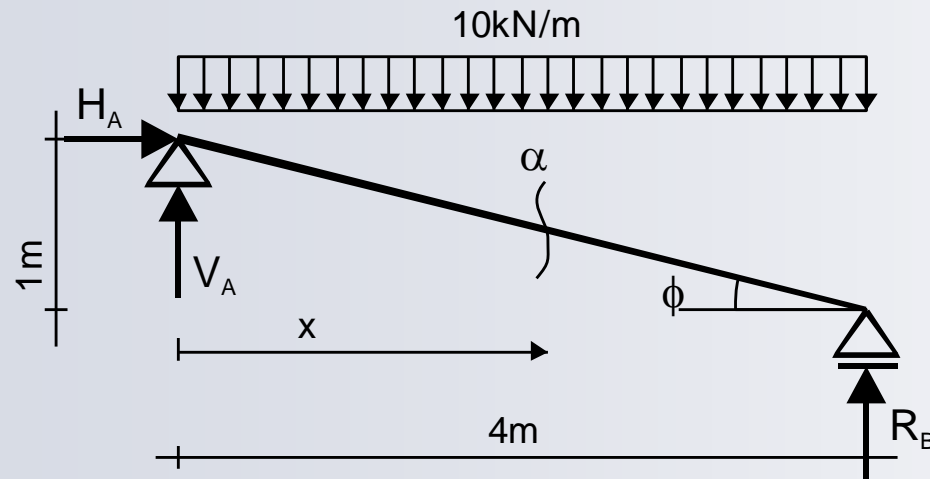
$$\begin{aligned} M_{\alpha 3}(x_0) &= \frac{1}{3} kN / m^2 \cdot x_0^3 - 7 kN / m \cdot x_0^2 + 37,039 kN \cdot x_0 - 53,039 kNm = \\ &= 5,145 kNm \end{aligned}$$

$$T_{\alpha 4} = 1 \frac{kN}{m^2} x^2 - 14 \frac{kN}{m} x + 40,783 kN = 0 \quad x_0 = 4,133 m$$

$$\begin{aligned} M_{\alpha 4}(x_0) &= \frac{1}{3} kN / m^2 \cdot x_0^3 - 7 kN / m \cdot x_0^2 + 40,783 kN \cdot x_0 - 68,015 kNm = \\ &= 4,502 kNm \end{aligned}$$



# Obciążenie na pręcie ukośnym – na jednostkę rzutu



$$\sin \phi = \frac{1m}{\sqrt{(4m)^2 + (1m)^2}} = 0,243$$

$$\cos \phi = \frac{4m}{\sqrt{(4m)^2 + (1m)^2}} = 0,970$$

$$\sum X : H_A = 0$$

$$\sum Y : V_A + R_B - 10 \frac{kN}{m} \cdot 4m = 0$$

$$\sum M_A : R_B \cdot 4m - 10 \frac{kN}{m} \cdot 4m \cdot \frac{1}{2} 4m = 0$$

$$H_A = 0$$

$$V_A = 20kN$$

$$R_B = 20kN$$

# Siły wewnętrzne

$$N_{\alpha} = V_A \cdot \sin \phi - q \cdot x \cdot \sin \phi =$$

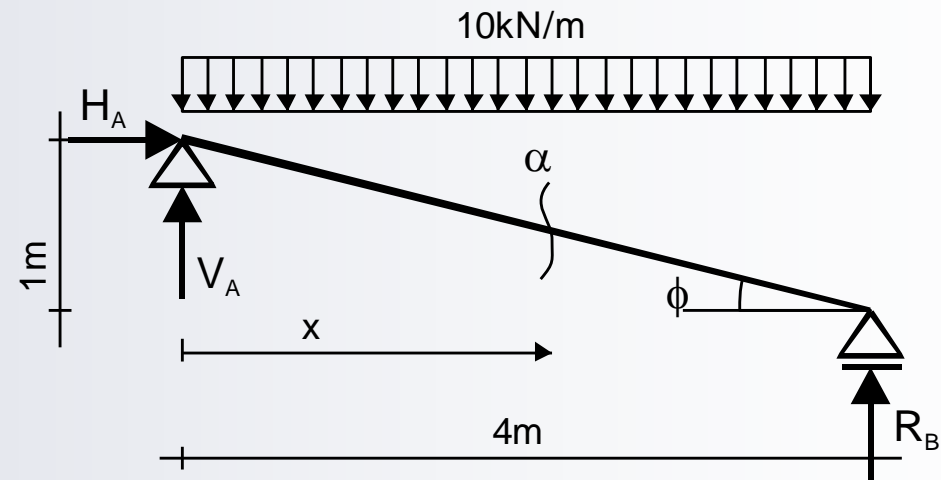
$$= 4,851kN - 2,425 \frac{kN}{m} \cdot x$$

$$T_{\alpha} = V_A \cdot \cos \phi - q \cdot x \cdot \cos \phi =$$

$$= 19,403kN - 9,701 \frac{kN}{m} \cdot x$$

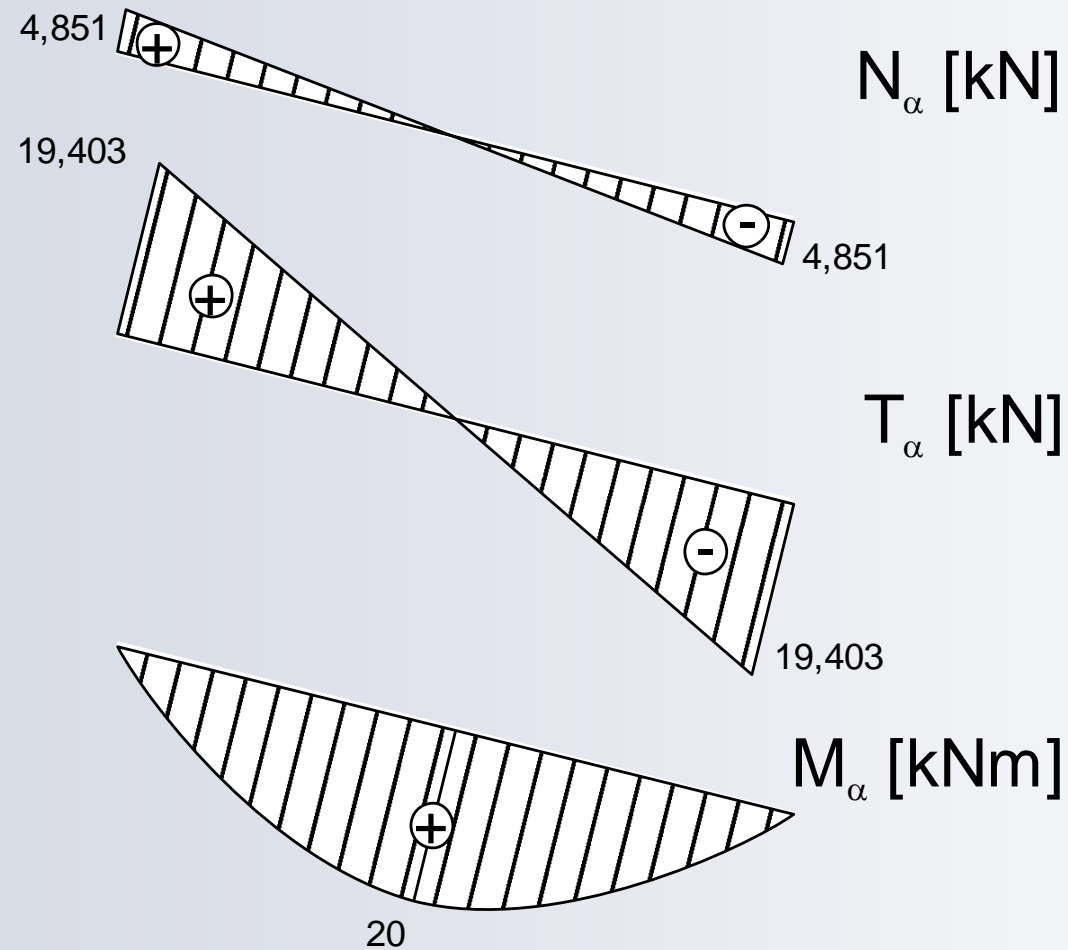
$$M_{\alpha} = V_A \cdot x - 10 \frac{kN}{m} \cdot x \cdot \frac{x}{2} =$$

$$= -5 \frac{kN}{m} \cdot x^2 + 20kN \cdot x$$

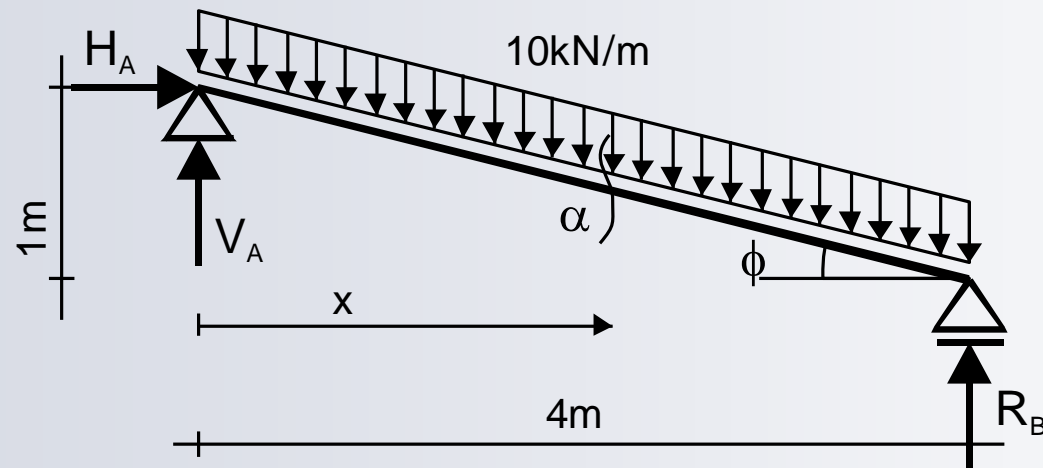


$x = 0$	$N_{\alpha} = 4,851kN$	$T_{\alpha} = 19,403kN$	$M_{\alpha} = 0$
$x = 2m$	$N_{\alpha} = 0$	$T_{\alpha} = 0$	$M_{\alpha} = 20kNm$
$x = 4m$	$N_{\alpha} = -4,849kN$	$T_{\alpha} = -19,401kN$	$M_{\alpha} = 0$

# Wykresy



# Obciążenie na pręcie ukośnym – na jednostkę długości pręta



$$\sum X : H_A = 0$$

$$\sum Y : V_A + R_B - 10 \frac{kN}{m} \cdot \sqrt{(4m)^2 + (1m)^2} = 0$$

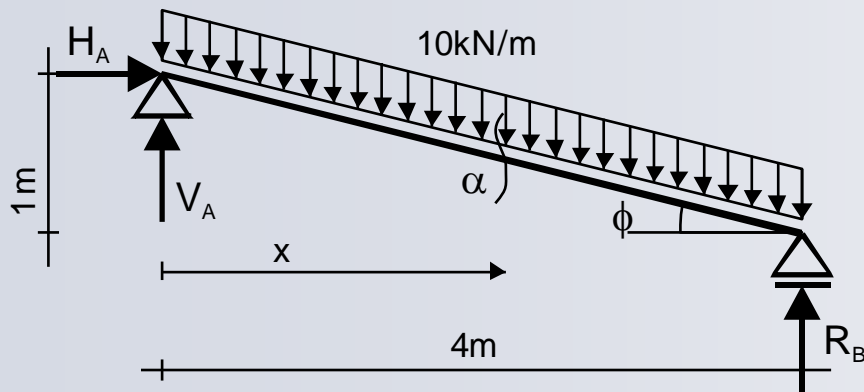
$$\sum M_A : R_B \cdot 4m - 10 \frac{kN}{m} \cdot \sqrt{(4m)^2 + (1m)^2} \cdot \frac{1}{2} 4m = 0$$

$$H_A = 0$$

$$V_A = 20,616kN$$

$$R_B = 20,616kN$$

# Siły wewnętrzne



$$M_{\alpha} = V_A \cdot x - 10 \frac{\text{kN}}{\text{m}} \cdot \sqrt{x^2 + \left(\frac{x}{4}\right)^2} \cdot \frac{x}{2} = -5,154 \frac{\text{kN}}{\text{m}} \cdot x^2 + 20,616 \text{kN} \cdot x$$

$$N_{\alpha} = V_A \cdot \sin \phi - q \cdot \sqrt{x^2 + \left(\frac{x}{4}\right)^2} \cdot \sin \phi =$$

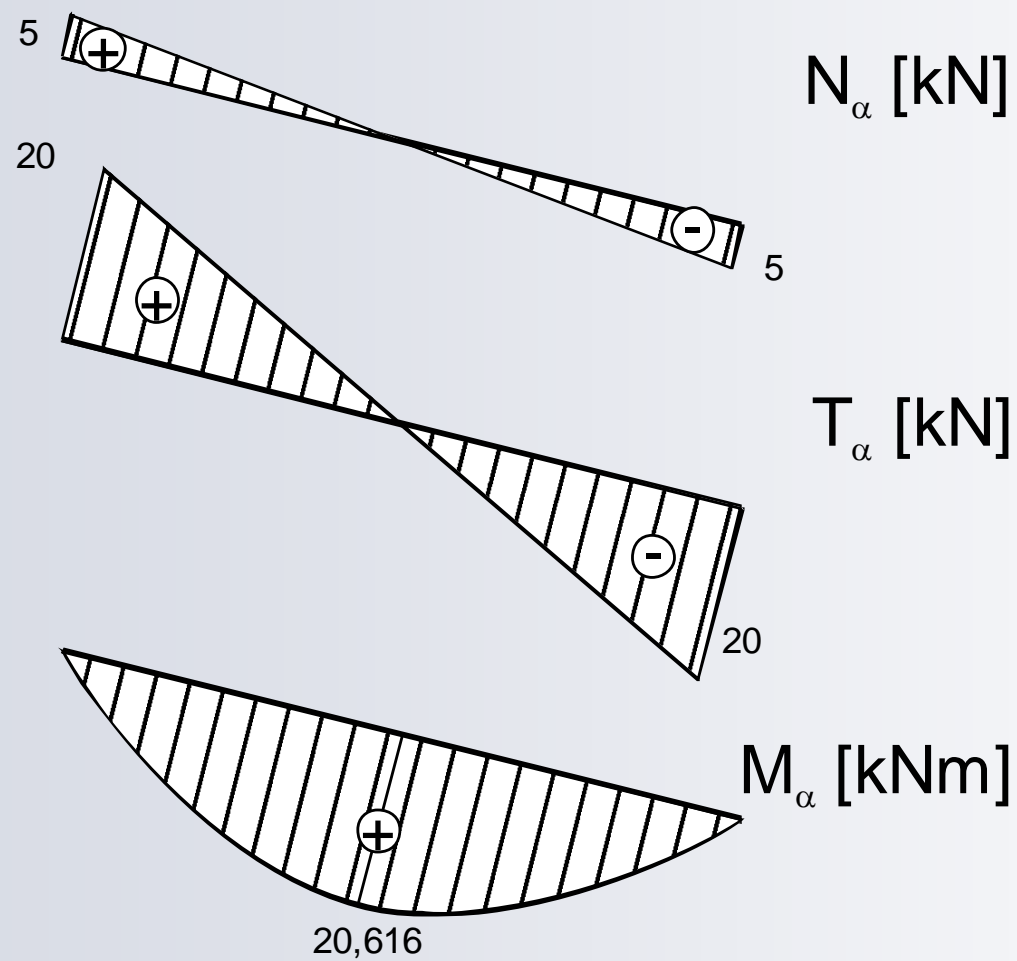
$$= 5 \text{kN} - 2,5 \frac{\text{kN}}{\text{m}} \cdot x$$

$$T_{\alpha} = V_A \cdot \cos \phi - q \cdot \sqrt{x^2 + \left(\frac{x}{4}\right)^2} \cdot \cos \phi =$$

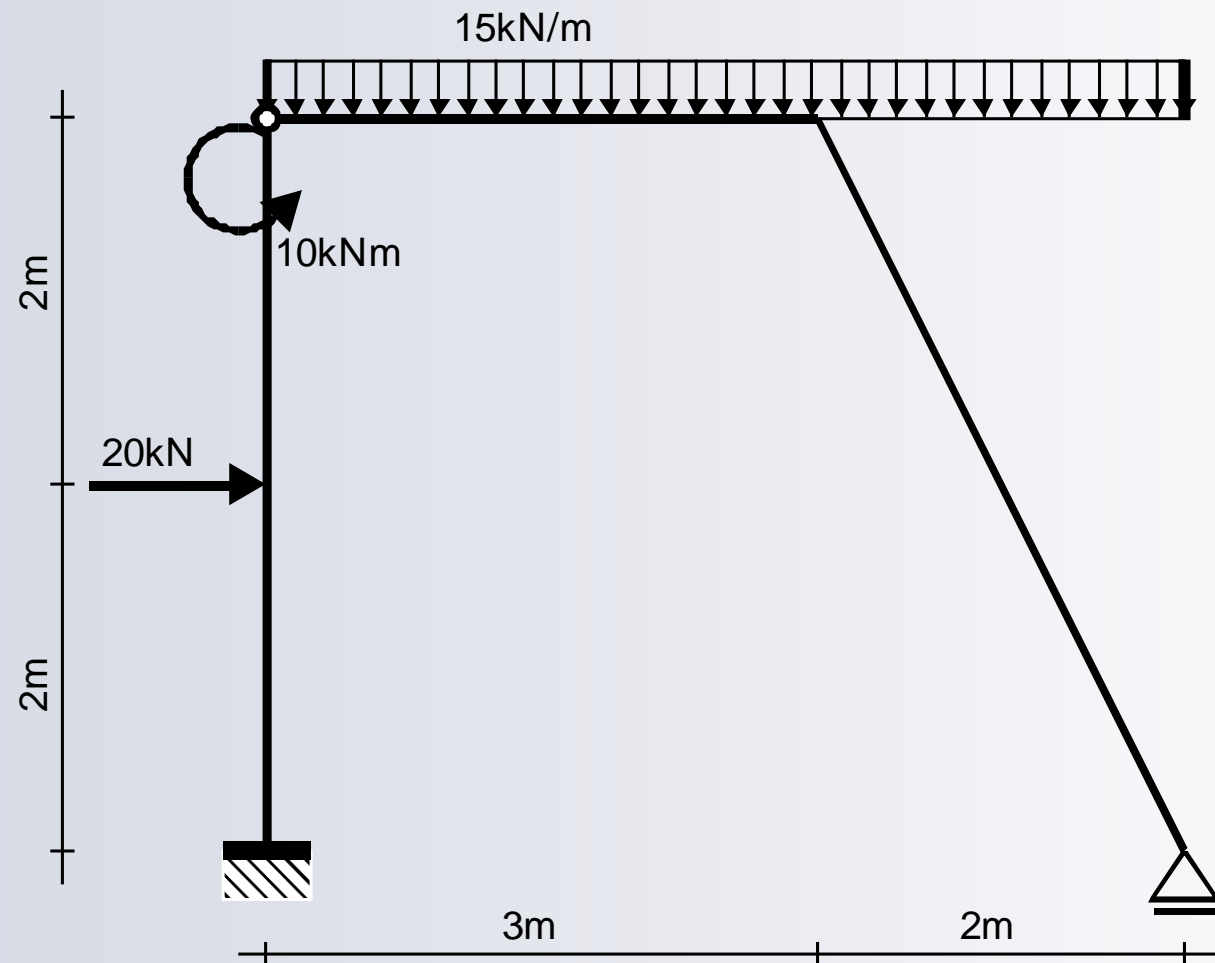
$$= 20 \text{kN} - 10 \frac{\text{kN}}{\text{m}} \cdot x$$

x = 0	N <sub>α</sub> = 5 kN	T <sub>α</sub> = 20 kN	M <sub>α</sub> = 0
x = 2 m	N <sub>α</sub> = 0	T <sub>α</sub> = 0	M <sub>α</sub> = 20,616 kNm
x = 4 m	N <sub>α</sub> = -5	T <sub>α</sub> = -20 kN	M <sub>α</sub> = 0

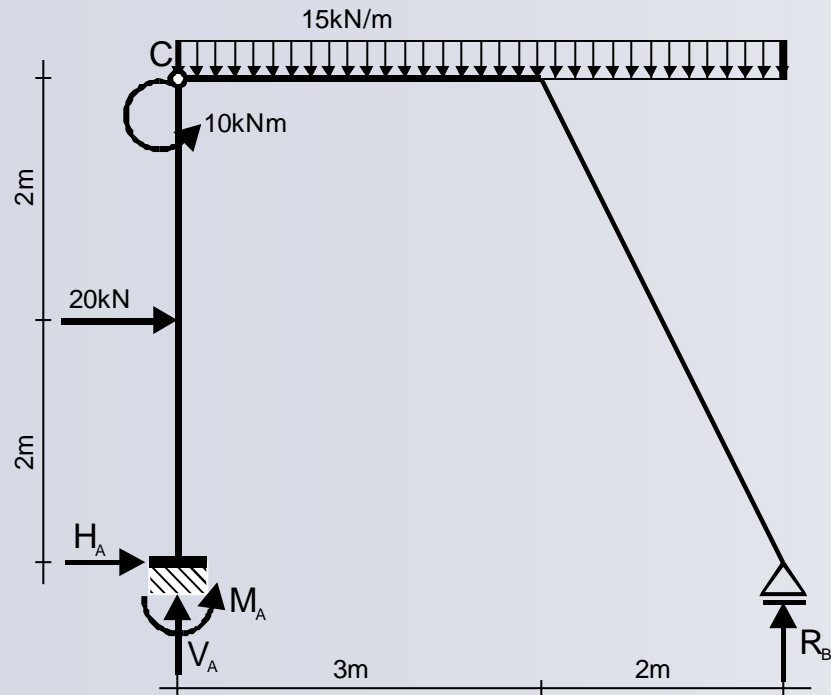
# Wykresy



# Przykład – rama



# Reakcije



$$\sum X : H_A + 20kN = 0$$

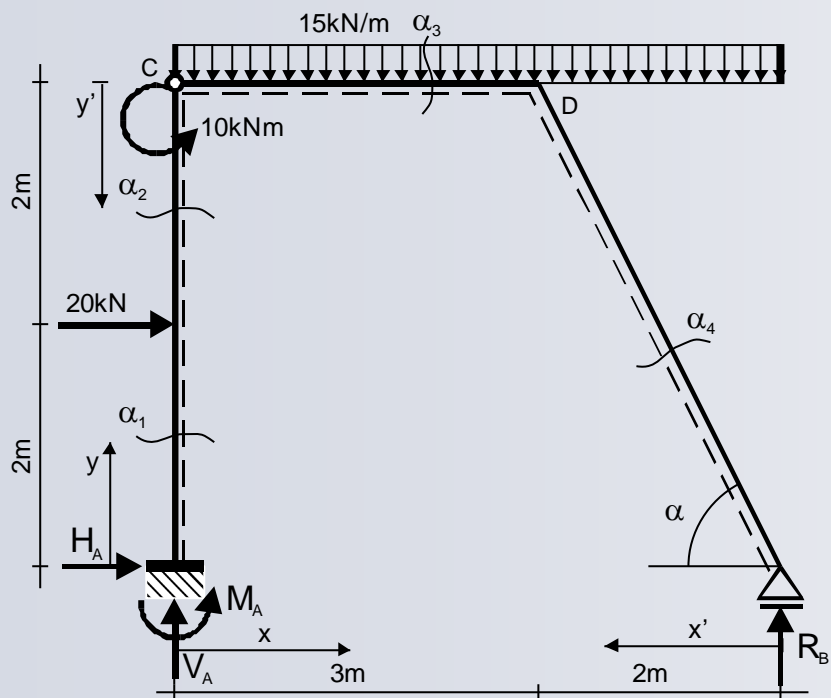
$$\sum Y : V_A + R_B - 15kN \cdot 5m = 0$$

$$\sum M_C^P : R_B \cdot 5m - 15kN / m \cdot 5m \cdot \frac{1}{2} 5m = 0$$

$$\sum M_A : M_A + R_B \cdot 5m + 10kNm - 20kN \cdot 2m - 15kN / m \cdot 5m \cdot \frac{1}{2} 5m = 0$$



# Przyjęcie przekrojów, przedziały



$$H_A = -20kN$$

$$R_B = 37,5kN$$

$$V_A = 37,5kN$$

$$M_A = 30kNm$$

$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$\alpha_2 - \alpha_2 \quad y \in \langle 2m; 4m \rangle$$

$$\alpha_3 - \alpha_3 \quad x \in \langle 0m; 3m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 3m; 5m \rangle \quad y \in \langle 0m; 4m \rangle$$

$$x' \in \langle 0m; 2m \rangle \quad y' \in \langle 0m; 4m \rangle$$

$$\text{punkt } D \quad x = 3m, \quad y = 4m, \quad x' = 2m, \quad y' = 0$$

$$\text{punkt } B \quad x = 5m, \quad y = 0, \quad x' = 0, \quad y' = 4m$$

# Przekrój

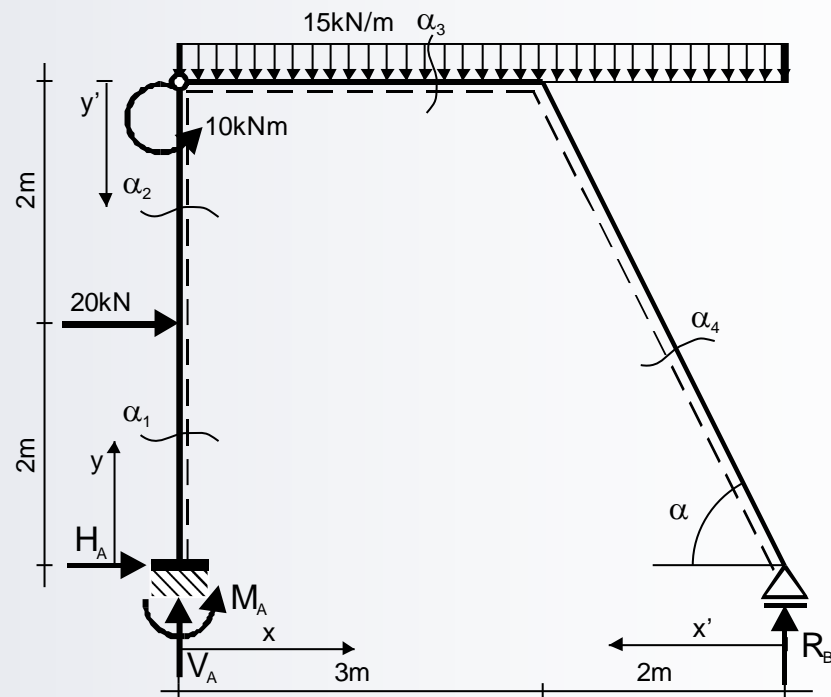
$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$N_{\alpha_1} = -V_A = -37,5kN$$

$$T_{\alpha_1} = -H_A = 20kN$$

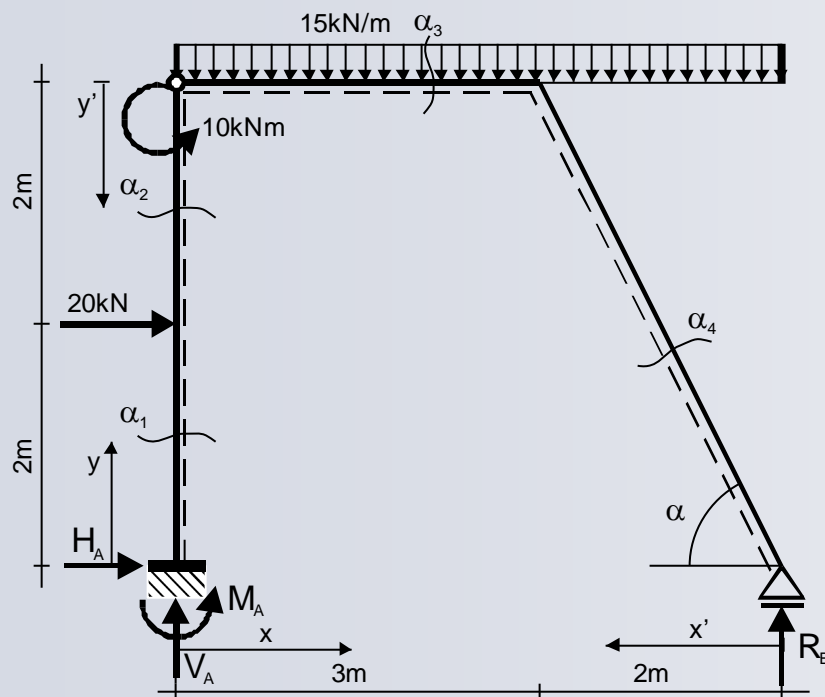
$$M_{\alpha_1} = -M_A - H_A \cdot y = \\ = 20kN \cdot y - 30kNm$$

$$\left| \begin{array}{l} y = 0 \quad M_{\alpha_1} = -30kNm \\ y = 2m \quad M_{\alpha_1} = 10kNm \end{array} \right.$$



# Przekrój

$$\alpha_2 - \alpha_2 \quad y \in \langle 2m; 4m \rangle$$



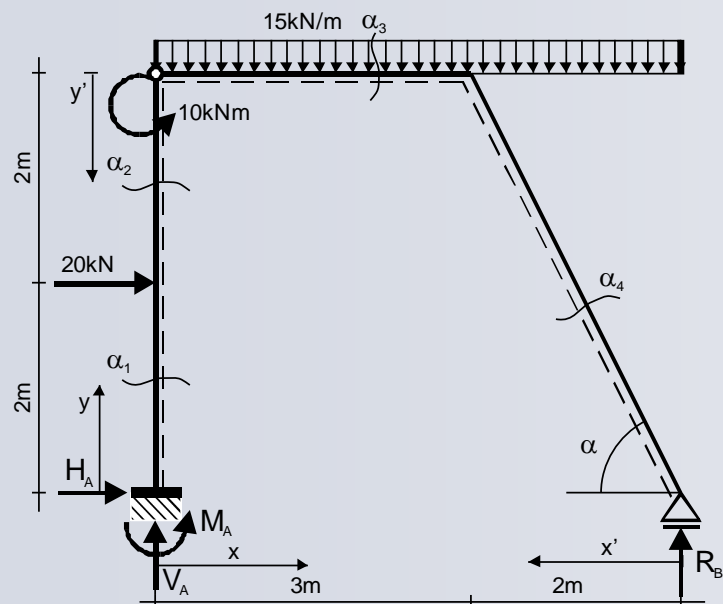
$$N_{\alpha_2} = -37,5kN$$

$$T_{\alpha_2} = -H_A - 20kN = 0$$

$$M_{\alpha_2} = 20kN \cdot y - 30kNm - 20kN \cdot (y - 2m) = 10kNm$$

# Przekrój

$$\alpha_3 - \alpha_3 \quad x \in \langle 0; 3m \rangle$$



$$N_{\alpha_3} = -H_A - 20kN = 20kN - 20kN = 0$$

$$T_{\alpha_3} = V_A - 15kN/m \cdot x = 37,5kN - 15kN/m \cdot x$$

$$\left| \begin{array}{l} x = 0m \quad T_{\alpha_3} = 37,5kN \\ x = 3m \quad T_{\alpha_3} = -7,5kN \end{array} \right.$$

$$\left| \begin{array}{l} x = 0m \quad T_{\alpha_3} = 37,5kN \\ x = 3m \quad T_{\alpha_3} = -7,5kN \end{array} \right.$$

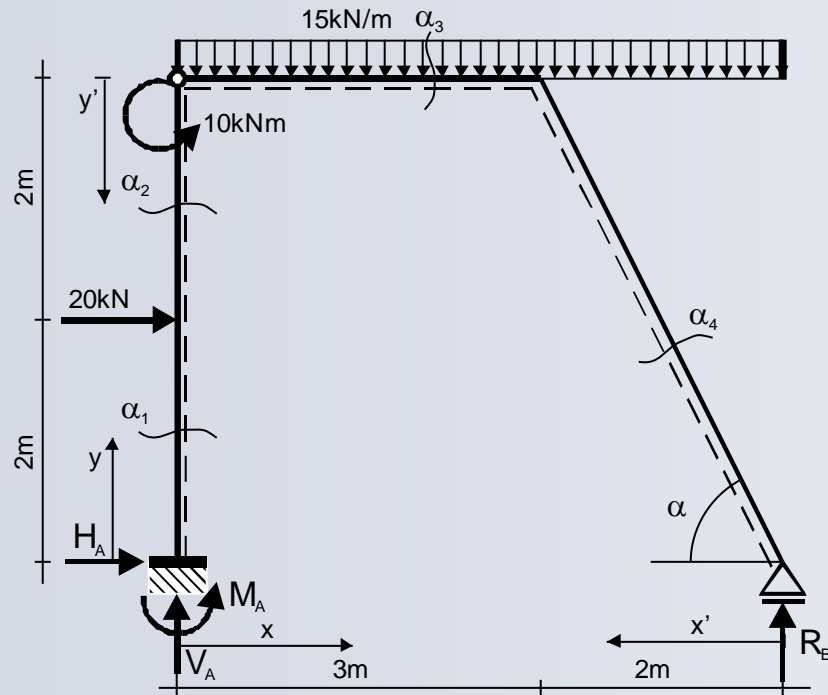
$$M_{\alpha_3} = V_A \cdot x - H_A \cdot 4m - M_A - 20kN \cdot 2m - 10kNm - 15kN/m \cdot x \cdot \frac{x}{2} =$$

$$= -7,5kN/m \cdot x^2 + 37,5kN \cdot x$$

$$\left| \begin{array}{l} x = 0m \quad M_{\alpha_3} = 0 \\ x = 3m \quad M_{\alpha_3} = 45kNm \end{array} \right.$$

$$\left| \begin{array}{l} x = 0m \quad M_{\alpha_3} = 0 \\ x = 3m \quad M_{\alpha_3} = 45kNm \end{array} \right.$$

# Ekstremum



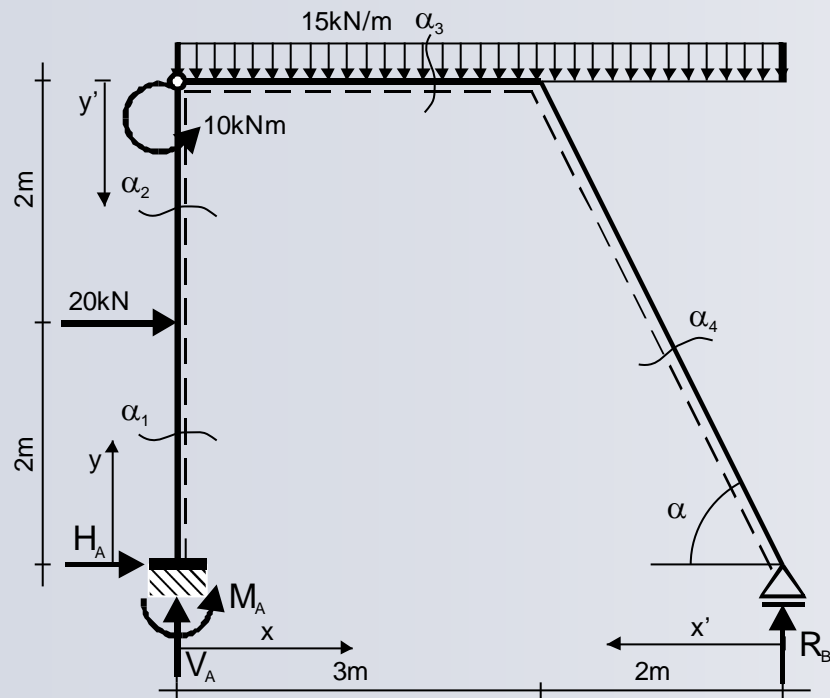
$$T_{\alpha_3} = 37,5kN - 15kN / m \cdot x = 0$$

$$x = 2,5m$$

$$M_{\alpha_3} = -7,5kN / m \cdot x^2 + 37,5kN \cdot x$$

$$M_{\alpha_3}(2,5m) = 46,875kNm$$

# Przekrój $\alpha_4 - \alpha_4$



$$\alpha_4 - \alpha_4 \quad x \in \langle 3m; 5m \rangle \quad y \in \langle 0m; 4m \rangle$$

$$x' \in \langle 0m; 2m \rangle \quad y' \in \langle 0m; 4m \rangle$$

$$\text{punkt } D \quad x = 3m, \quad y = 4m, \quad x' = 2m, \quad y' = 0$$

$$\text{punkt } B \quad x = 5m, \quad y = 0, \quad x' = 0, \quad y' = 4m$$

$$\sin \alpha = \frac{4m}{\sqrt{(4m)^2 + (2m)^2}} = 0,894$$

$$\cos \alpha = \frac{2m}{\sqrt{(4m)^2 + (2m)^2}} = 0,447$$

# Przekrój $\alpha_4 - \alpha_4$ (z lewej strony)

$$N_{\alpha 4} = -H_A \cos \alpha - 20kN \cos \alpha + V_A \sin \alpha - q \cdot x \sin \alpha =$$

$$= 37,5kN \sin \alpha - 15 \frac{kN}{m} x \sin \alpha =$$

$$= -13,41 \frac{kN}{m} x + 33,525kN$$

$$\left| \begin{array}{l} x = 3m \\ x = 5m \end{array} \right. \quad N_{\alpha 4} = -6,705kN$$

$$\left| \begin{array}{l} x = 3m \\ x = 5m \end{array} \right. \quad N_{\alpha 4} = -33,525kN$$

$$T_{\alpha 4} = H_A \sin \alpha + 20kN \sin \alpha + V_A \cos \alpha - q \cdot x \cos \alpha =$$

$$= 37,5kN \cos \alpha - 15 \frac{kN}{m} x \cos \alpha =$$

$$= -6,705 \frac{kN}{m} x + 16,763kN$$

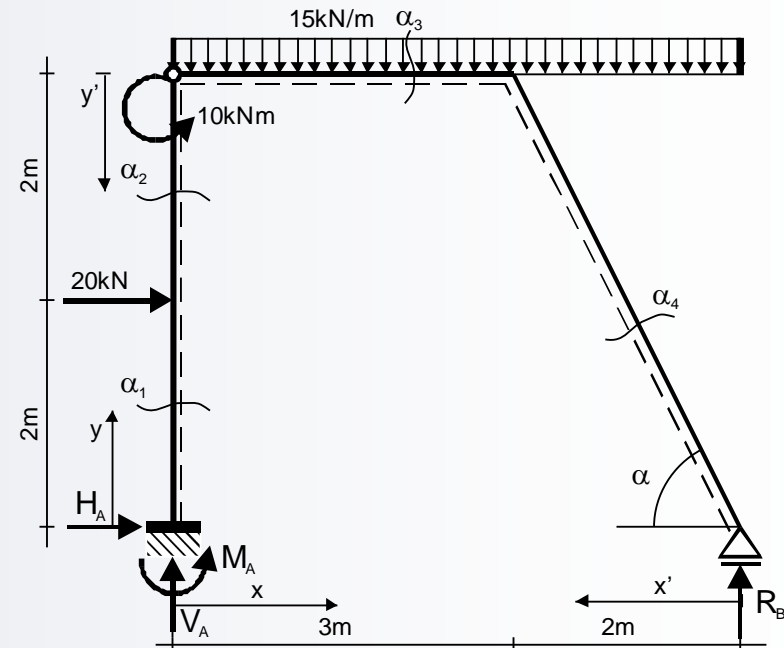
$$\left| \begin{array}{l} x = 3m \\ x = 5m \end{array} \right. \quad N_{\alpha 4} = -3,352kN$$

$$\left| \begin{array}{l} x = 3m \\ x = 5m \end{array} \right. \quad N_{\alpha 4} = -16,762kN$$

# Przekrój $\alpha_4 - \alpha_4$ (z lewej strony)

$$\begin{aligned}
 M_{\alpha_4} &= V_A \cdot x - H_A \cdot y - 20\text{kN} \cdot (y - 2\text{m}) - M_A - 15\text{kN/m} \cdot x \cdot \frac{x}{2} - 10\text{kNm} = \\
 &= 37,5\text{kN} \cdot x + 20\text{kN} \cdot y - 20\text{kN} \cdot y + 40\text{kNm} + \\
 &\quad - 30\text{kNm} - 7,5 \frac{\text{kN}}{\text{m}} x^2 - 10\text{kNm} = \\
 &= -7,5 \frac{\text{kN}}{\text{m}} x^2 + 37,5\text{kN} \cdot x
 \end{aligned}$$

$$\left| \begin{array}{ll}
 x = 3\text{m} & M_{\alpha_4} = 45\text{kNm} \\
 x = 5\text{m} & M_{\alpha_4} = 0
 \end{array} \right.$$





# Przekrój $\alpha_4 - \alpha_4$ (z prawej strony)

$$N_{\alpha_4} = -R_B \sin \alpha + q \cdot x' \sin \alpha = -37,5kN \sin \alpha + 15 \frac{kN}{m} x' \sin \alpha =$$

$$= 13,41 \frac{kN}{m} x' - 33,525kN$$

$$\left| \begin{array}{l} x' = 0 \quad N_{\alpha_4} = -33,525kN \\ x' = 2m \quad N_{\alpha_4} = -6,705kN \end{array} \right.$$

$$\left| \begin{array}{l} x' = 0 \quad N_{\alpha_4} = -33,525kN \\ x' = 2m \quad N_{\alpha_4} = -6,705kN \end{array} \right.$$

$$T_{\alpha_4} = -R_B \cos \alpha + q \cdot x' \cos \alpha = -37,5kN \cos \alpha + 15 \frac{kN}{m} x' \cos \alpha =$$

$$= 6,705 \frac{kN}{m} x' - 16,763kN$$

$$\left| \begin{array}{l} x' = 0 \quad N_{\alpha_4} = -16,763kN \\ x' = 2m \quad N_{\alpha_4} = -3,353kN \end{array} \right.$$

$$\left| \begin{array}{l} x' = 0 \quad N_{\alpha_4} = -16,763kN \\ x' = 2m \quad N_{\alpha_4} = -3,353kN \end{array} \right.$$

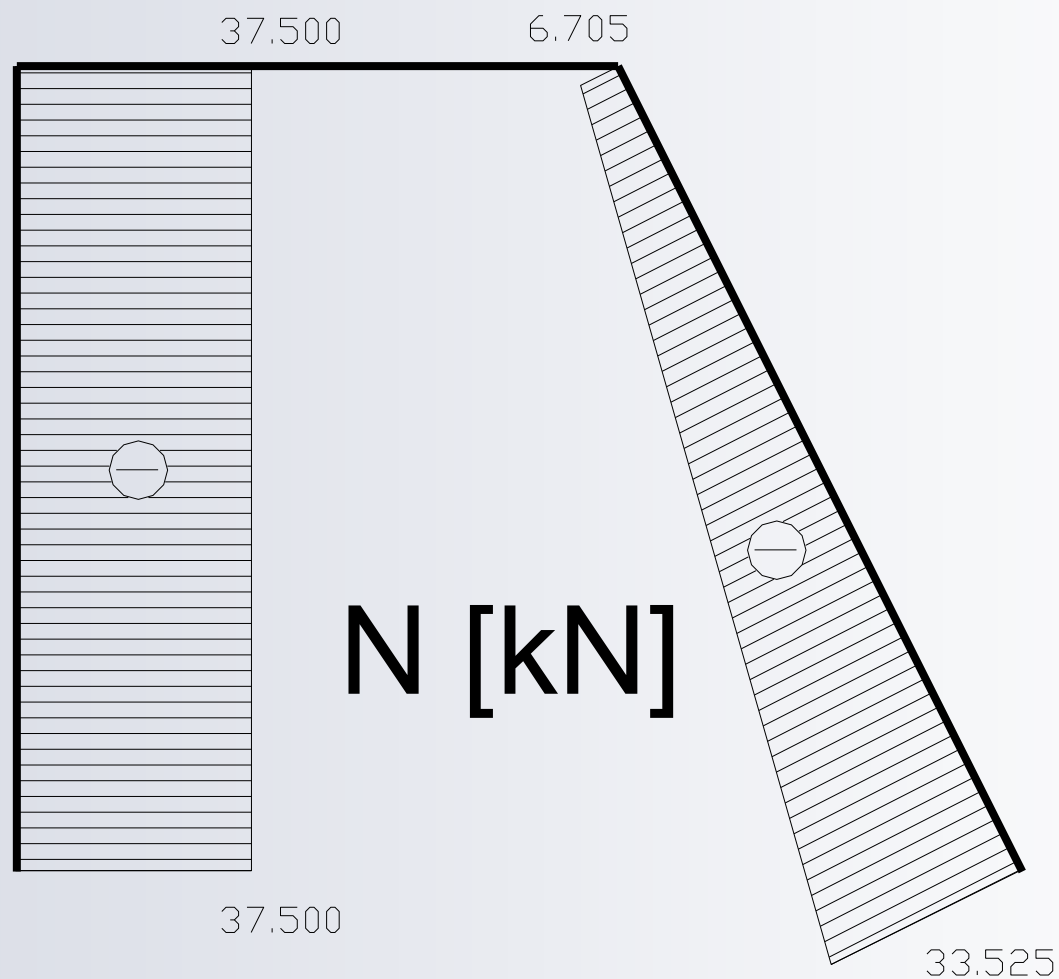
$$M_{\alpha_4} = R_B \cdot x' - 15kN / m \cdot x' \cdot \frac{x'}{2} =$$

$$= -7,5 \frac{kN}{m} x'^2 + 37,5kN \cdot x'$$

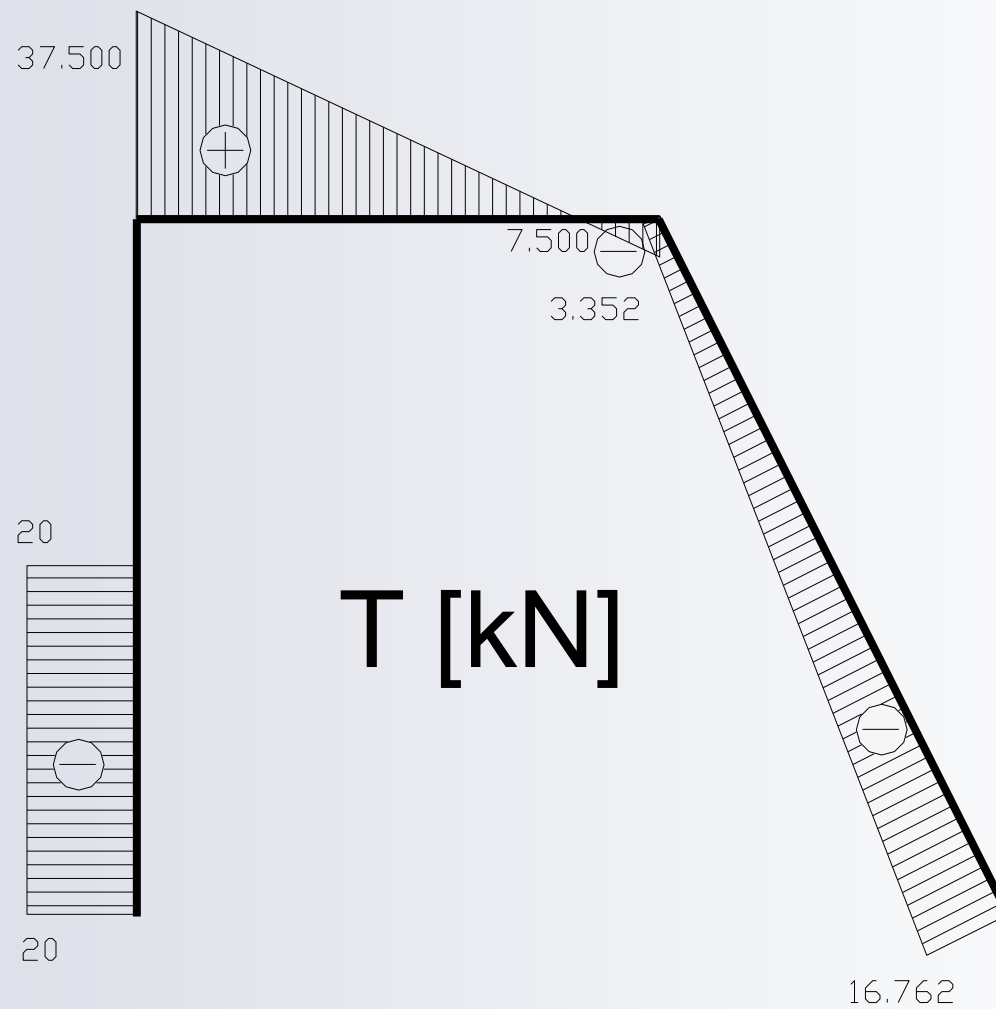
$$\left| \begin{array}{l} x' = 0 \quad M_{\alpha_4} = 0 \\ x' = 2m \quad M_{\alpha_4} = 45kNm \end{array} \right.$$

$$\left| \begin{array}{l} x' = 0 \quad M_{\alpha_4} = 0 \\ x' = 2m \quad M_{\alpha_4} = 45kNm \end{array} \right.$$

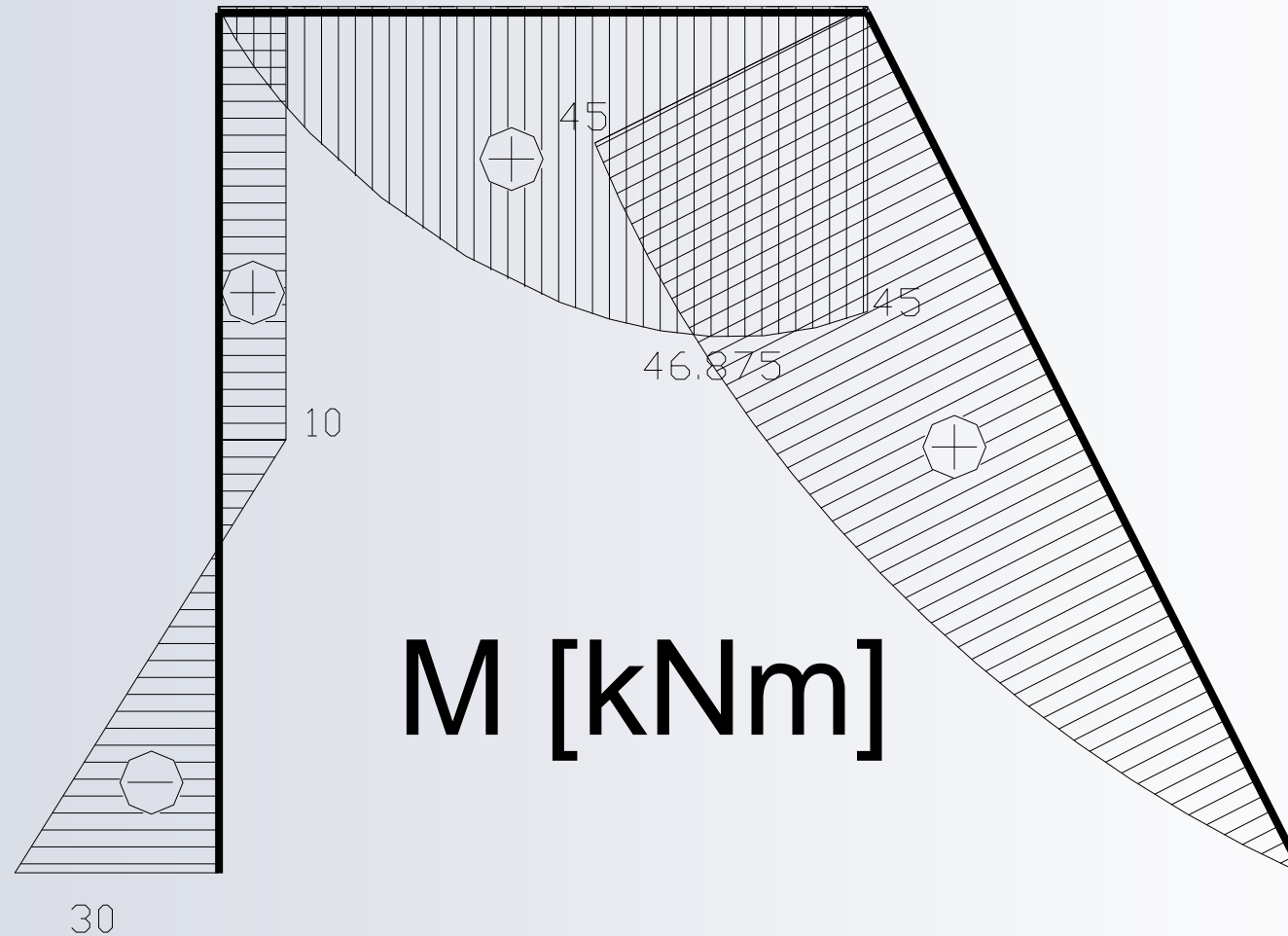
# Wykres sił normalnych



# Wykres sił tnących



# Wykres momentów zginających

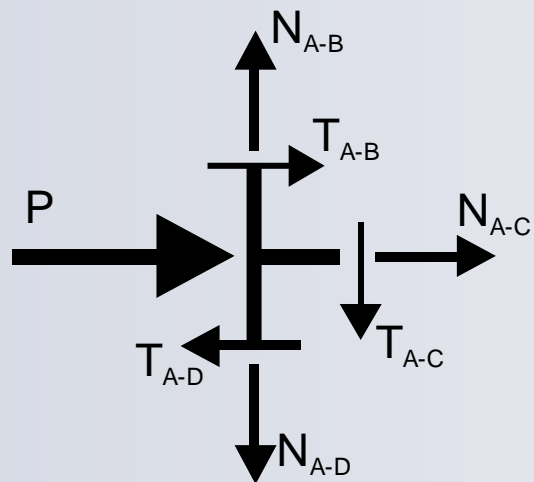


# Kontrola wyników

- Sprawdzenie równowagi ogólnej
  - Suma rzutów sił na nową oś, nierównoległą do wcześniej wykorzystywanych.
  - Suma momentów względem dowolnego punktu na płaszczyźnie. Punkt powinien być dobrany tak, aby jak najwięcej reakcji znalazło się w równaniu.

# Kontrola wyników

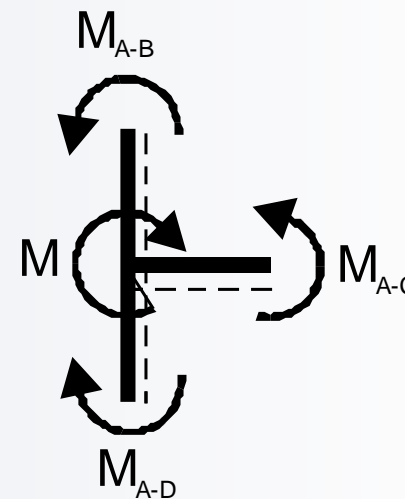
- Sprawdzenie równowagi w węzłach:



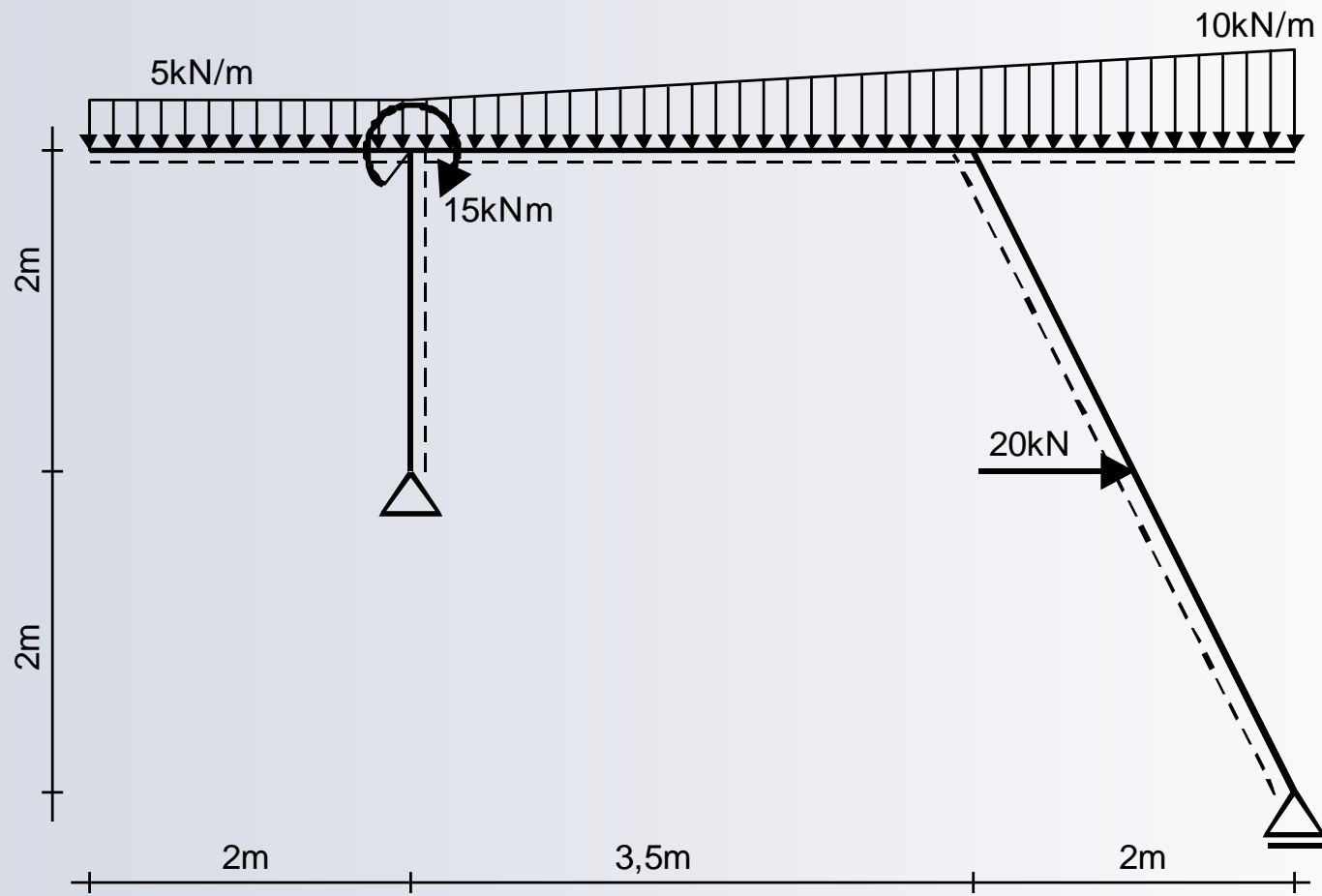
$$\sum X : T_{A-B} + N_{A-C} - T_{A-D} + P = 0$$

$$\sum Y : N_{A-B} - T_{A-C} - N_{A-D} = 0$$

$$\sum M_A : M_{A-B} + M_{A-C} - M_{A-D} - M = 0$$



# Przykład – rama

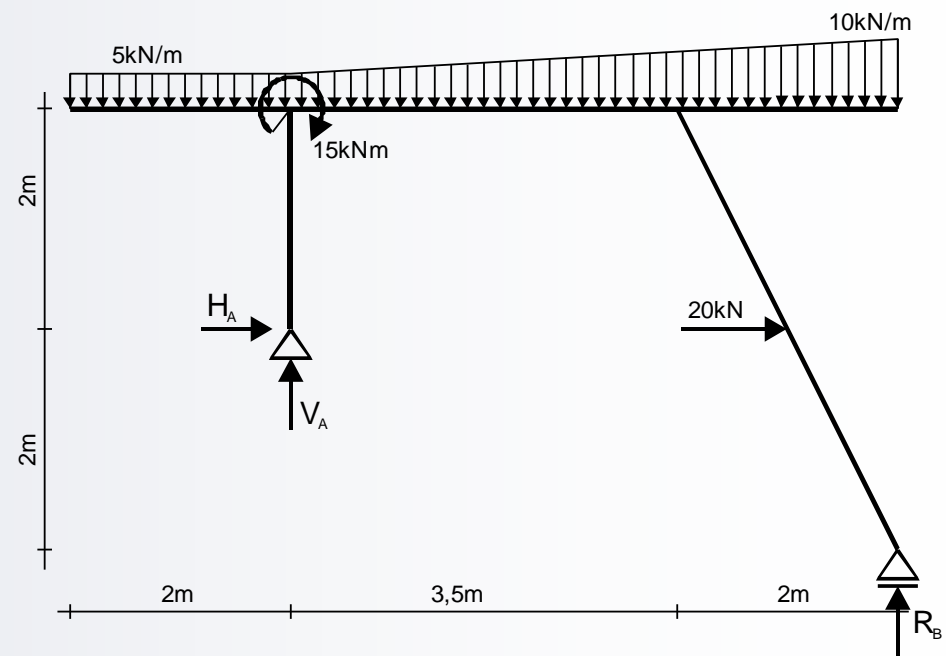


# Reakcije

$$\sum X : H_A + 20kN = 0$$

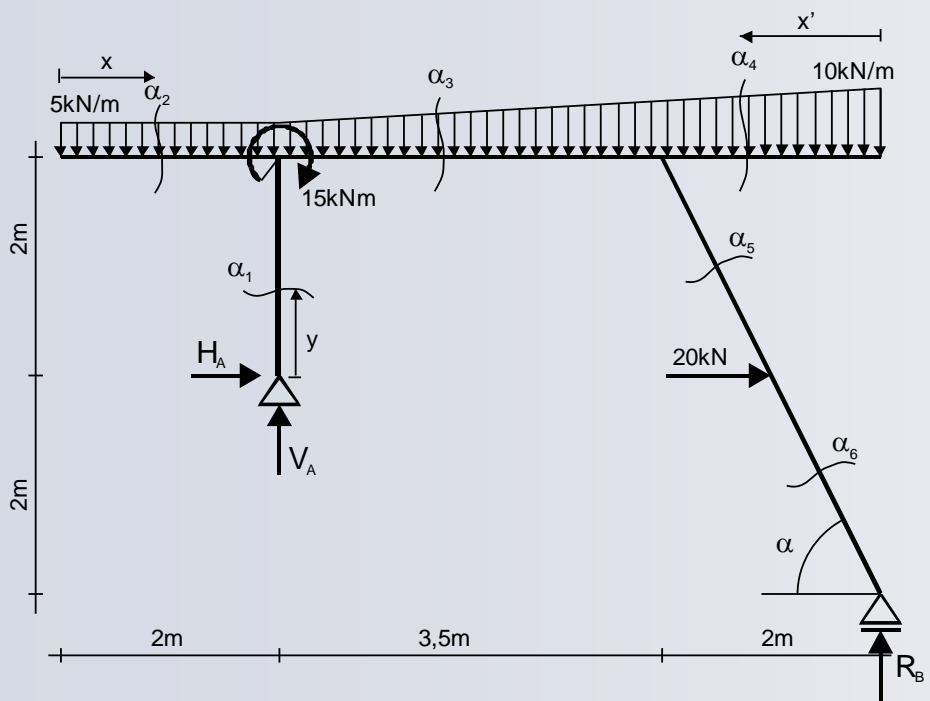
$$\sum Y : V_A + R_B - 5 \frac{kN}{m} \cdot 7,5m - \frac{1}{2} 5 \frac{kN}{m} \cdot 5,5m = 0$$

$$\begin{aligned} \sum M_A : R_B \cdot 5,5m - 15kNm + 5 \frac{kN}{m} \cdot 2m \cdot \frac{1}{2} 2m - 5 \frac{kN}{m} \cdot 5,5m \cdot \frac{1}{2} 5,5m + \\ - \frac{1}{2} 5 \frac{kN}{m} \cdot 5,5m \cdot \frac{2}{3} 5,5m = 0 \end{aligned}$$





# Przyjęcie przekrojów, przedziały



$$H_A = -20kN$$

$$R_B = 23,826kN$$

$$V_A = 27,424kN$$

$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$\alpha_2 - \alpha_2 \quad x \in \langle 0; 2m \rangle$$

$$\alpha_3 - \alpha_3 \quad x \in \langle 2m; 5,5m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 5,5m; 7,5m \rangle$$

$$x' \in \langle 0; 2m \rangle$$

$$\alpha_5 - \alpha_5 \quad x \in \langle 5,5m; 6,5m \rangle \quad y \in \langle 0; 2m \rangle$$

$$x' \in \langle 1m; 2m \rangle$$

$$\alpha_6 - \alpha_6 \quad x \in \langle 6,5m; 7,5m \rangle \quad y \in \langle -2m; 0 \rangle$$

$$x' \in \langle 0; 1m \rangle$$

# Przekrój

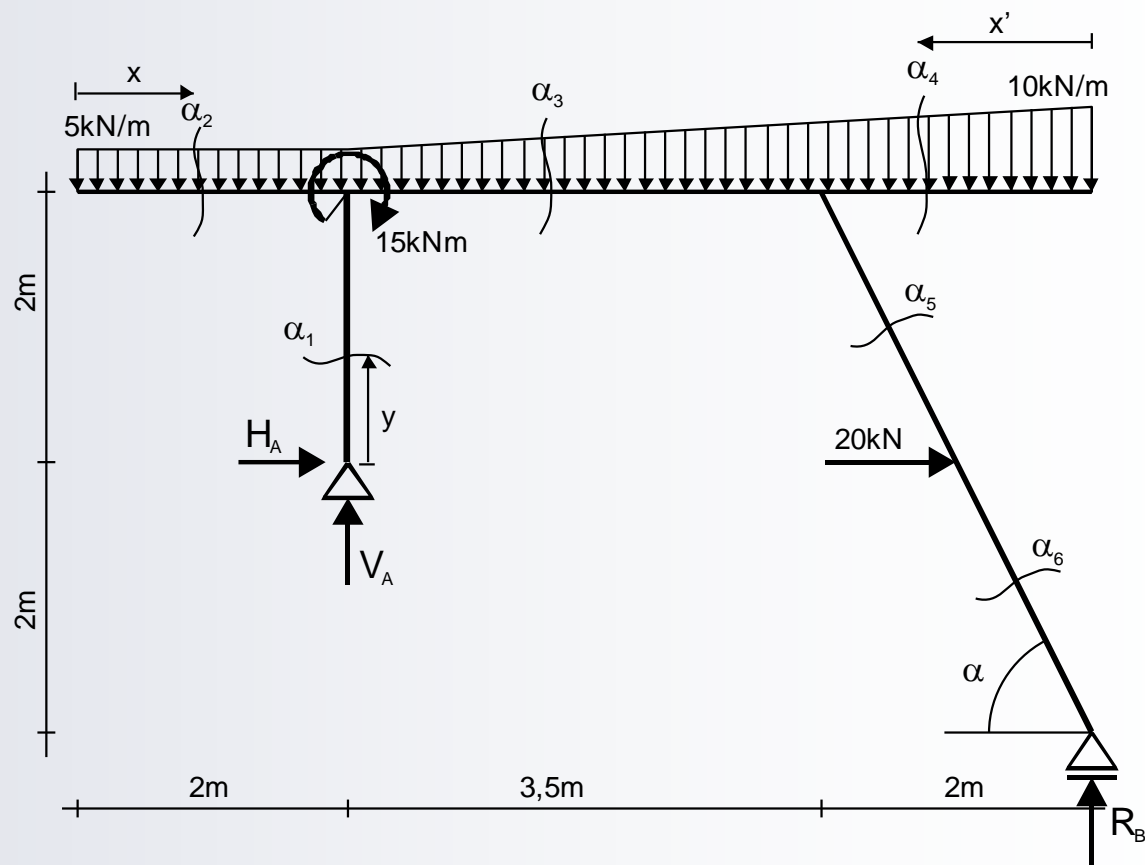
$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$N_{\alpha_1} = -V_A = -27,424kN$$

$$T_{\alpha_1} = -H_A = 20kN$$

$$M_{\alpha_1} = -H_A \cdot y = 20kN \cdot y$$

$$\left| \begin{array}{l} y = 0 \quad M_{\alpha_1} = 0 \\ y = 2m \quad M_{\alpha_1} = 40kNm \end{array} \right.$$



# Przekrój

$$\alpha_2 - \alpha_2 \quad x \in \langle 0; 2m \rangle$$

$$N_{\alpha_2} = 0$$

$$T_{\alpha_2} = -5 \frac{kN}{m} \cdot x$$

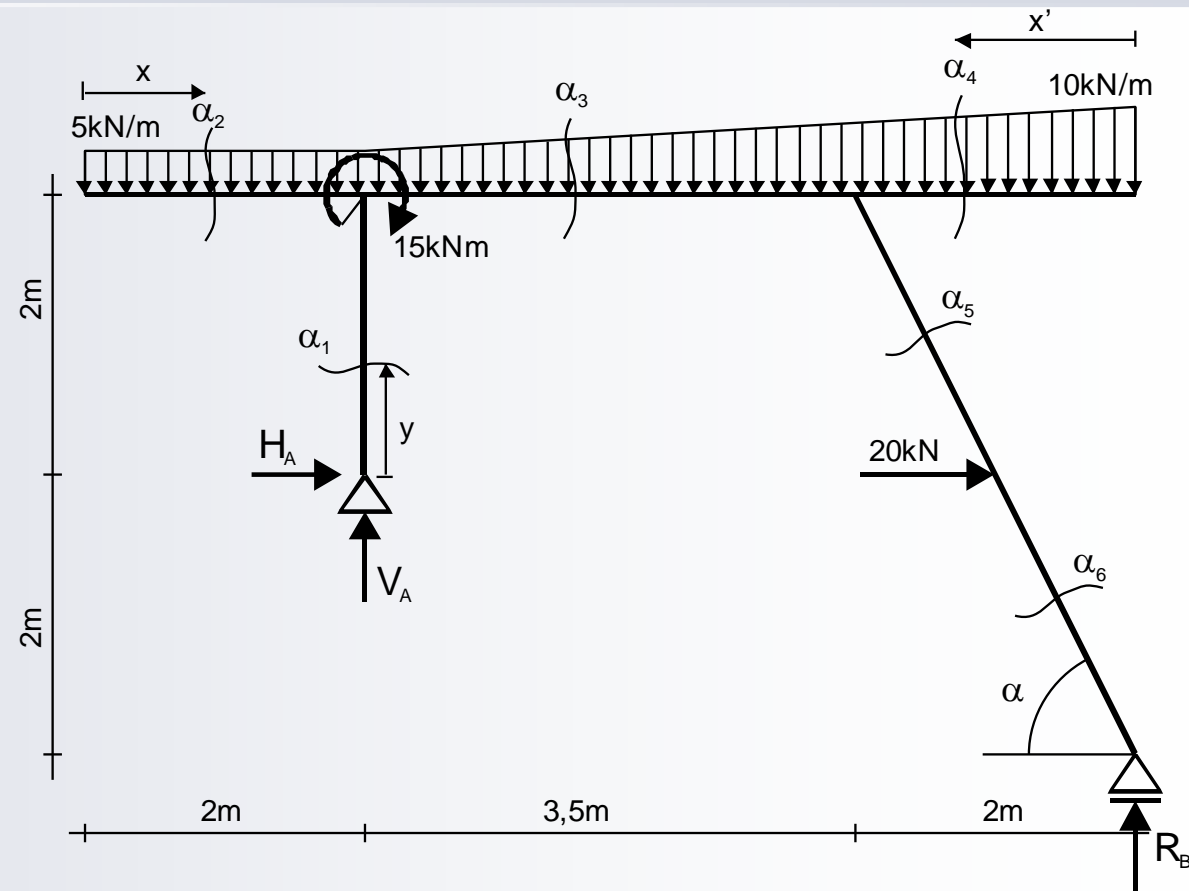
$$\left| \begin{array}{l} x = 0 \quad T_{\alpha_2} = 0 \end{array} \right.$$

$$\left| \begin{array}{l} x = 2m \quad T_{\alpha_2} = -10kN \end{array} \right.$$

$$M_{\alpha_2} = -5 \frac{kN}{m} \cdot x \cdot \frac{x}{2} = -2,5 \frac{kN}{m} \cdot x^2$$

$$\left| \begin{array}{l} x = 0 \quad M_{\alpha_2} = 0 \end{array} \right.$$

$$\left| \begin{array}{l} x = 2m \quad M_{alpha_2} = -10kNm \end{array} \right.$$



# Przekrój

$$\alpha_3 - \alpha_3 \quad x \in \langle 2m; 5,5m \rangle$$

$$N_{\alpha_3} = -H_A = 20kN$$

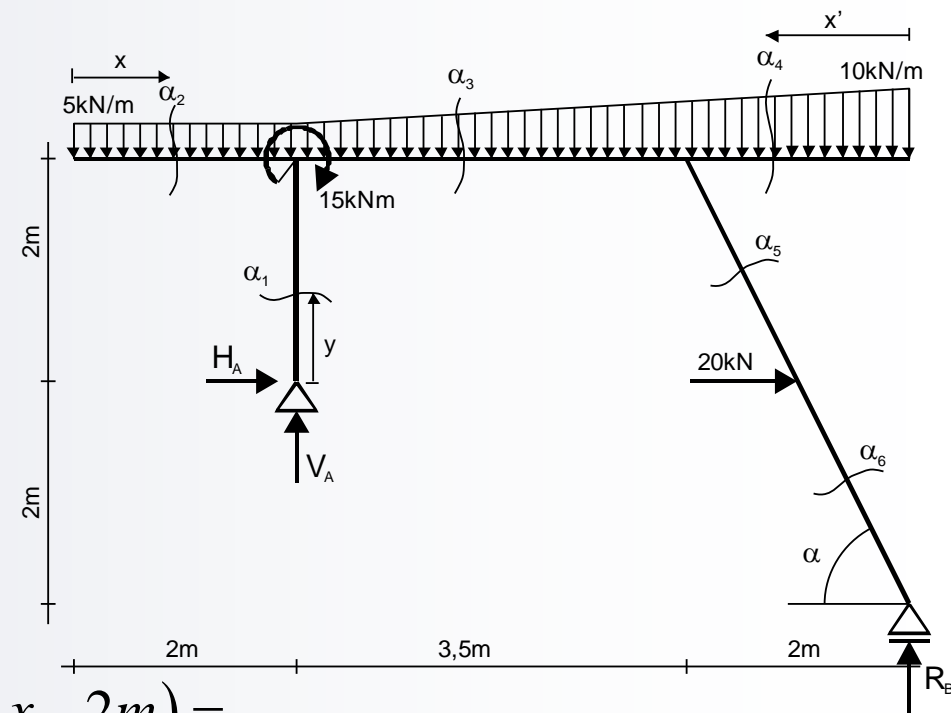
$$\frac{5 \frac{kN}{m}}{q'} = \frac{5,5m}{x-2m}$$

$$q' = 0,909 \frac{kN}{m^2} (x-2m) = 0,909 \frac{kN}{m^2} x - 1,818 \frac{kN}{m}$$

$$T_{\alpha_3} = V_A - 5 \frac{kN}{m} \cdot x - \frac{1}{2} q' (x-2m) =$$

$$= 27,424kN - 5 \frac{kN}{m} \cdot x - \frac{1}{2} \left( 0,909 \frac{kN}{m^2} x - 1,818 \frac{kN}{m} \right) (x-2m) =$$

$$= -0,455 \frac{kN}{m^2} x^2 - 3,182 \frac{kN}{m} x + 25,606kN$$



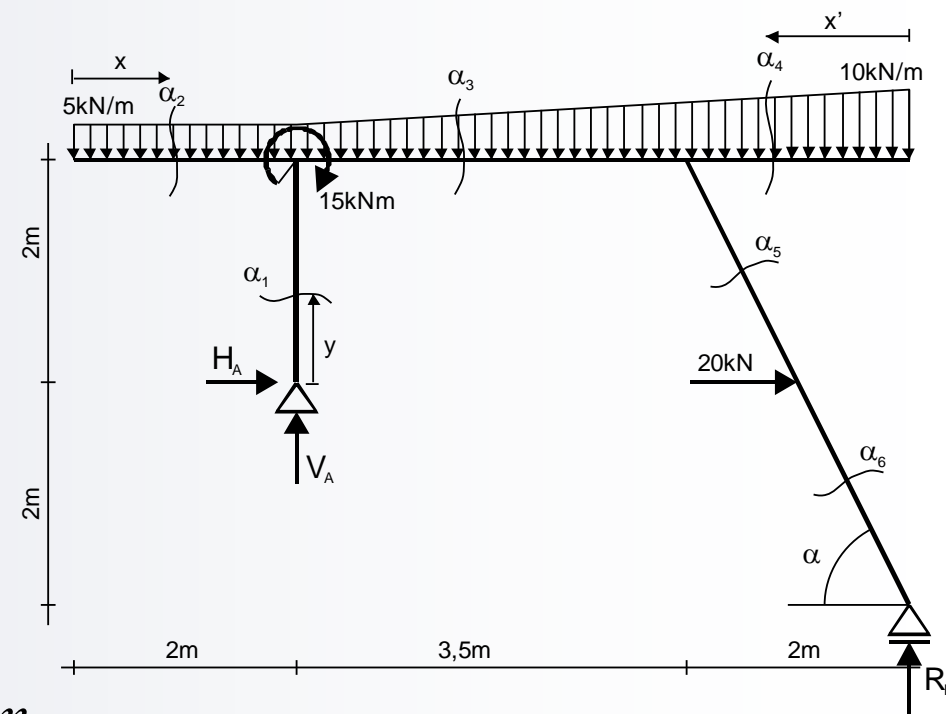
$$\left| \begin{array}{l} x = 2m \quad T_{\alpha_3} = 17,422kN \\ x = 5,5m \quad T_{\alpha_3} = -5,659kN \end{array} \right.$$

# Przekrój

$$\alpha_3 - \alpha_3 \quad x \in \langle 2m; 5,5m \rangle$$

$$\begin{aligned}
 M_{\alpha_3} &= V_A \cdot (x - 2m) - H_A \cdot 2m - 5 \frac{kN}{m} \cdot x \cdot \frac{x}{2} + \\
 &\quad - \frac{1}{2} q' \cdot (x - 2m) \cdot \frac{1}{3} (x - 2m) + 15kNm = \\
 &= 27,424kN \cdot (x - 2m) + 20kN \cdot 2m - 2,5 \frac{kN}{m} \cdot x^2 + \\
 &\quad - \frac{1}{6} \left( 0,909 \frac{kN}{m^2} x - 1,818 \frac{kN}{m} \right) \cdot (x - 2m)^2 + 15kNm = \\
 &= -0,156 \frac{kN}{m^2} x^3 - 1,591 \frac{kN}{m} x^2 + 25,606kN \cdot x + 1,364kNm
 \end{aligned}$$

$$\begin{aligned}
 & \left| \begin{array}{l} x = 2m \quad M_{\alpha_3} = 44,964kNm \\ x = 5,5m \quad M_{\alpha_3} = 68,115kNm \end{array} \right.
 \end{aligned}$$



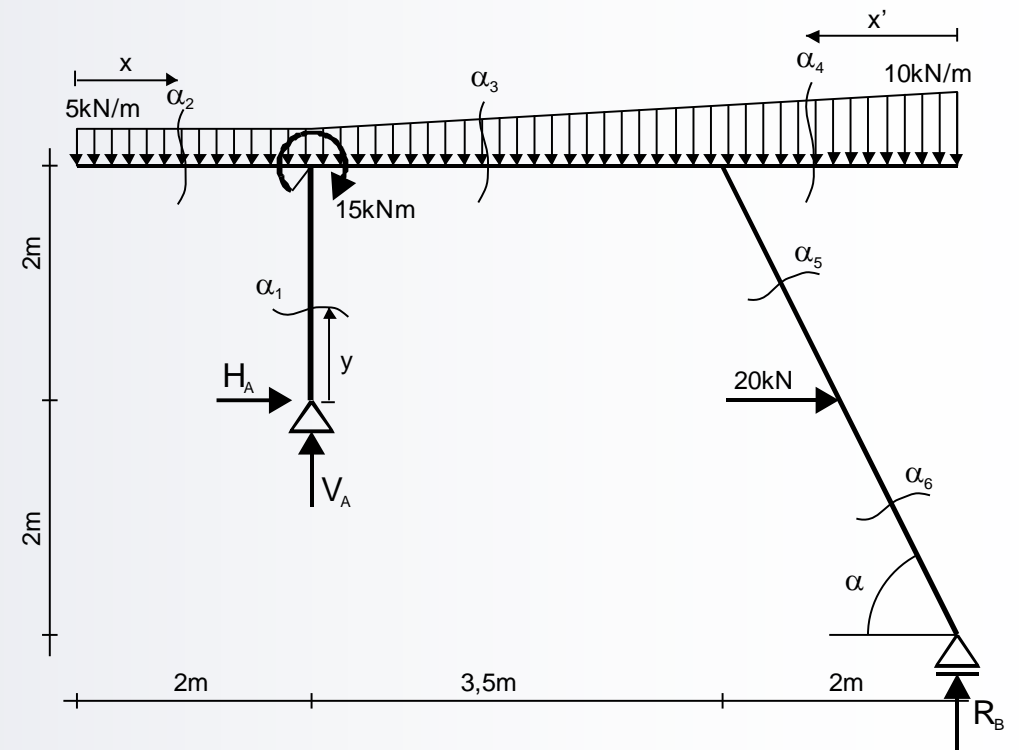
# Ekstremum

$$T_{\alpha_3} = -0,455 \frac{kN}{m^2} x^2 - 3,182 \frac{kN}{m} x + 25,606 kN = 0$$

$$x = 4,78m$$

$$M_{\alpha_3} = -0,156 \frac{kN}{m^2} x^3 - 1,591 \frac{kN}{m} x^2 + 25,606 kN \cdot x + 1,364 kNm$$

$$M_{\alpha_3}(4.78m) = 70,371 kNm$$



# Przekrój $\alpha_4 - \alpha_4$ $x \in \langle 5,5m; 7,5m \rangle$

$$N_{\alpha_4} = 0$$

$$T_{\alpha_4} = 5 \frac{kN}{m} \cdot (7,5m - x) + q'(7,5m - x) + \frac{1}{2} \left( 5 \frac{kN}{m} - q' \right) (7,5m - x) =$$

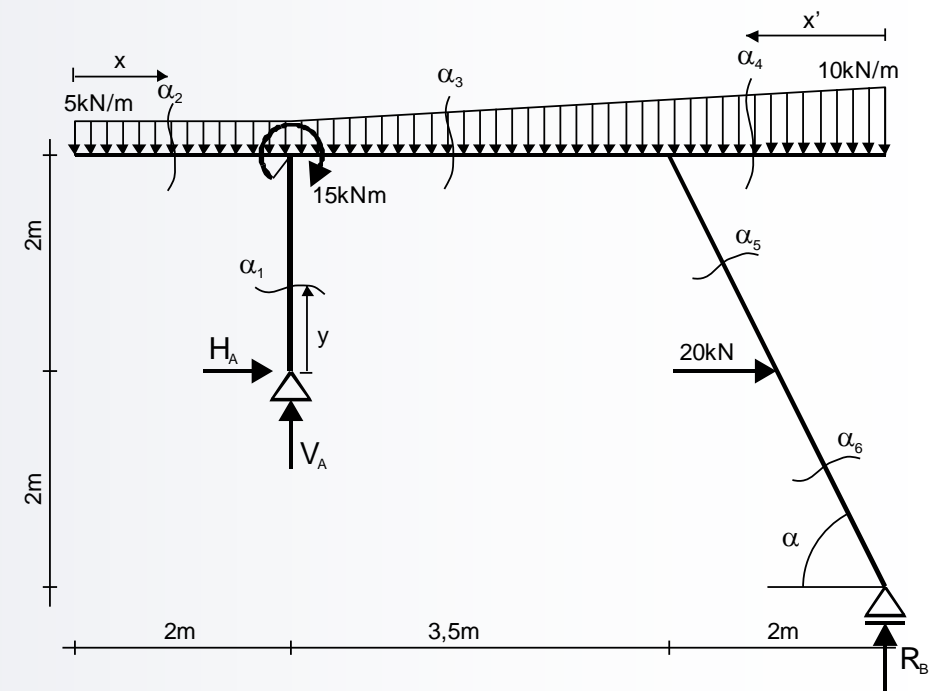
$$= 37,5kN - 5 \frac{kN}{m} \cdot x - \left( 0,909 \frac{kN}{m^2} x - 1,818 \frac{kN}{m} \right) (7,5m - x) +$$

$$+ \frac{1}{2} \left( 5 \frac{kN}{m} - 0,909 \frac{kN}{m^2} x + 1,808 \frac{kN}{m} \right) (7,5m - x) =$$

$$= -0,454 \frac{kN}{m^2} x^2 - 3,182 \frac{kN}{m} x + 49,433kN$$

$$x = 5,5m \quad T_{\alpha_4} = 18,199kN$$

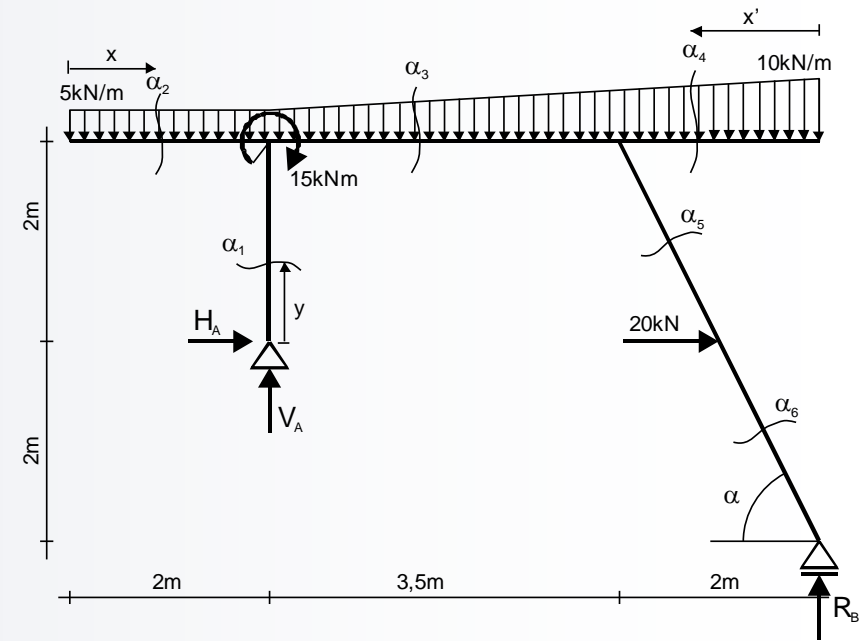
$$x = 7,5m \quad T_{\alpha_4} = 0,031kN \approx 0$$



# Przekrój $\alpha_4 - \alpha_4$ $x \in \langle 5,5m; 7,5m \rangle$

$$\begin{aligned}
 M_{\alpha_4} &= -5 \frac{kN}{m} \cdot (7,5m - x) \frac{1}{2} (7,5m - x) - q' (7,5m - x) \frac{1}{2} (7,5m - x) + \\
 &\quad - \frac{1}{2} \left( 5 \frac{kN}{m} - q' \right) (7,5m - x) \frac{2}{3} (7,5m - x) = \\
 &= -5 \frac{kN}{m} \cdot \frac{1}{2} (7,5m - x)^2 - \left( 0,909 \frac{kN}{m^2} - 1,818 \frac{kN}{m} \right) \frac{1}{2} (7,5m - x)^2 + \\
 &\quad - \frac{1}{2} \left( 5 \frac{kN}{m} - \left( 0,909 \frac{kN}{m^2} - 1,818 \frac{kN}{m} \right) \right) \frac{2}{3} (7,5m - x)^2 = \\
 &= -0,152 \frac{kN}{m^2} x^3 - 1,591 \frac{kN}{m} x^2 + 49,433 kN \cdot x - 217,331 kNm
 \end{aligned}$$

$$\left| \begin{array}{ll}
 x = 5,5m & M_{\alpha_4} = -18,886 kNm \\
 x = 7,5m & M_{\alpha_4} = -0,202 kNm \approx 0
 \end{array} \right.$$





# Przekrój $\alpha_4 - \alpha_4$

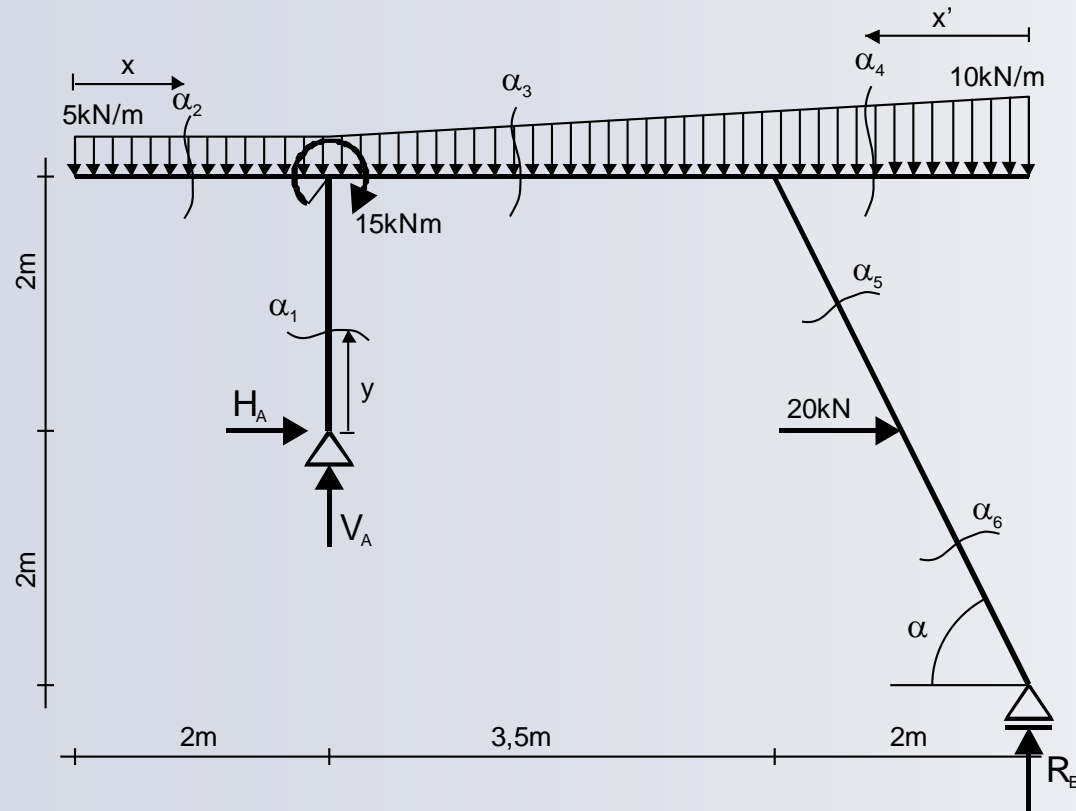
- Zwiększenie dokładności przy  $x$  w najwyższej potędze (większa liczba cyfr znaczących):

$$T_{\alpha_4} = -0,4545 \frac{kN}{m^2} x^2 - 3,182 \frac{kN}{m} x + 49,433 kN$$

$$M_{\alpha_4} = -0,1515 \frac{kN}{m^2} x^3 - 1,591 \frac{kN}{m} x^2 + 49,433 kN \cdot x - 217,331 kNm$$

$$\left| \begin{array}{lll} x = 5,5m & T_{\alpha_4} = 18,183 kN & M_{\alpha_4} = -18,783 kNm \\ x = 7,5m & T_{\alpha_4} = 0,002 kN \approx 0 & M_{\alpha_4} = 0,009 kNm \end{array} \right.$$

# Przekrój $\alpha_6 - \alpha_6$ $x' \in \langle 0; 1m \rangle$



$$\sin \alpha = \frac{4m}{\sqrt{(4m)^2 + (2m)^2}} = 0,894$$

$$\cos \alpha = \frac{2m}{\sqrt{(4m)^2 + (2m)^2}} = 0,447$$

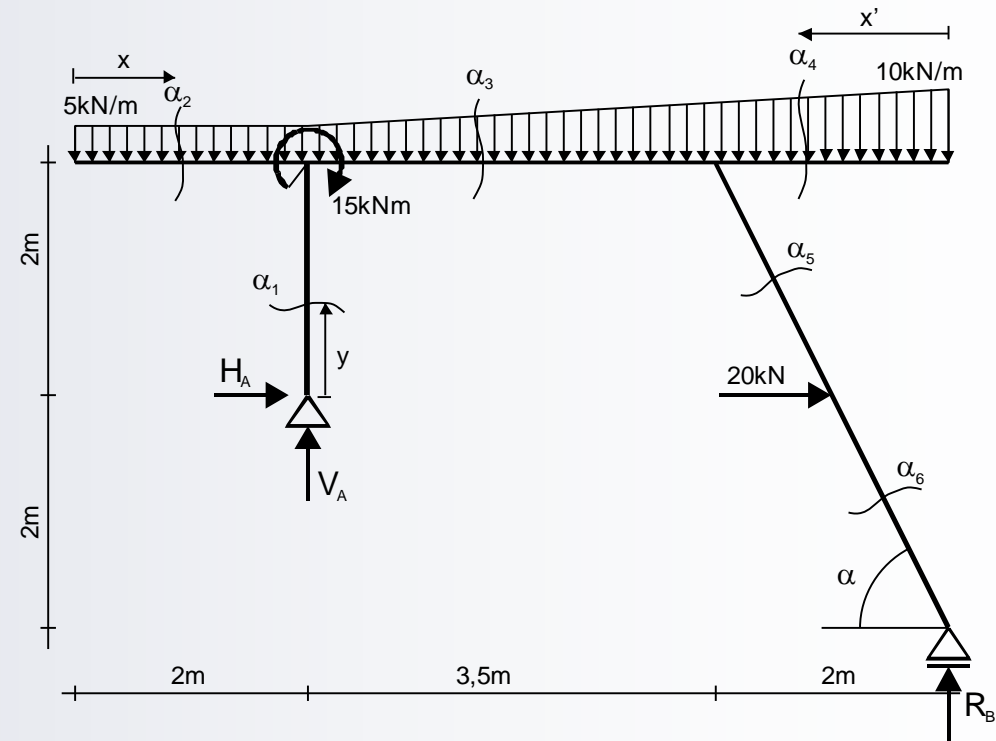
# Przekrój $\alpha_6 - \alpha_6$ $x' \in \langle 0; 1m \rangle$

$$N_{\alpha_6} = -R_B \cdot \sin \alpha = -23,826kN \cdot 0,894 = -21,300kN$$

$$T_{\alpha_6} = -R_B \cdot \cos \alpha = -23,826kN \cdot 0,447 = -10,650kN$$

$$M_{\alpha_6} = R_B \cdot x' = 23,826kN \cdot x'$$

$x' = 0$	$M_{\alpha_6} = 0$
$x' = 1m$	$M_{\alpha_6} = 23,826kNm$



# Przekrój $\alpha_5 - \alpha_5$ $x' \in \langle 1m; 2m \rangle$

$$N_{\alpha_5} = -R_B \cdot \sin \alpha + 20 \cos \alpha =$$

$$= -23,826kN \cdot 0,894 + 20kN \cdot 0,447 = -12,360kN$$

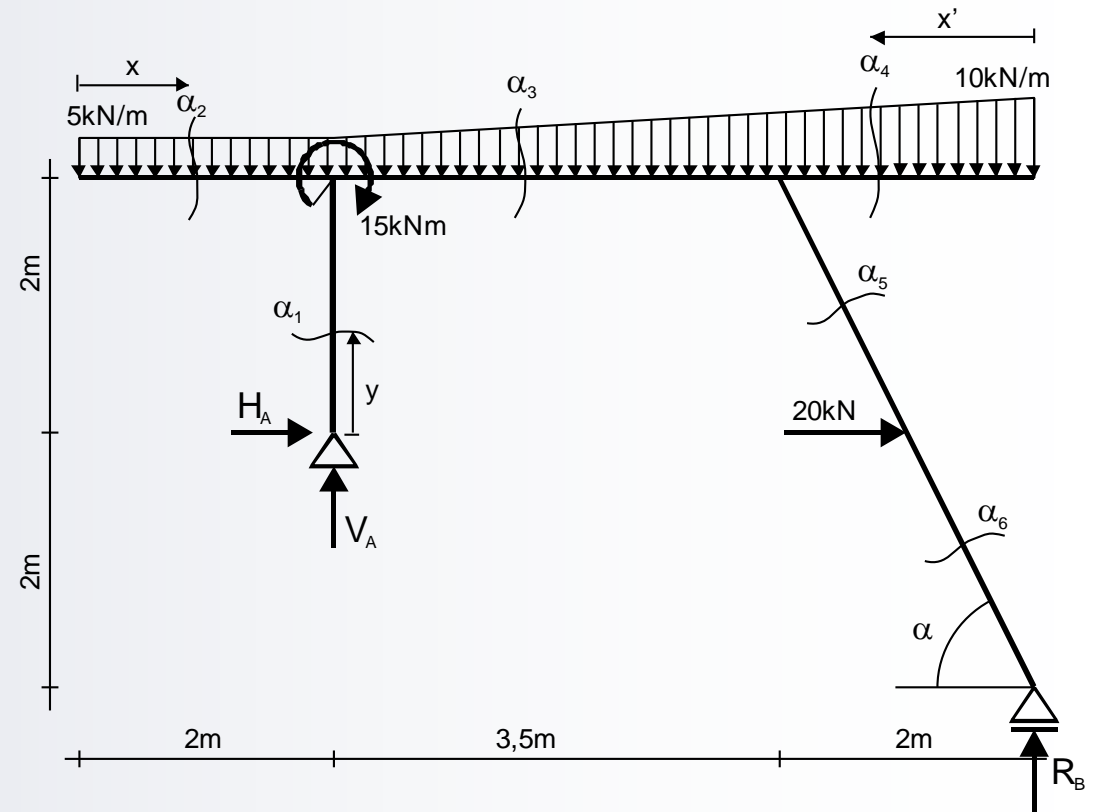
$$T_{\alpha_5} = -R_B \cdot \cos \alpha - 20 \sin \alpha =$$

$$= -23,826kN \cdot 0,447 - 20kN \cdot 0,894 = -28,530kN$$

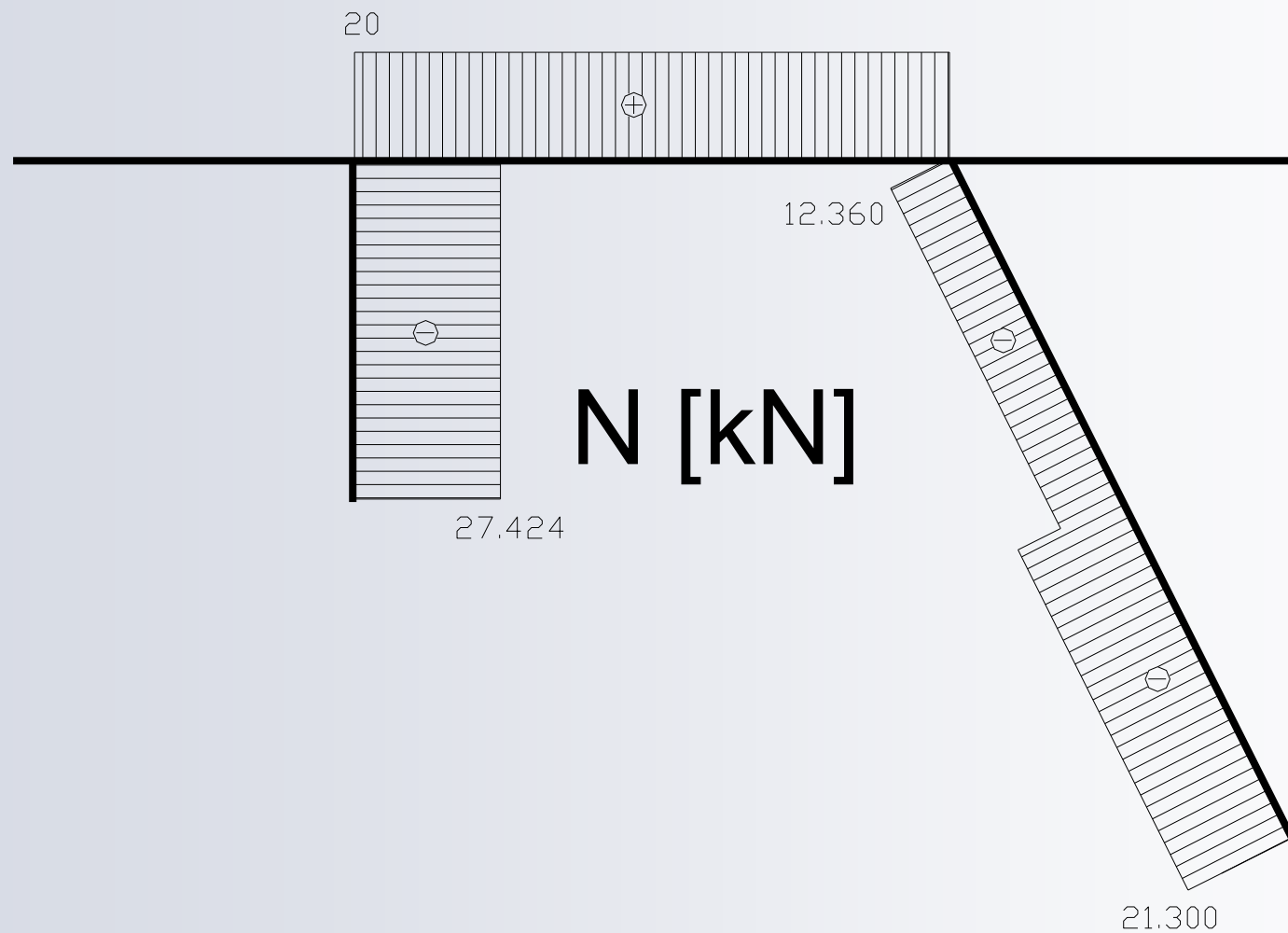
$$M_{\alpha_5} = R_B \cdot x' + 20 \cdot y =$$

$$= 23,826kN \cdot x' + 20kN \cdot y$$

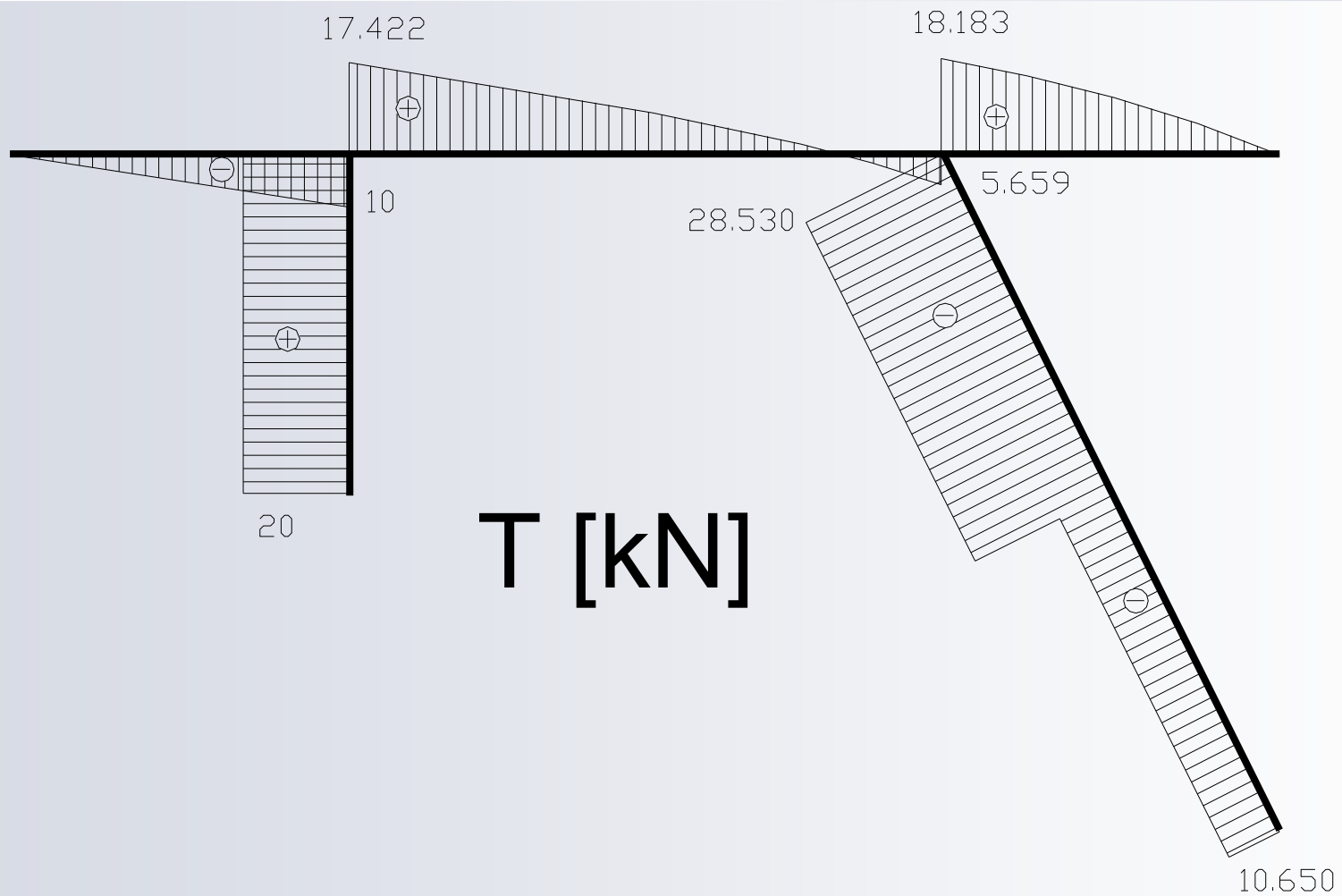
$x' = 1m$	$y = 0$	$M_{\alpha_5} = 23,826kNm$
$x' = 2m$	$y = 2m$	$M_{\alpha_5} = 87,652kNm$



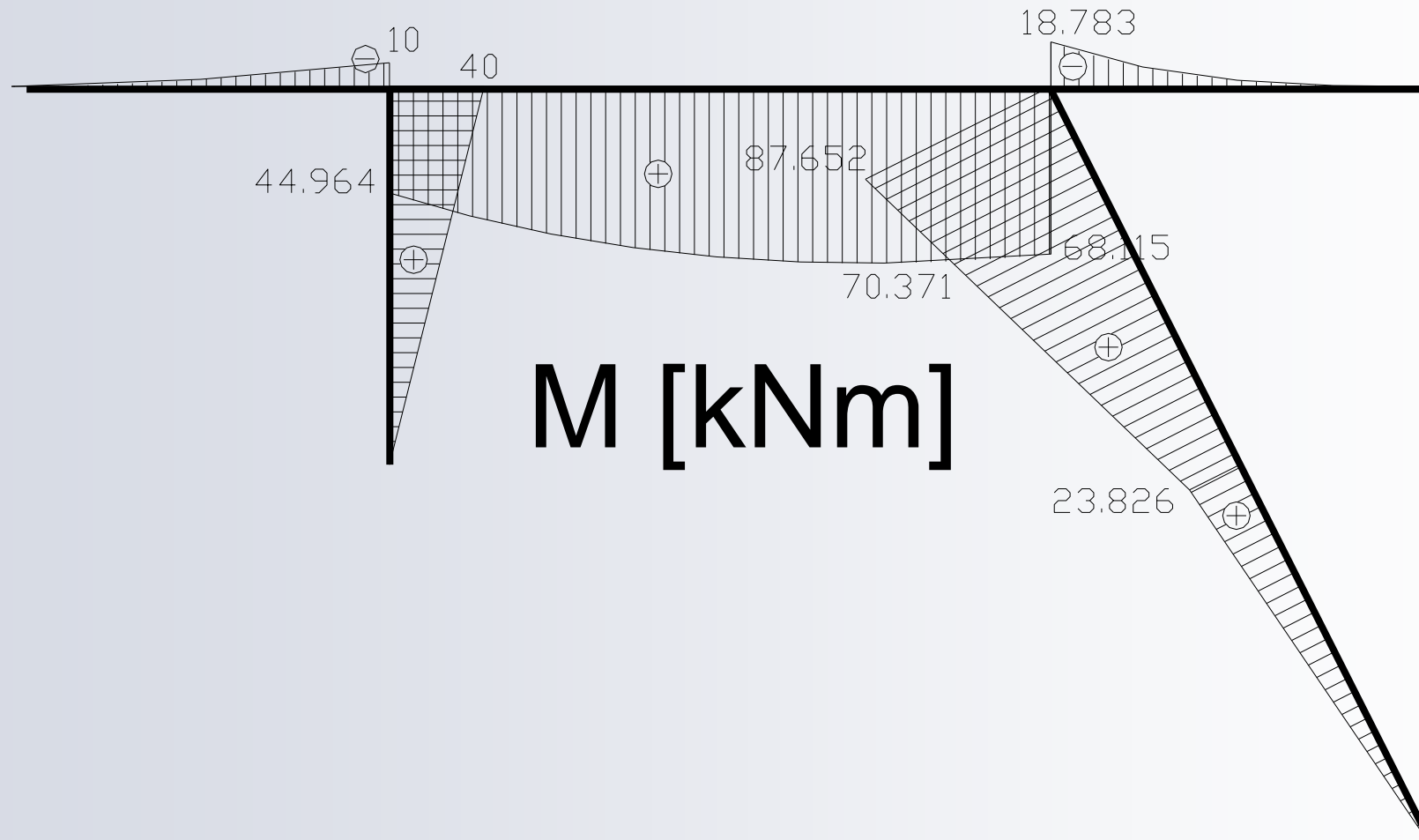
# Wykres sił normalnych



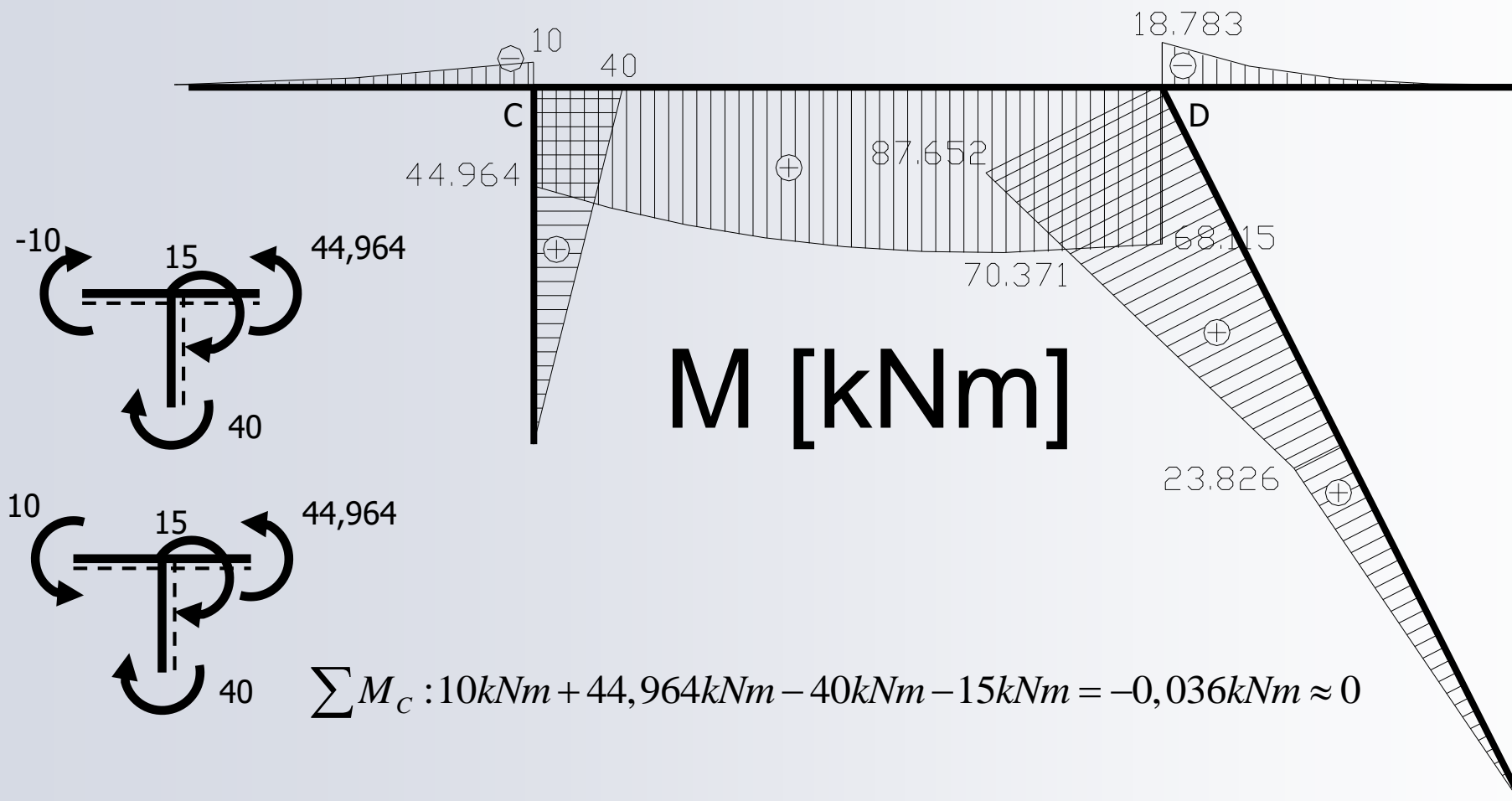
# Wykres sił tnących



# Wykres momentów zginających

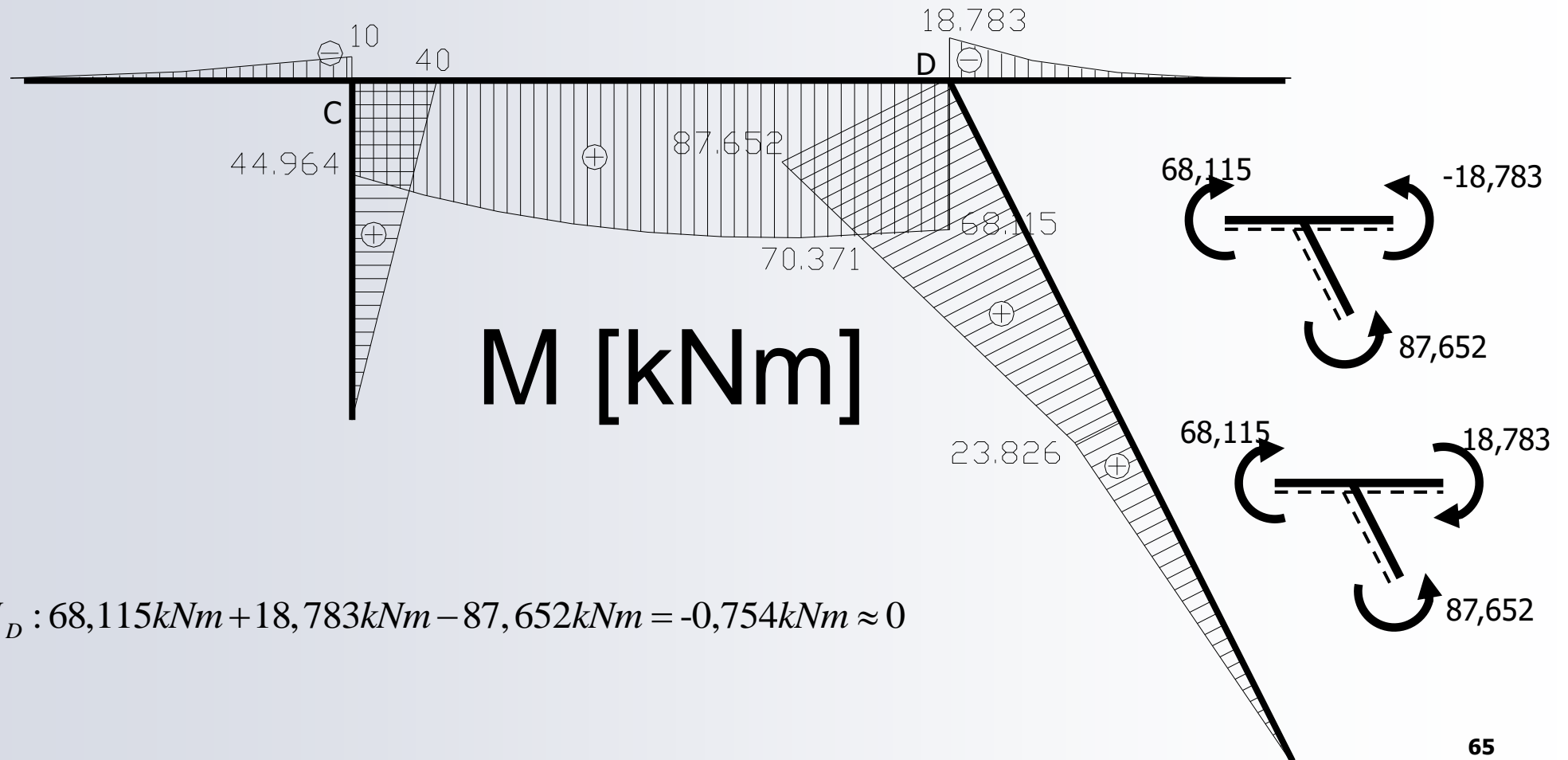


# Sprawdzenie w węźle C – momenty węzłowe



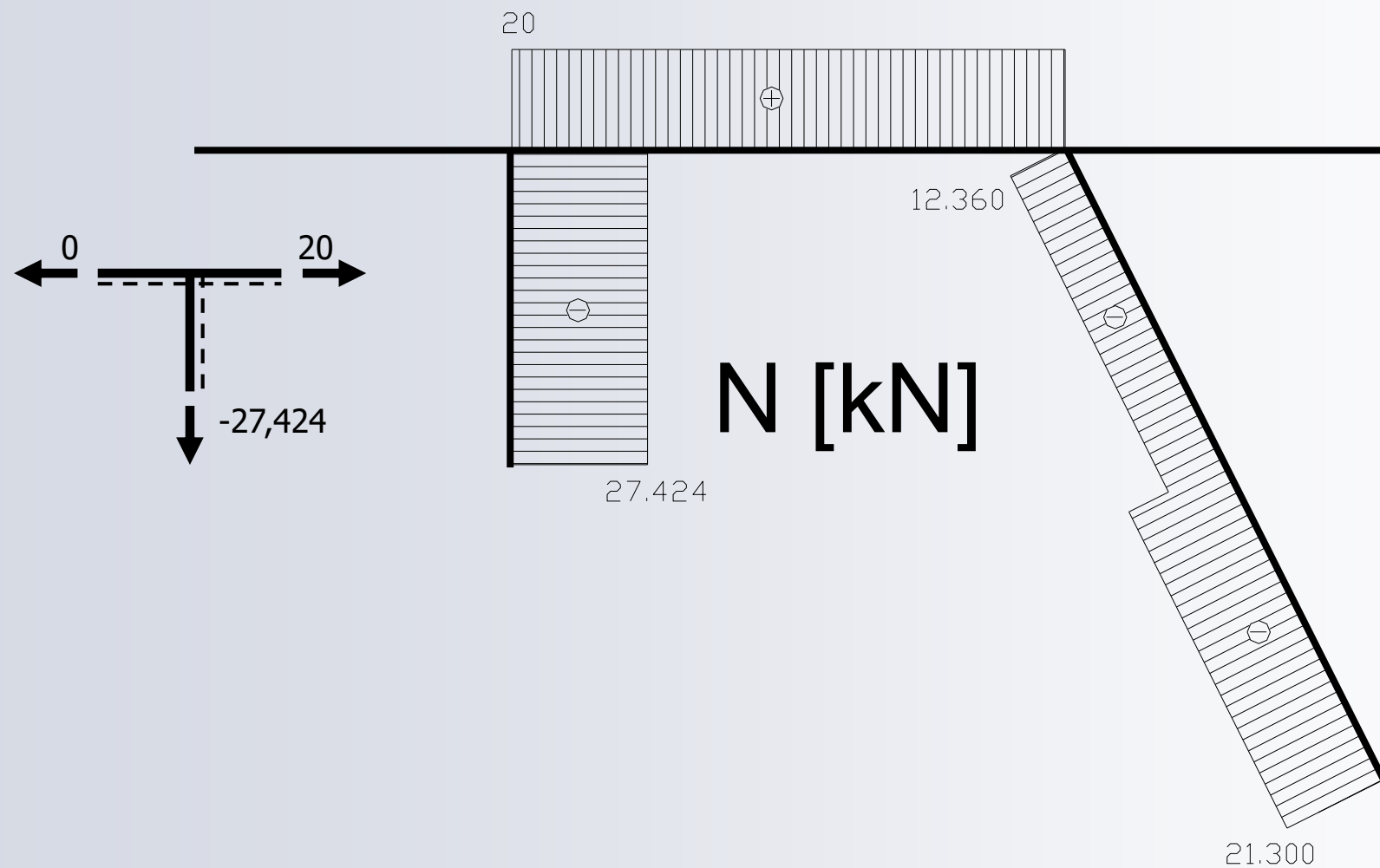


# Sprawdzenie w węźle D – momenty węzłowe

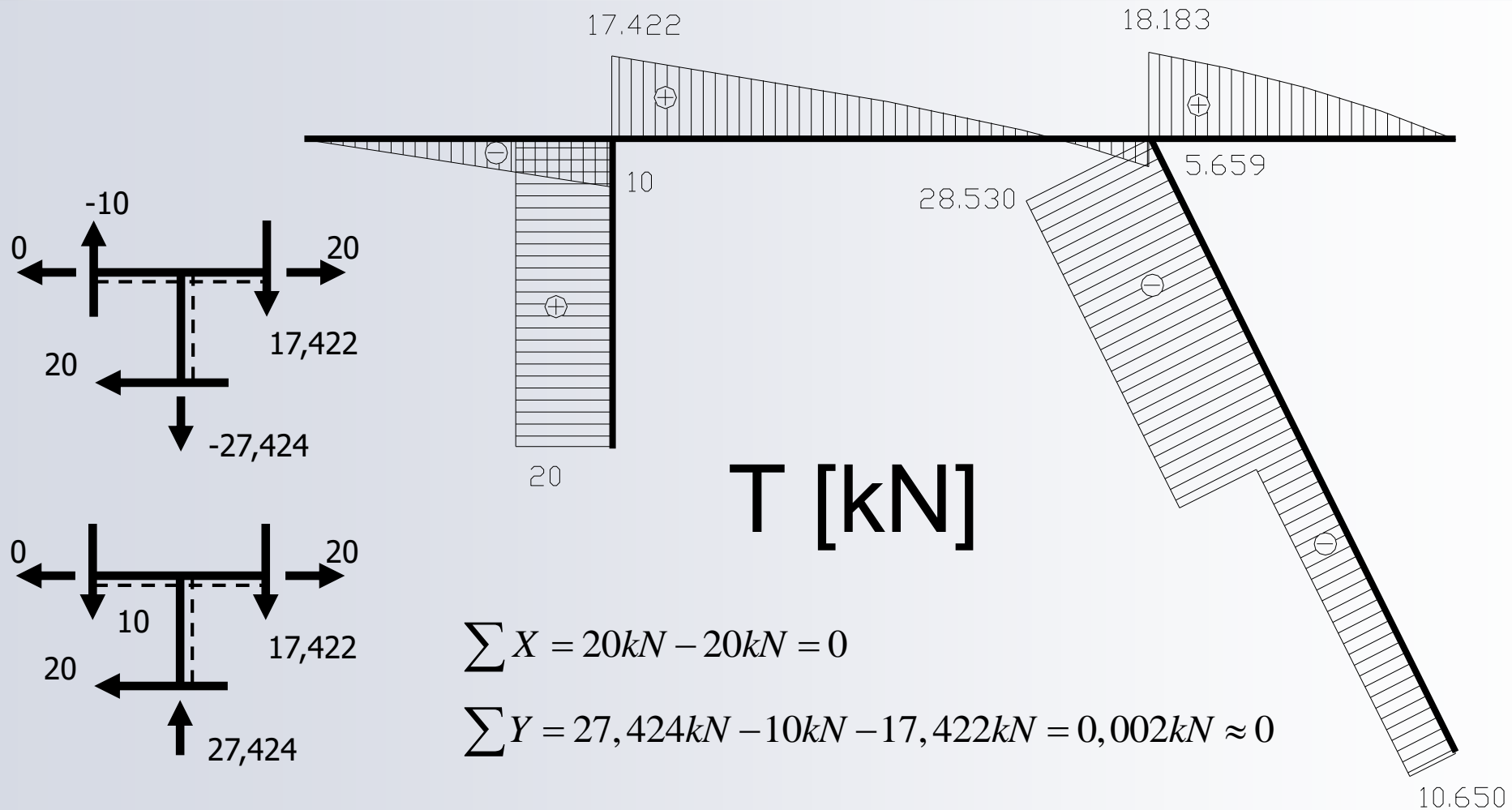


$$\sum M_D : 68,115kNm + 18,783kNm - 87,652kNm = -0,754kNm \approx 0$$

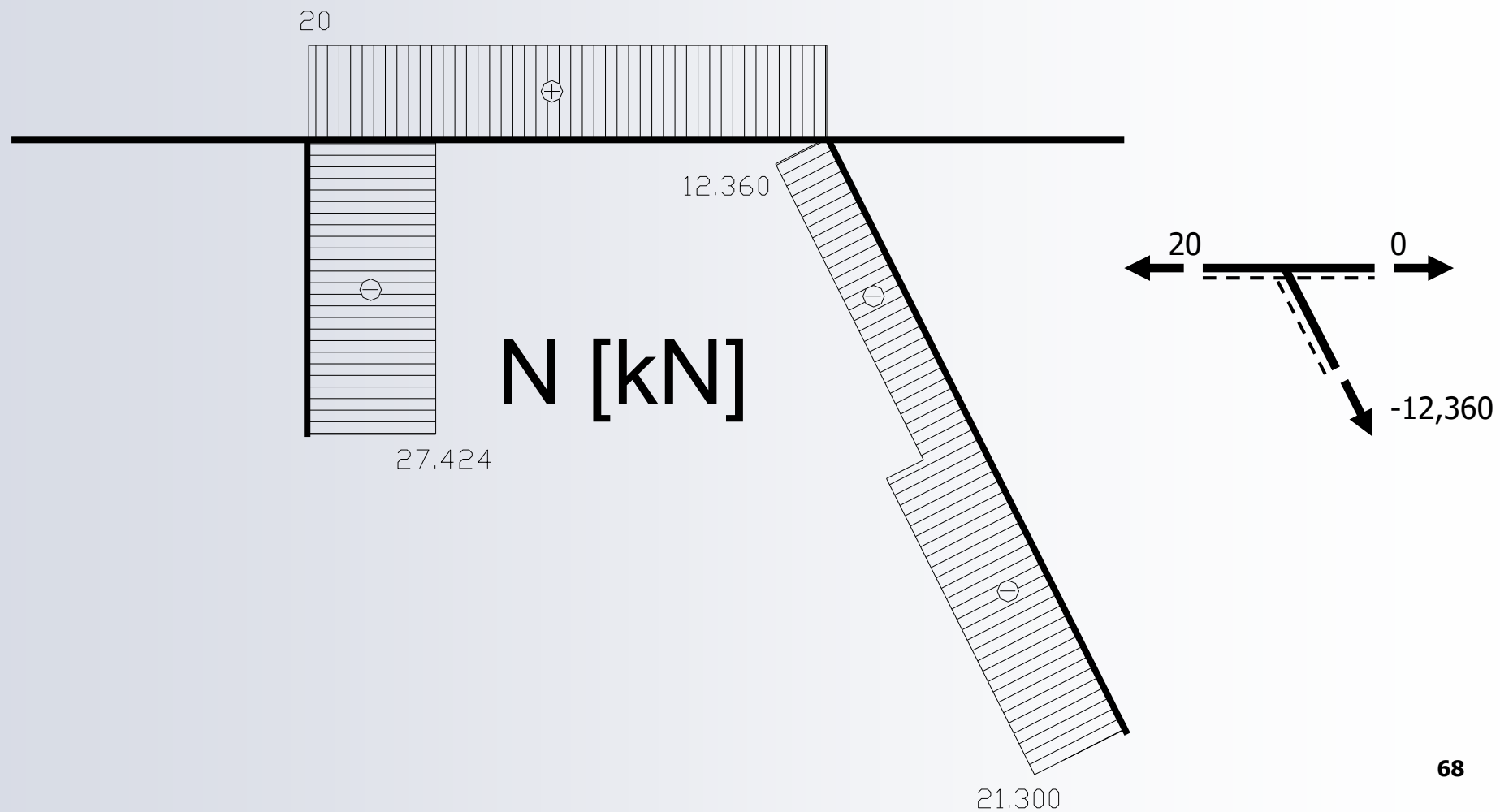
# Węzeł C – normalne



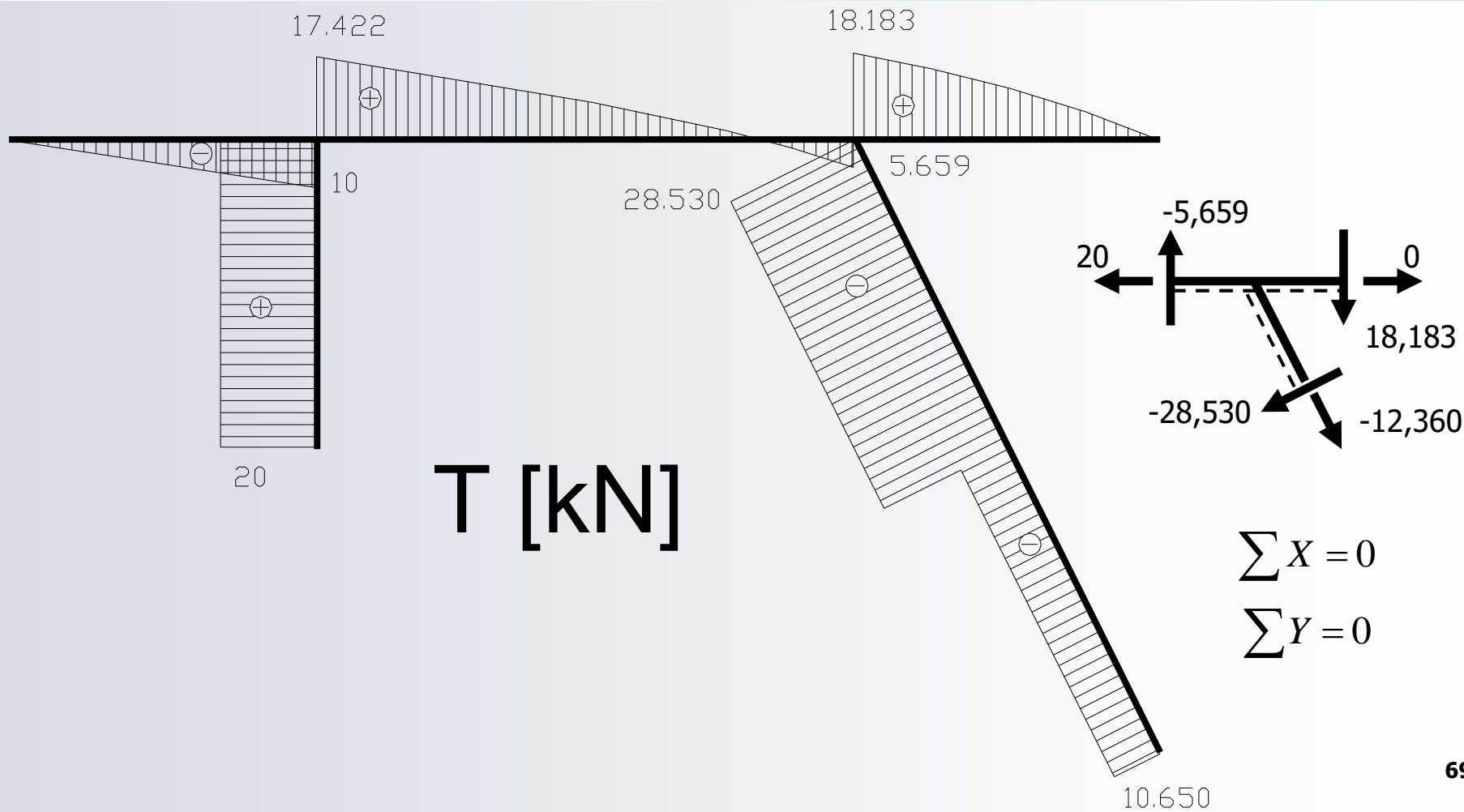
# Sprawdzenie w węźle C – normalne i tnące



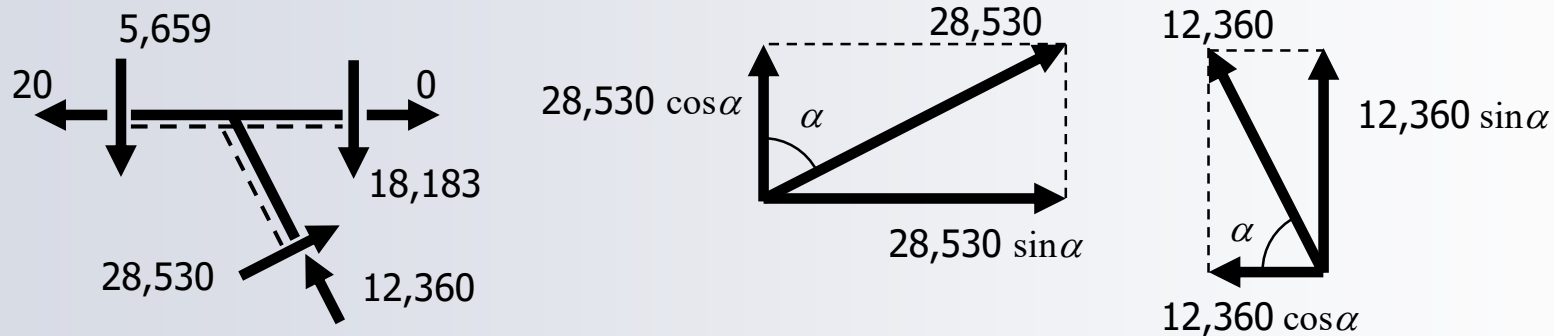
# Węzeł D – normalne



# Sprawdzenie w węźle D – normalne i tnące



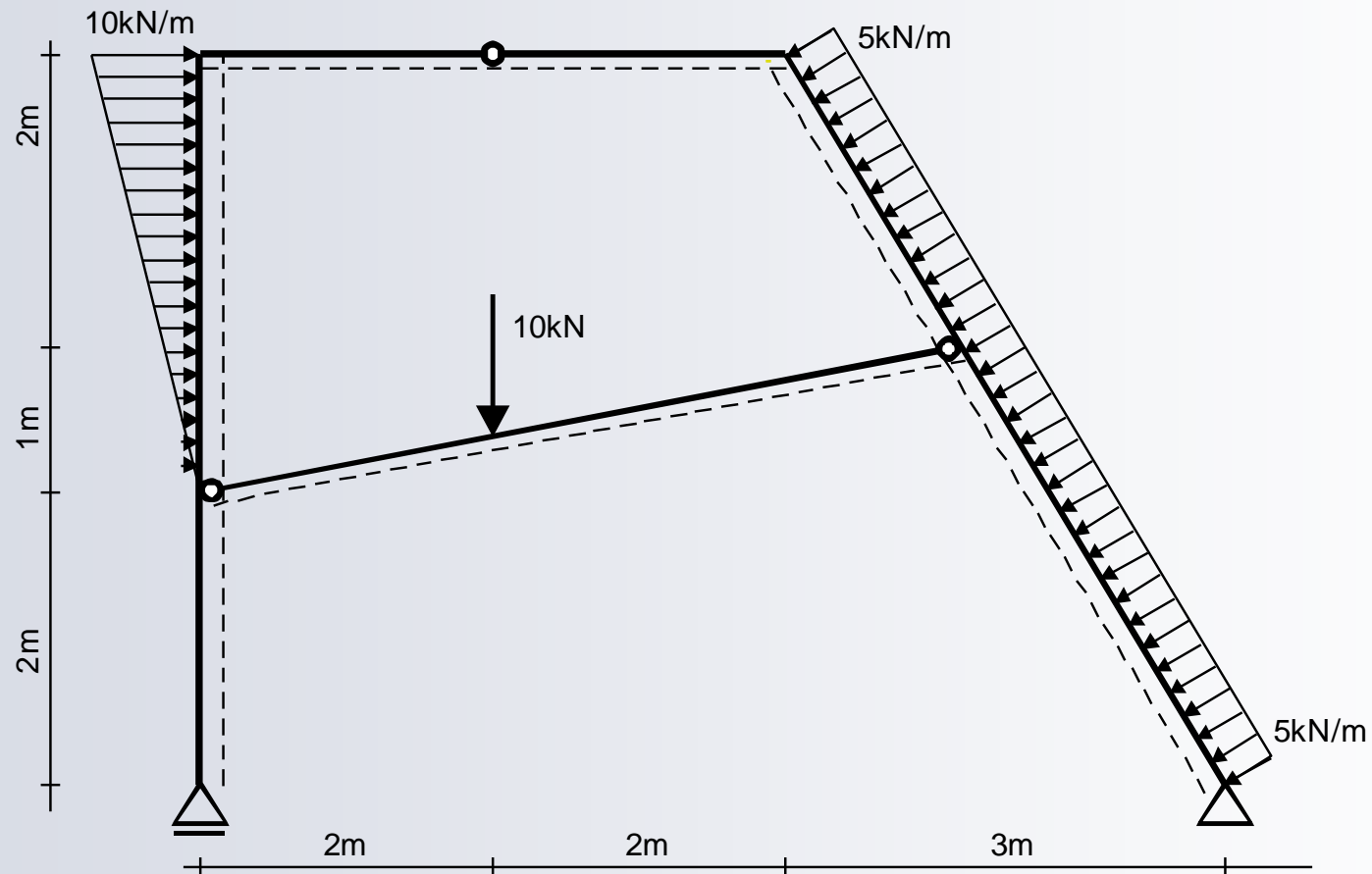
# Sprawdzenie w węźle D – normalne i tnące



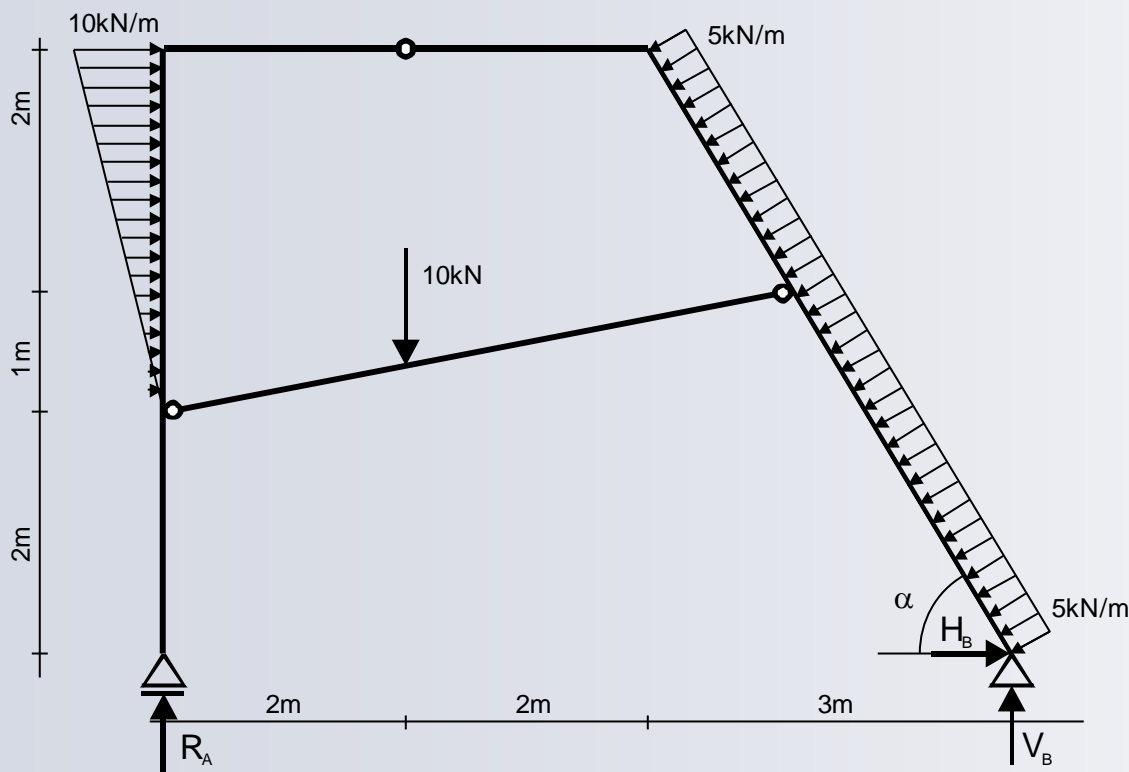
$$\begin{aligned}\sum X &= -20kN + 28,530kN \sin \alpha - 12,360kN \cos \alpha = \\ &= -20kN + 28,530kN \cdot 0,894 - 12,360kN \cdot 0,447 = -0,019kN \approx 0\end{aligned}$$

$$\begin{aligned}\sum Y &= -5,659kN - 18,183kN + 28,530kN \cos \alpha + 12,360kN \sin \alpha = \\ &= -5,659kN - 18,183kN + 28,530kN \cdot 0,447 + 12,360kN \cdot 0,894 = 0,039kN \approx 0\end{aligned}$$

# Przykład – rama ze ściągiem



# Reakcje podporowe



$$\sin \alpha = \frac{5m}{\sqrt{(3m)^2 + (5m)^2}} = 0,857$$

$$\cos \alpha = \frac{3m}{\sqrt{(3m)^2 + (5m)^2}} = 0,514$$



# Reakcje podporowe

$$\sum X : H_B + \frac{1}{2} 10 \frac{kN}{m} \cdot 3m - 5 \frac{kN}{m} \cdot \sqrt{(3m)^2 + (5m)^2} \sin \alpha = 0$$

$$H_B = 9,986kN$$

$$\sum Y : R_A + V_B - 10kN - 5 \frac{kN}{m} \cdot \sqrt{(3m)^2 + (5m)^2} \cos \alpha = 0$$

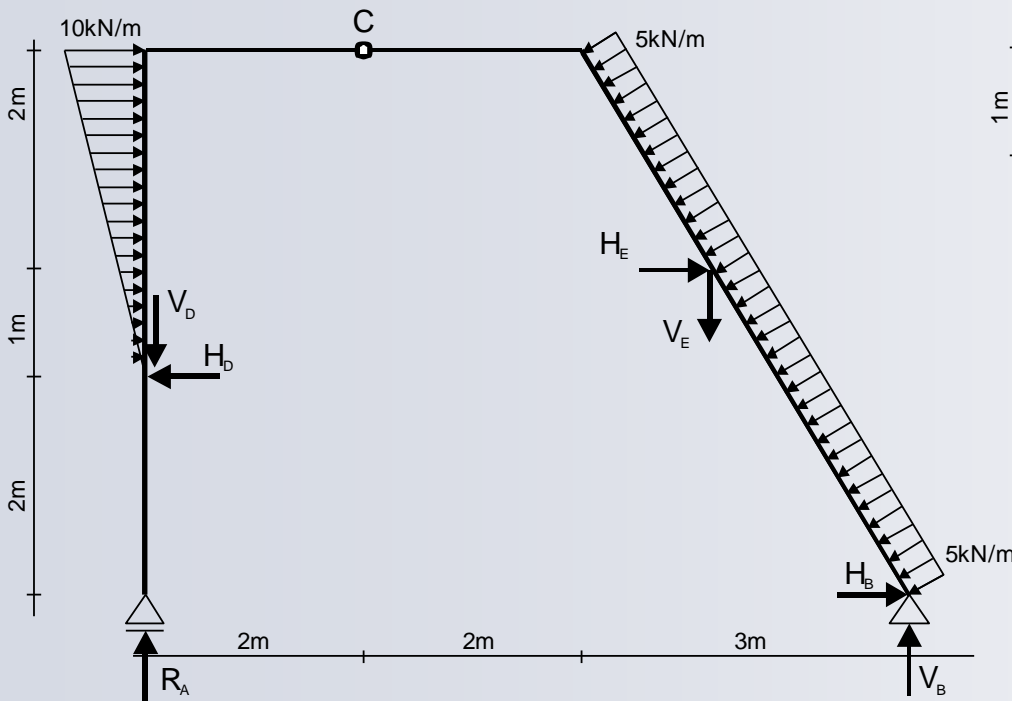
$$V_B = 14,272kN$$

$$\sum M_B : R_A \cdot 7m + \frac{1}{2} 10 \frac{kN}{m} \cdot 3m \cdot \left( 2m + \frac{2}{3} 3m \right) - 10kN \cdot 5m +$$

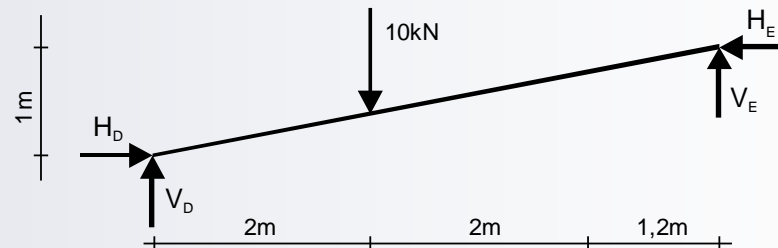
$$R_A = 10,714kN$$

$$- 5 \frac{kN}{m} \cdot \sqrt{(3m)^2 + (5m)^2} \cdot \frac{1}{2} \sqrt{(3m)^2 + (5m)^2} = 0$$

# Reakcje w przegubach ściągu



$$\sum M_C^I : R_A \cdot 2m - V_D \cdot 2m + H_D \cdot 3m - \frac{1}{2} 10 \frac{kN}{m} \cdot 3m \cdot \frac{1}{3} 3m = 0$$



$$\sum X : H_D - H_E = 0$$

$$\sum Y : V_D + V_E - 10kN = 0$$

$$\sum M_D : V_E \cdot 5,2m + H_E \cdot 1m - 10kN \cdot 2m = 0$$

$$V_D = 6,586kN$$

$$V_E = 3,414kN$$

$$H_D = 2,247kN$$

$$H_E = 2,247kN$$

# Przyjęcie przekrojów, przedziały

$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$\alpha_2 - \alpha_2 \quad y \in \langle 2m; 5m \rangle$$

$$\alpha_3 - \alpha_3 \quad x \in \langle 0; 4m \rangle$$

$$\alpha_4 - \alpha_4 \quad x \in \langle 4m; 5,2m \rangle \quad y \in \langle 3m; 5m \rangle$$

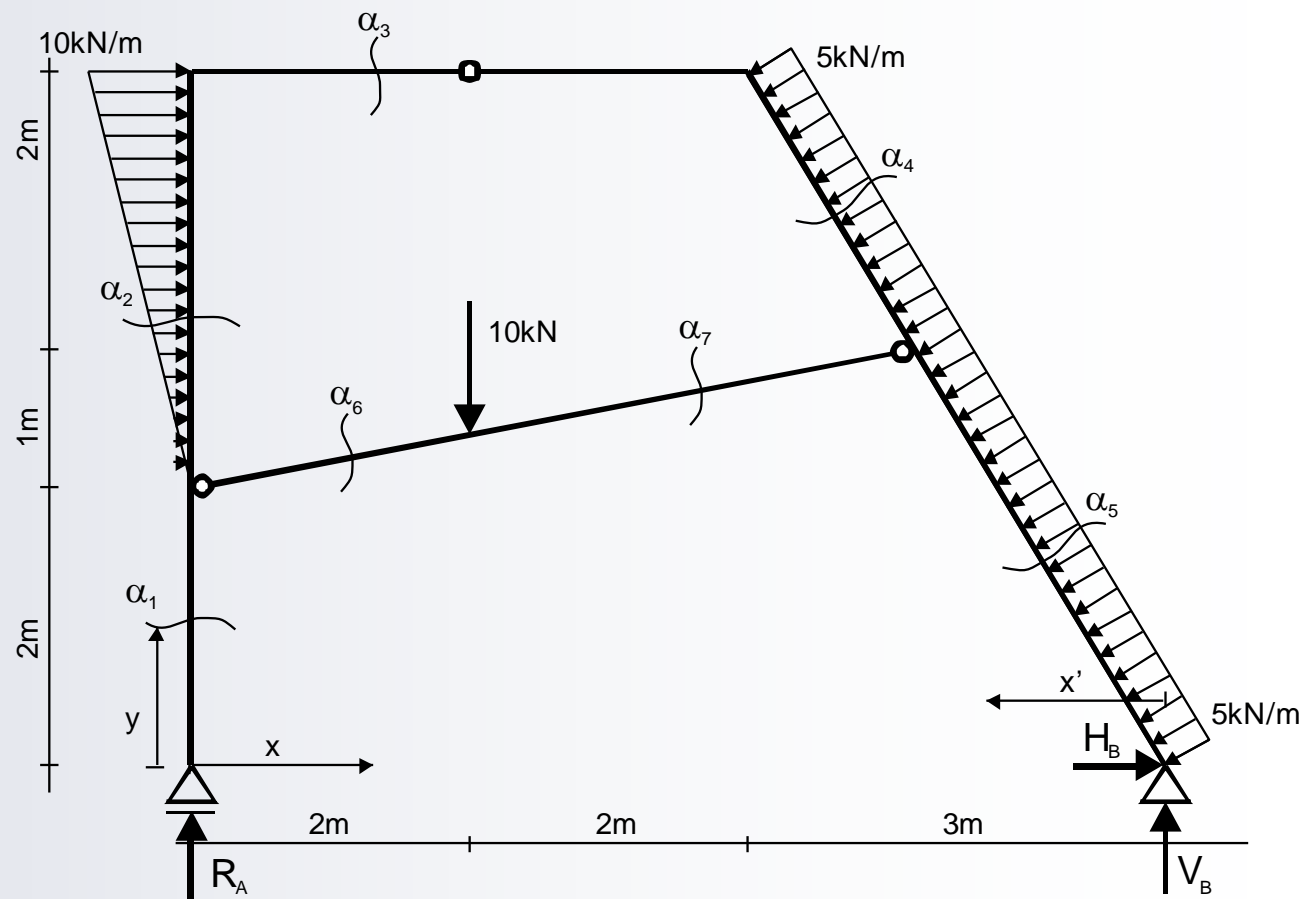
$$x' \in \langle 1,8m; 3m \rangle$$

$$\alpha_5 - \alpha_5 \quad x \in \langle 5,2m; 7m \rangle \quad y \in \langle 0; 3m \rangle$$

$$x' \in \langle 0; 1,8m \rangle$$

$$\alpha_6 - \alpha_6 \quad x \in \langle 0; 2m \rangle \quad y \in \langle 2m; 2,6m \rangle$$

$$\alpha_7 - \alpha_7 \quad x \in \langle 2m; 5,2m \rangle \quad y \in \langle 2,6m; 3m \rangle$$



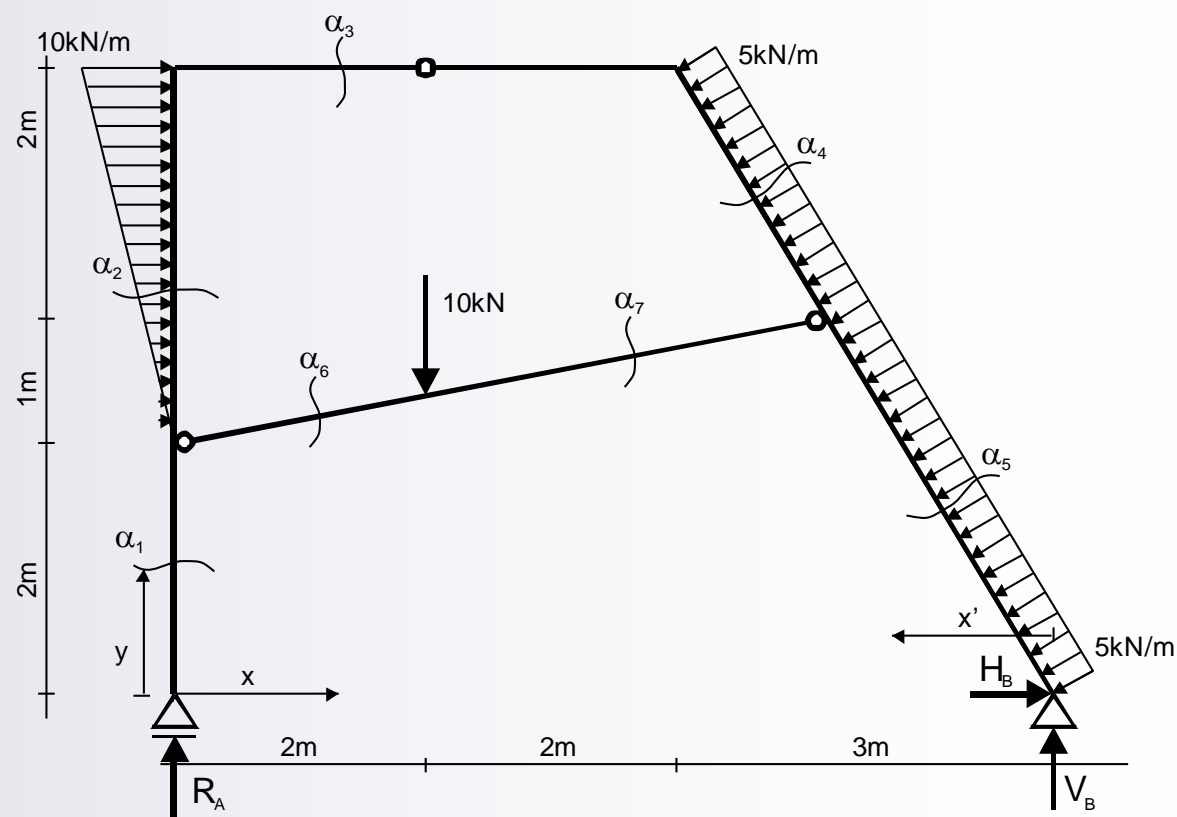
# Przekrój

$$\alpha_1 - \alpha_1 \quad y \in \langle 0; 2m \rangle$$

$$N_{\alpha_1} = -R_A = -10,714kN$$

$$T_{\alpha_1} = 0$$

$$M_{\alpha_1} = 0$$



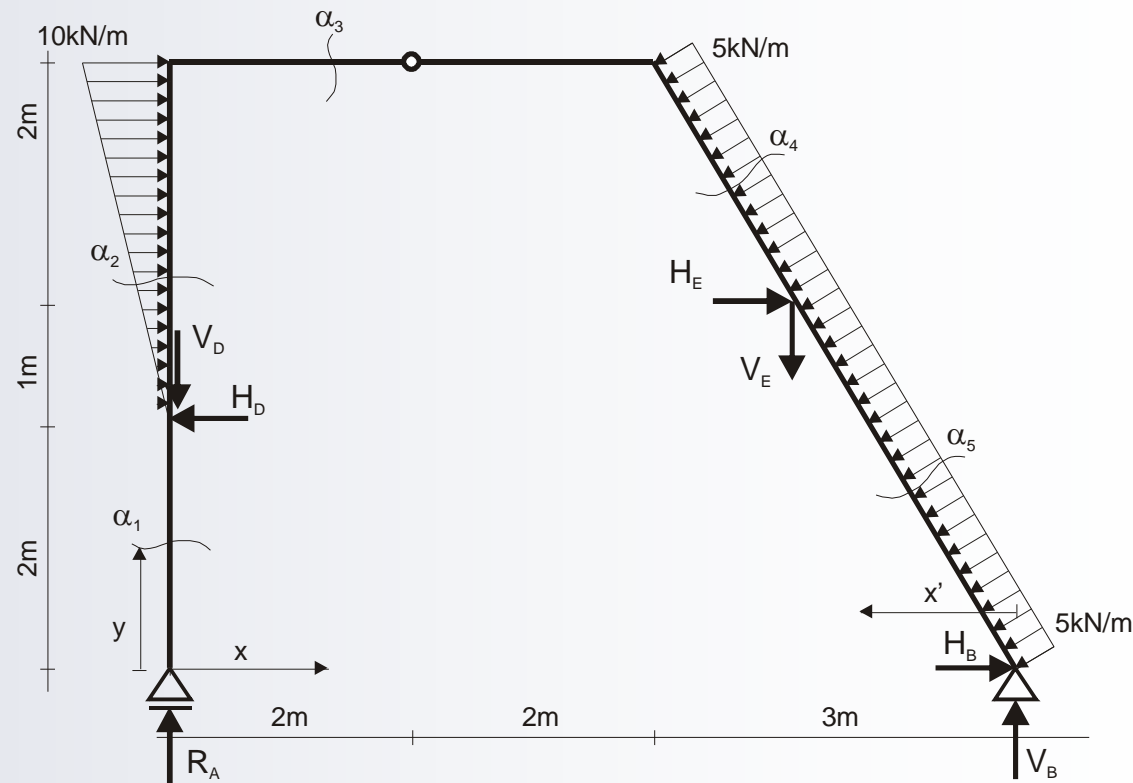
# Przekrój

$$\alpha_2 - \alpha_2 \quad y \in \langle 2m; 5m \rangle$$

$$N_{\alpha_2} = -R_A + V_D = -10,714kN + 6,586kN = -4,128kN$$

$$\frac{10 \frac{kN}{m}}{q'} = \frac{3m}{y-2m}$$

$$q' = 3,333 \frac{kN}{m^2} (y-2m)$$



# Przekrój

$$\alpha_2 - \alpha_2 \quad y \in \langle 2m; 5m \rangle$$

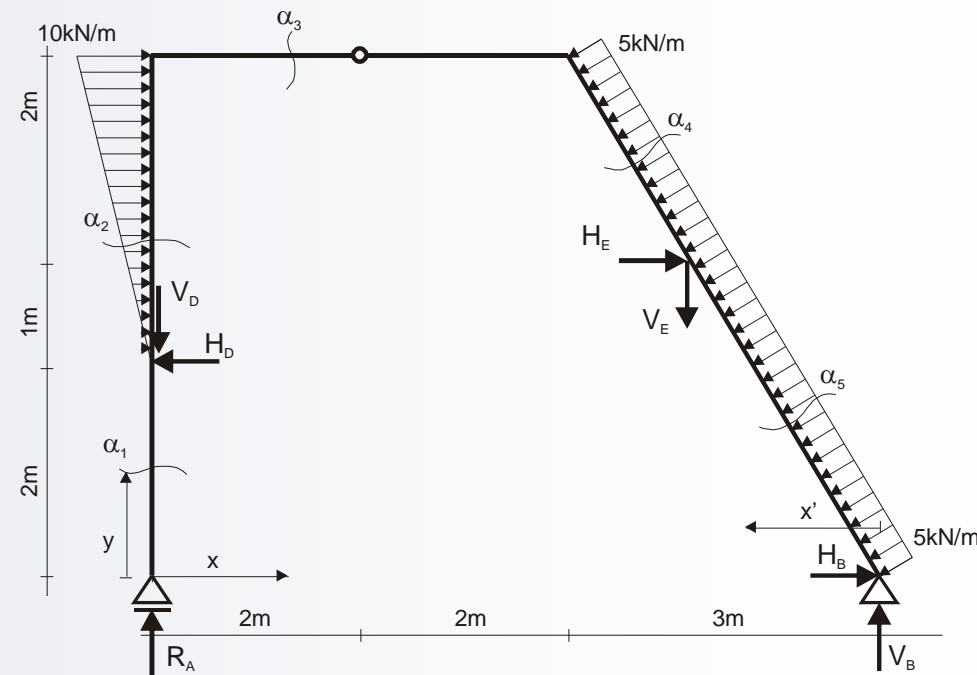
$$\begin{aligned} T_{\alpha_2} &= H_D - \frac{1}{2} q' \cdot (y - 2m) \\ &= 2,247kN - \frac{1}{2} 3,333 \frac{kN}{m^2} \cdot (y - 2m)^2 = \\ &= -1,667 \frac{kN}{m^2} y^2 + 6,666kN \cdot y - 4.419kN \end{aligned}$$

$$\left| \begin{array}{l} y = 2m \quad T_{\alpha_2} = 2,245kN \\ y = 5m \quad T_{\alpha_2} = -12,764kN \end{array} \right.$$

$$\begin{aligned} M_{\alpha_2} &= H_D \cdot (y - 2m) - \frac{1}{2} q' \cdot (y - 2m) \cdot \frac{1}{3} (y - 2m) = \\ &= 2,247kN \cdot (y - 2m) - \frac{1}{6} 3,333 \frac{kN}{m^2} (y - 2m)^3 = \end{aligned}$$

$$= -0,5555 \frac{kN}{m^2} \cdot y^3 + 3,333 \frac{kN}{m} \cdot y^2 - 4,419kN \cdot y - 0,050kNm$$

$$\left| \begin{array}{l} y = 2m \quad M_{\alpha_2} = 0 \\ y = 5m \quad M_{\alpha_2} = -8,258kNm \end{array} \right.$$



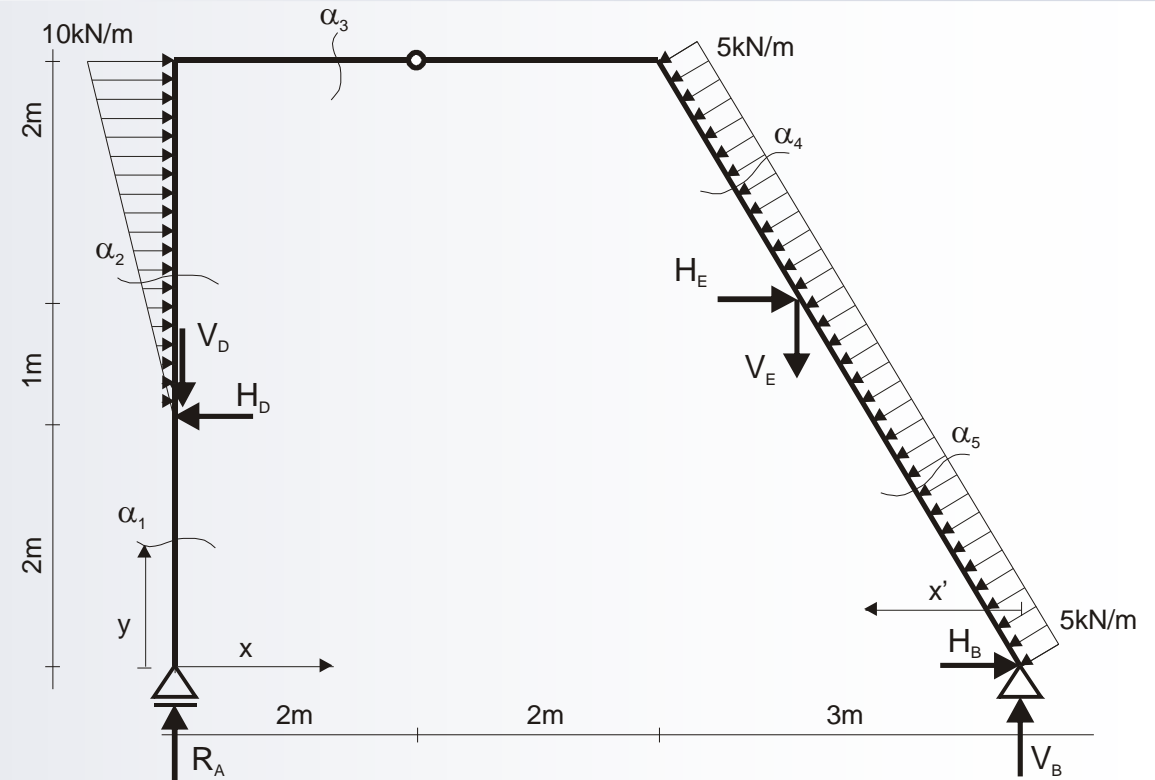
# Ekstremum

$$T_{\alpha_2} = -1,667 \frac{kN}{m^2} y^2 + 6,666 kN \cdot y - 4.419 kN = 0$$

$$y = 3.16m$$

$$M_{\alpha_2} = -0,5555 \frac{kN}{m^2} \cdot y^3 + 3,333 \frac{kN}{m} \cdot y^2 - 4,419 kN \cdot y - 0,050 kNm$$

$$M_{\alpha_2}(3,16m) = 1.739 kNm$$



# Przekrój

$$\alpha_3 - \alpha_3 \quad x \in \langle 0; 4m \rangle$$

$$\begin{aligned} N_{\alpha_3} &= H_D - \frac{1}{2} 10 \frac{kN}{m} \cdot 3m = \\ &= 2,247kN - 15kN = -12,753kN \end{aligned}$$

$$T_{\alpha_3} = R_A - V_D = 10,714kN - 6,586kN = 4,128kN$$

$$\begin{aligned} M_{\alpha_3} &= R_A \cdot x - V_D \cdot x + H_D \cdot 3m - \frac{1}{2} 10 \frac{kN}{m} \cdot 3m \cdot \frac{1}{3} 3m = \\ &= 10,714kN \cdot x - 6,586kN \cdot x + 2,247kN \cdot 3m - 15kNm = \\ &= 4,128kN \cdot x - 8,259kNm \end{aligned}$$

$$x = 0m \quad M_{\alpha_3} = -8,259kNm$$

$$x = 2m \quad M_{\alpha_3} = -0,003kNm \approx 0$$

$$x = 4m \quad M_{\alpha_3} = 8,253kNm$$



# Przekrój

$$\alpha_5 - \alpha_5 \quad x' \in \langle 0m; 1,8m \rangle$$

$$\begin{aligned} N_{\alpha 5} &= H_B \cos \alpha - V_B \sin \alpha = \\ &= 9,986kN \cdot 0,514 - 14,272kN \cdot 0,857 = -7,098kN \end{aligned}$$

$$\begin{aligned} T_{\alpha 5} &= -H_B \sin \alpha - V_B \cos \alpha + 5 \frac{kN}{m} \sqrt{x'^2 + y^2} = \\ &= -9,986kN \cdot 0,857 - 14,272kN \cdot 0,514 + 5 \frac{kN}{m} \sqrt{x'^2 + y^2} = \\ &= -15,894kN + 5 \frac{kN}{m} \sqrt{x'^2 + y^2} = \\ &= -15,894kN + 9,720 \frac{kN}{m} x' \end{aligned} \quad \left| \begin{array}{l} x' = 0m \quad T_{\alpha 5} = -15,894kN \\ x' = 1,8m \quad T_{\alpha 5} = 1,602kN \end{array} \right.$$

# Przekrój

$$\alpha_5 - \alpha_5 \quad x' \in \langle 0m; 1,8m \rangle$$

$$\begin{aligned} M_{\alpha 5} &= V_B \cdot x' + H_B \cdot y - 5 \frac{kN}{m} \sqrt{x'^2 + y^2} \cdot \frac{1}{2} \sqrt{x'^2 + y^2} = \\ &= 14,272kN \cdot x' + 9,986kN \cdot y - 2,5 \frac{kN}{m} (x'^2 + y^2) = \\ &= 14,272kN \cdot x' + 9,986kN \cdot 1,667x' - 2,5 \frac{kN}{m} (x'^2 + (1,667x')^2) = \\ &= -9,447 \frac{kN}{m} x'^2 + 30,919kN \cdot x' \end{aligned}$$

$$\left| \begin{array}{ll} x' = 0 & M_{\alpha 5} = 0 \\ x' = 1,8m & M_{\alpha 5} = 25,046kNm \end{array} \right.$$

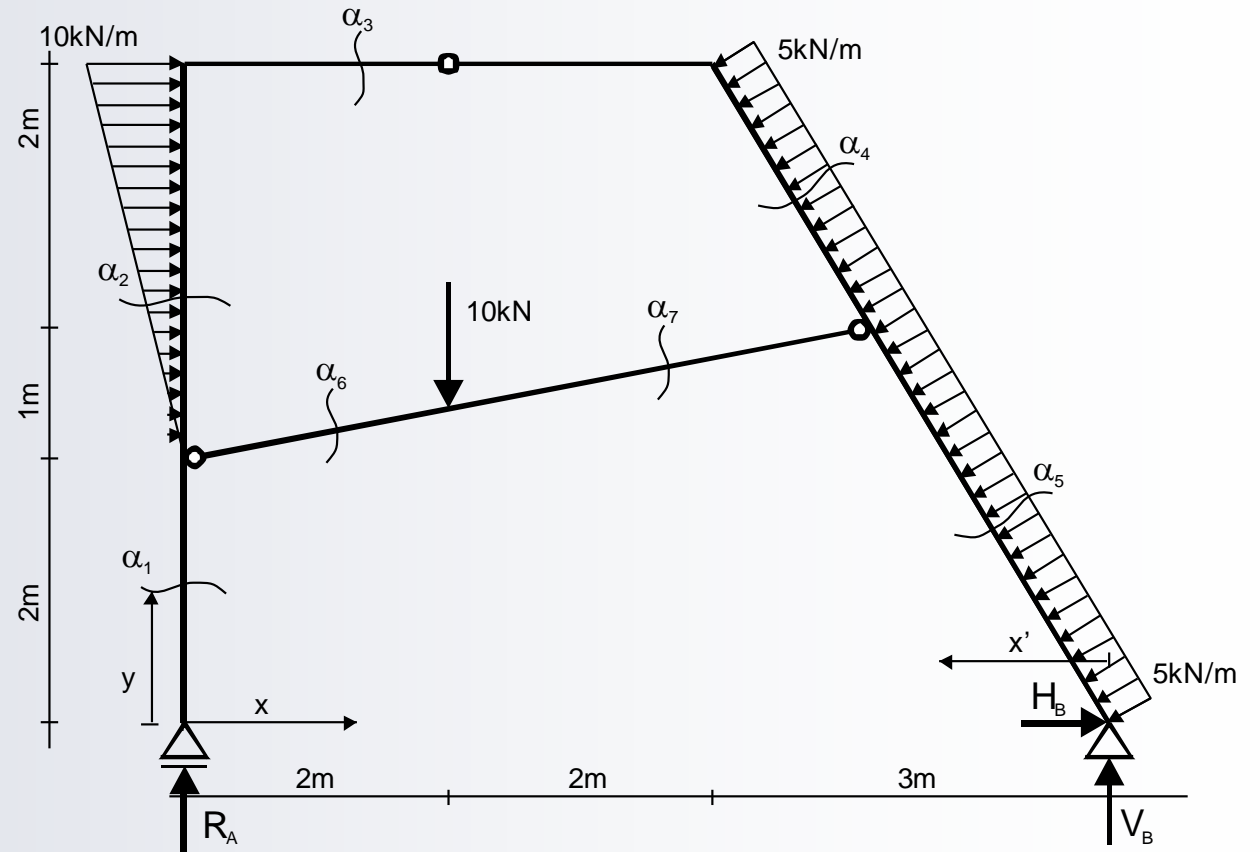
# Ekstremum

$$T_{\alpha_5} = -15,894kN + 9,720 \frac{kN}{m} x' = 0$$

$$x' = 1.635m$$

$$M_{\alpha_5} = -9,447 \frac{kN}{m} x'^2 + 30,919kN \cdot x'$$

$$M_{\alpha_5}(1.635m) = 25.299kNm$$



# Przekrój $\alpha_4 - \alpha_4$ $x' \in \langle 1,8m; 3m \rangle$

$$\begin{aligned}N_{\alpha_4} &= -7,098kN + H_E \cos \alpha + V_E \sin \alpha = \\ &= -7,098kN + 2,247kN \cdot 0,514 + 3,414kN \cdot 0,857 = \\ &= -3,008kN\end{aligned}$$

$$\begin{aligned}T_{\alpha_4} &= -15,894kN + 9,720 \frac{kN}{m} x' - H_E \sin \alpha + V_E \cos \alpha = \\ &= -15,894kN + 9,720 \frac{kN}{m} x' - 2,247kN \cdot 0,857 + 3,414kN \cdot 0,514 =\end{aligned}$$

$$= 9,720 \frac{kN}{m} x' - 16,065kN$$

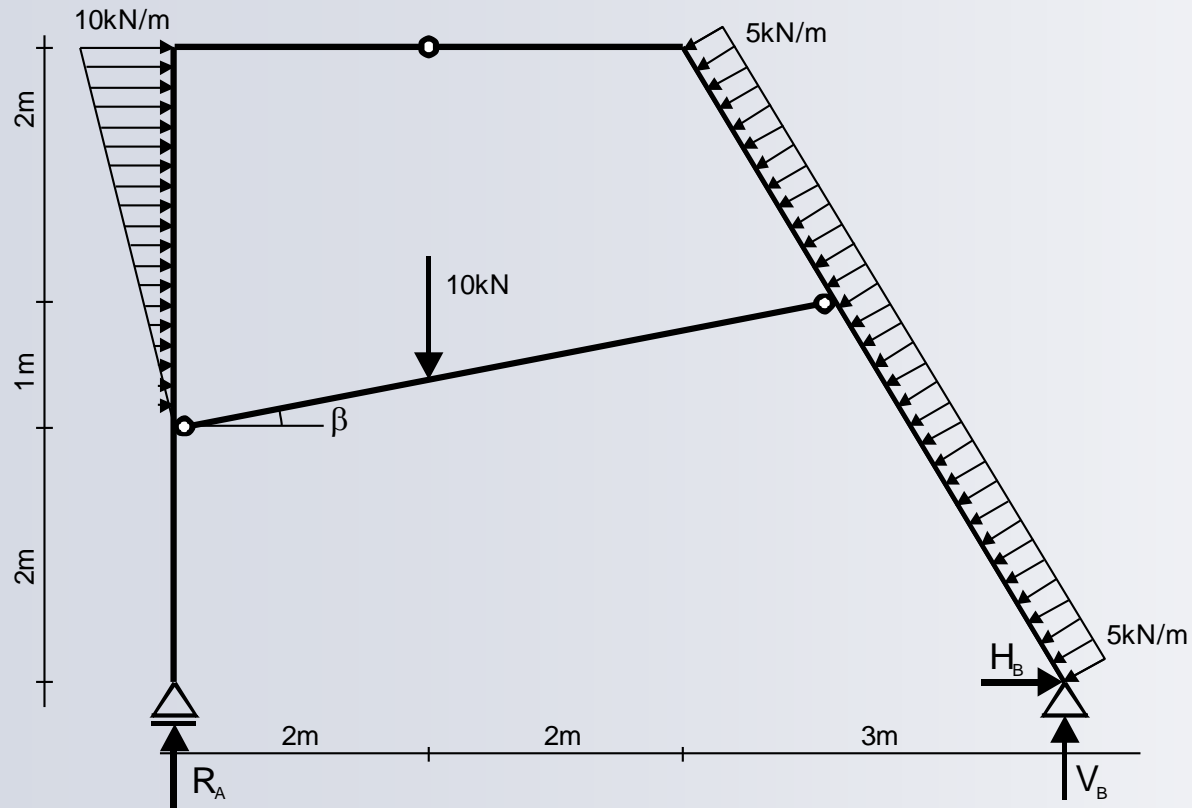
$$\left| \begin{array}{l} x' = 0 \quad T_{\alpha_4} = 1,431kN \\ x' = 1,8m \quad T_{\alpha_4} = 13,095kN \end{array} \right.$$

# Przekrój $\alpha_4 - \alpha_4$ $x' \in \langle 1,8m; 3m \rangle$

$$\begin{aligned} M_{\alpha_4} &= -9,447 \frac{kN}{m} x'^2 + 30,919kN \cdot x' + H_E \cdot (y - 3m) - V_E \cdot (x' - 1,8m) = \\ &= -9,447 \frac{kN}{m} x'^2 + 30,919kN \cdot x' + 2,247kN (1,667x' - 3m) + \\ &\quad - 3,414kN \cdot (x' - 1,8m) = \\ &= -9,447 \frac{kN}{m} x'^2 + 31,251kN \cdot x' - 0,596kNm \end{aligned}$$

$$\left| \begin{array}{l} x' = 1,8m \quad M_{\alpha_4} = 25,048kNm \\ x' = 3m \quad M_{\alpha_4} = 8,134kNm \end{array} \right.$$

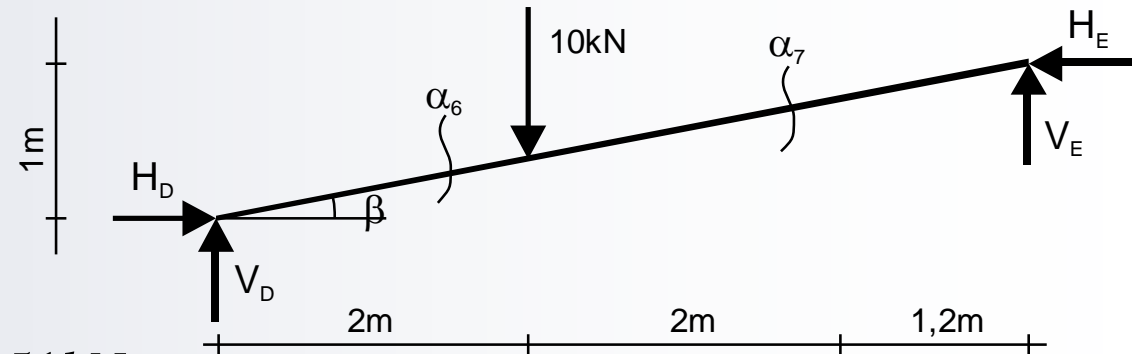
# Przekrój $\alpha_6 - \alpha_6$ $x \in \langle 0; 2m \rangle$



$$\sin \beta = \frac{1m}{\sqrt{(1m)^2 + (5,2m)^2}} = 0,189$$

$$\cos \beta = \frac{5,2m}{\sqrt{(1m)^2 + (5,2m)^2}} = 0,982$$

# Przekrój $\alpha_6 - \alpha_6$ $x \in \langle 0; 2m \rangle$



$$N_{\alpha_6} = -H_D \cos \beta - V_D \sin \beta =$$

$$= -2,247kN \cdot 0,982 - 6,586kN \cdot 0,189 = -3,451kN$$

$$T_{\alpha_6} = -H_D \sin \beta + V_D \cos \beta =$$

$$= -2,247kN \cdot 0,189 + 6,586kN \cdot 0,982 = 6,043kN$$

$$M_{\alpha_6} = -H_D \cdot (y - 2m) + V_D \cdot x =$$

$$= -2,247kN \cdot \left( \frac{x}{5,2} + 2m - 2m \right) + 6,586kN \cdot x =$$

$$= -0,432x + 6,586x = 6,154x$$

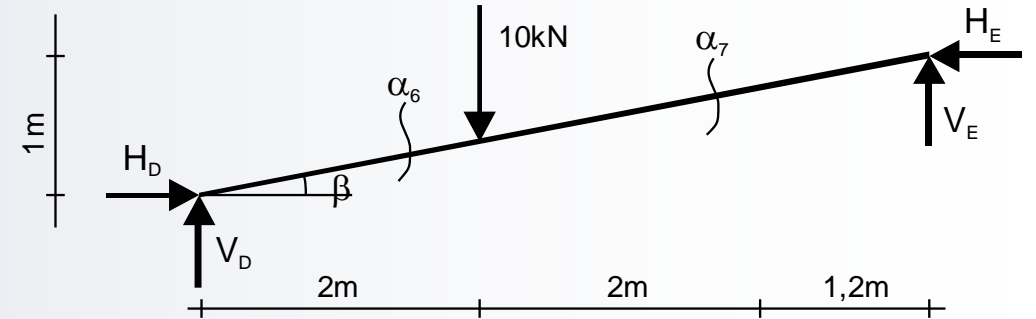
$$\left| \begin{array}{ll} x = 0m & M_{\alpha_6} = 0 \\ x = 2m & M_{\alpha_6} = 12,308kNm \end{array} \right.$$

# Przekrój $\alpha_7 - \alpha_7$ $x \in \langle 2m; 5,2m \rangle$

$$\begin{aligned} N_{\alpha_7} &= -3,451kN + 10kN \sin \beta = \\ &= -3,451kN + 10kN \cdot 0,189 = -1,561kN \end{aligned}$$

$$\begin{aligned} T_{\alpha_7} &= 6,043kN - 10kN \cos \beta = \\ &= 6,043kN - 10kN \cdot 0,982 = -3,777kN \end{aligned}$$

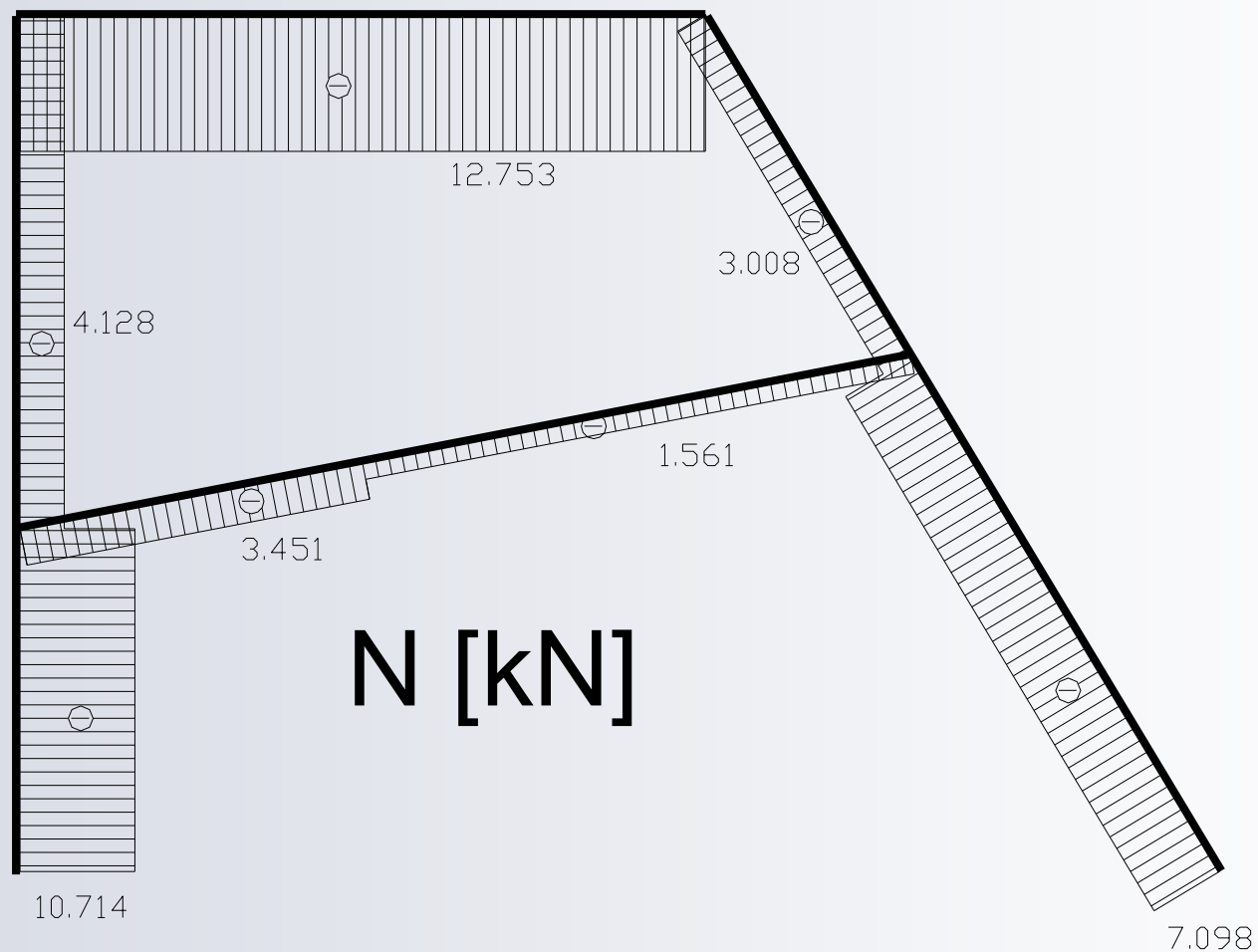
$$\begin{aligned} M_{\alpha_7} &= 6,154x - 10kN \cdot (x - 2m) = \\ &= -3,846x + 20kNm \end{aligned}$$



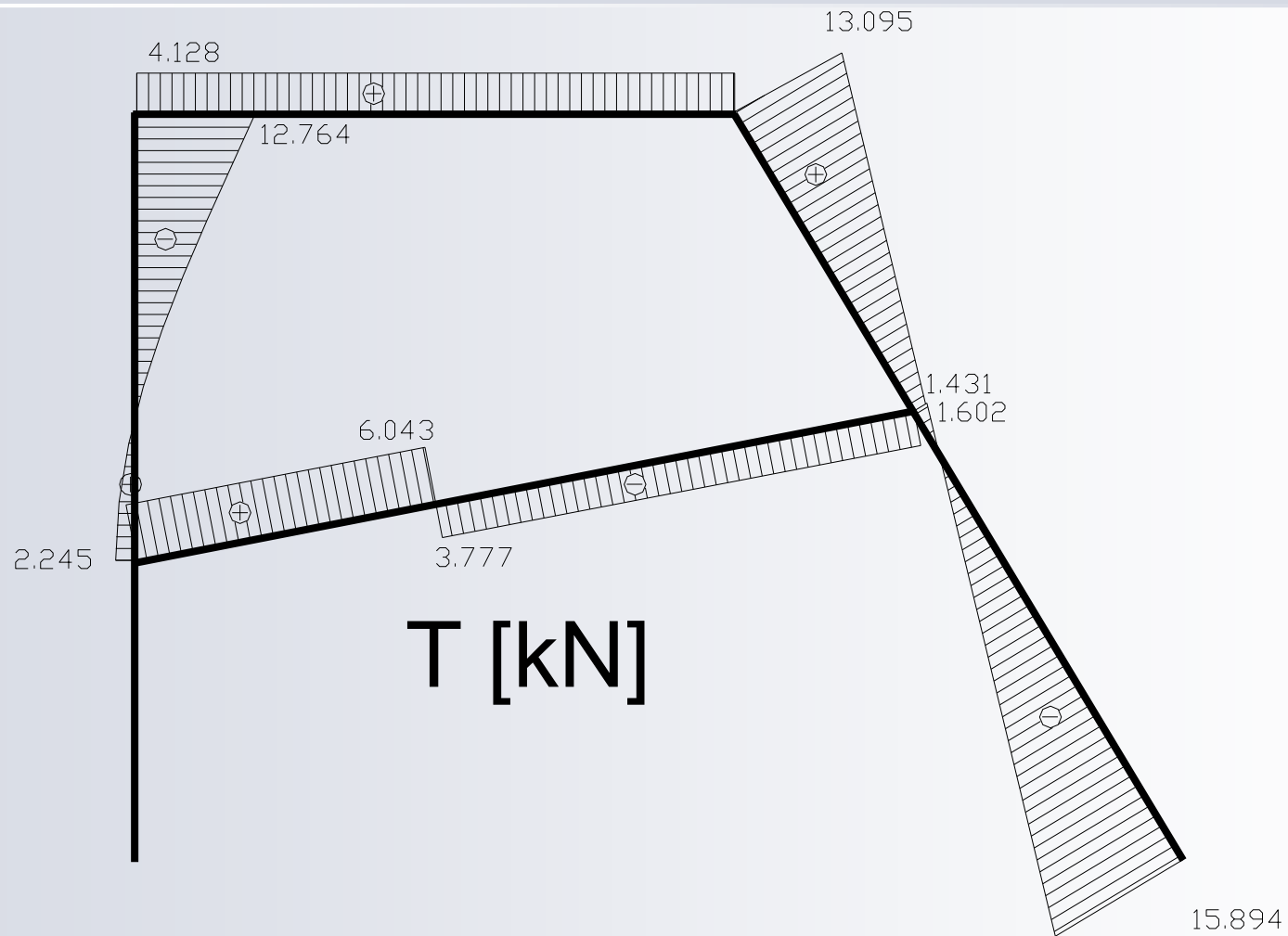
$$\left| \begin{array}{ll} x = 2m & M_{\alpha_7} = 12,308kNm \\ x = 5,2m & M_{\alpha_7} = 0,001kNm \approx 0 \end{array} \right.$$



# Wykres sił normalnych



# Wykres sił tnących



# Wykres momentów zginających

