Mechanika teoretyczna

Wykład nr 6

Układy przestrzenne.

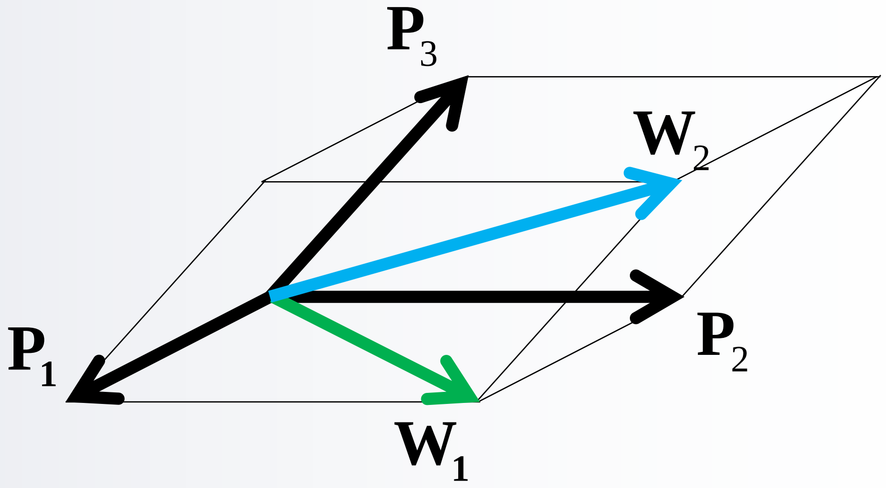
Wypadkowa przestrzennego układu sił zbieżnych

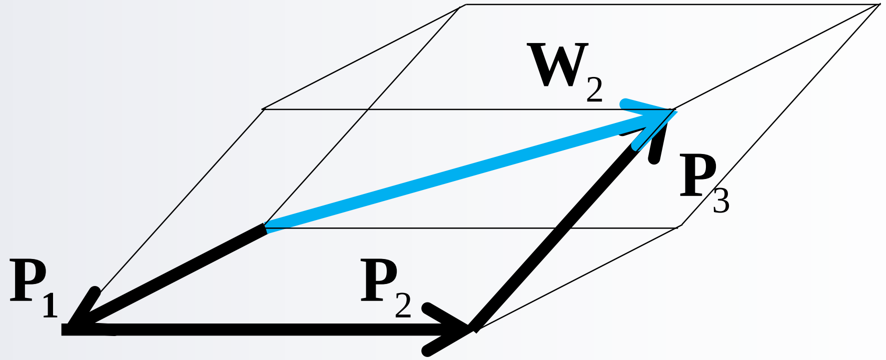
- Metoda graficzna:

$$\mathbf{W_1} = \mathbf{P_1} + \mathbf{P}_2$$

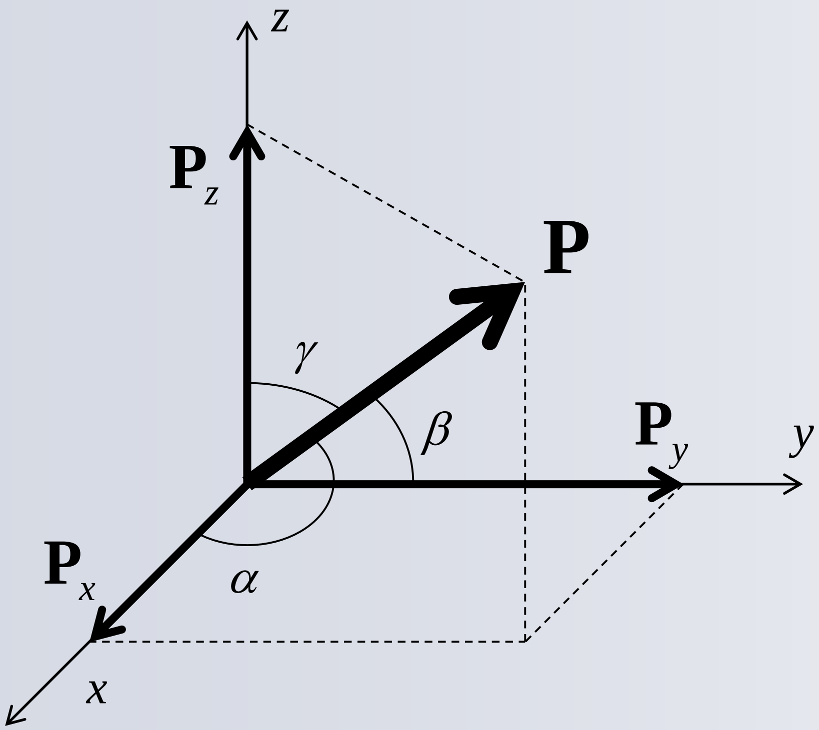
$$\mathbf{W}_2 = \mathbf{W_1} + \mathbf{P}_3 = \mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3$$

$$\mathbf{W} = \sum_{i=1}^{n} \mathbf{P}_i$$





Analityczna metoda wyznaczania wypadkowej



$$P = \sqrt{P_x^{\,2} + P_y^{\,2} + P_z^{\,2}}$$

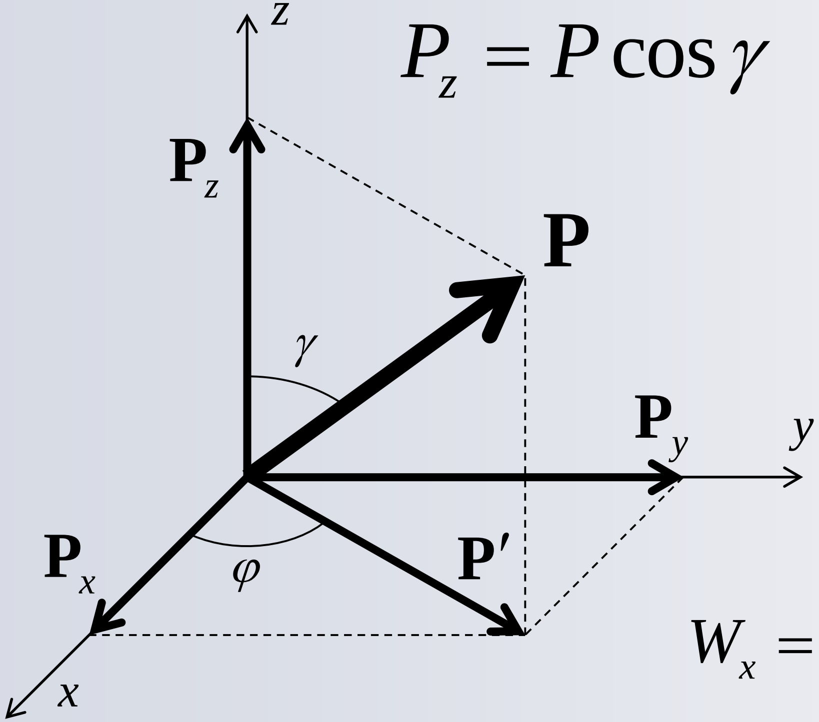
$$\cos\alpha = \frac{P_x}{P} = \frac{P_x}{\sqrt{P_x^{\,2} + P_y^{\,2} + P_z^{\,2}}}$$

$$\cos\beta = \frac{P_y}{P} = \frac{P_y}{\sqrt{P_x^{\,2} + P_y^{\,2} + P_z^{\,2}}}$$

$$\cos\gamma = \frac{P_z}{P} = \frac{P_z}{\sqrt{P_x^{\,2} + P_y^{\,2} + P_z^{\,2}}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Analityczna metoda wyznaczania wypadkowej



$$P_z = P\cos\gamma \qquad P' = P\sin\gamma$$

$$P_x = P'\cos\varphi = P\sin\gamma\cos\varphi$$

$$P_y = P'\sin\varphi = P\sin\gamma\sin\varphi$$

$$\mathbf{W} = \sum_{i=1}^{n} \mathbf{P}_i$$

$$W_x = \sum_{i=1}^{n} P_{ix} \qquad W_y = \sum_{i=1}^{n} P_{iy} \qquad W_z = \sum_{i=1}^{n} P_{iz}$$

$$W = \sqrt{W_x^{\ 2} + W_y^{\ 2} + W_z^{\ 2}} \qquad \cos\alpha = \frac{W_x}{W} \qquad \cos\beta = \frac{W_y}{W} \qquad \cos\gamma = \frac{W_z}{W}$$

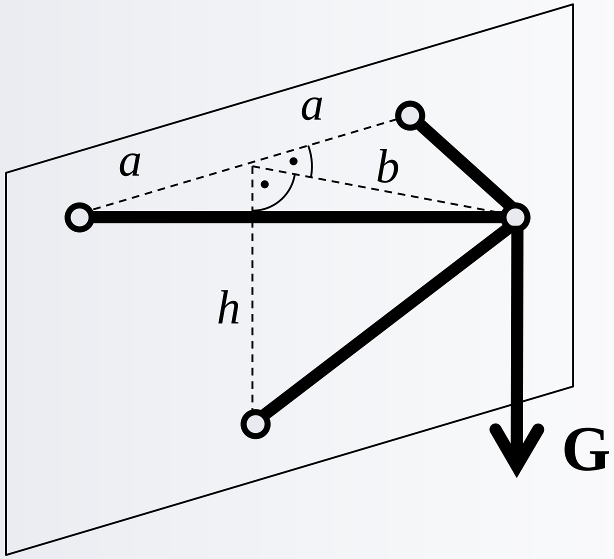
Warunki równowagi przestrzennego układu zbieżnego

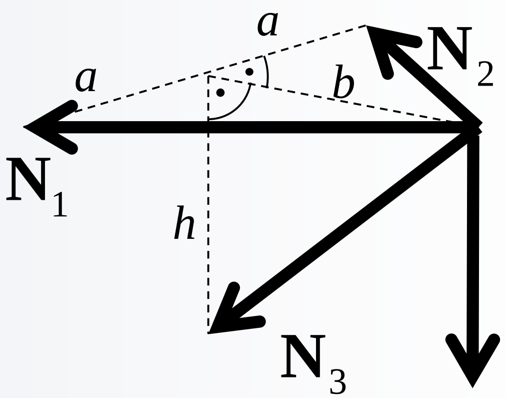
$$\mathbf{W} = \sum_{i=1}^{n} \mathbf{P}_i = \mathbf{0}$$

$$W_x = 0 \qquad \sum_{i=1}^{n} P_{ix} = \sum X = 0$$

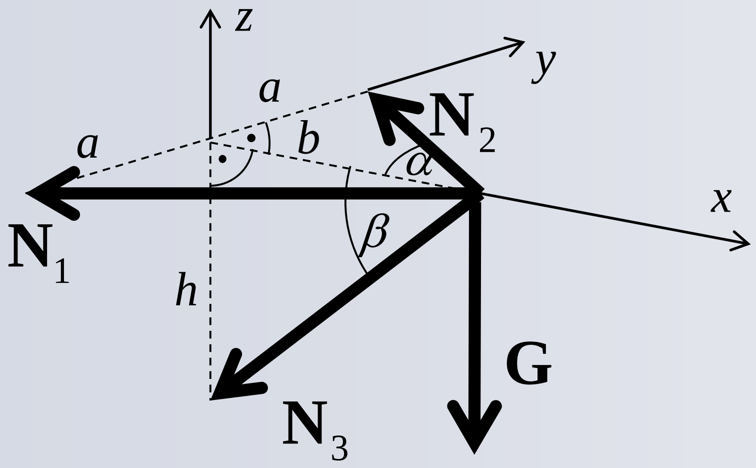
$$W_y = 0 \qquad \sum_{i=1}^{n} P_{iy} = \sum Y = 0$$

$$W_z = 0 \qquad \sum_{i=1}^{n} P_{iz} = \sum Z = 0$$





Układ zbieżny - przykład



$$\sum X = 0 \qquad N_1 \cos\alpha + N_2 \cos\alpha + N_3 \cos\beta = 0$$

$$\sum Y = 0 \qquad -N_1 \sin\alpha + N_2 \sin\alpha = 0 \qquad N_1 = N_2$$

$$\sum Z = 0 \qquad N_3 \sin\beta + G = 0 \qquad N_3 = -\frac{G}{\sin\beta} = -\frac{G}{h}\sqrt{h^2 + b^2}$$

$$2N_1 \frac{b}{\sqrt{a^2 + b^2}} - \frac{G}{h}\sqrt{h^2 + b^2}\,\frac{b}{\sqrt{h^2 + b^2}} = 0$$

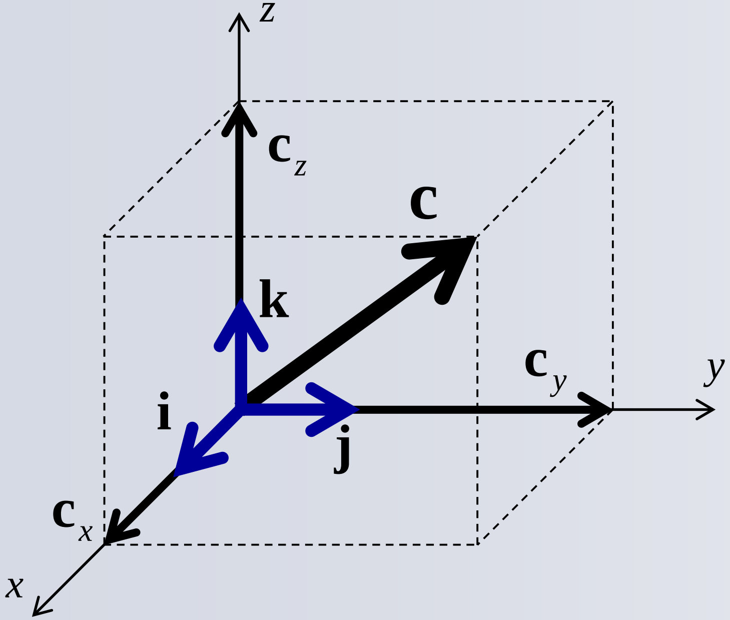
$$\sin\alpha = \frac{a}{\sqrt{a^2 + b^2}} \qquad \cos\alpha = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\sin\beta = \frac{h}{\sqrt{h^2 + b^2}} \qquad \cos\beta = \frac{b}{\sqrt{h^2 + b^2}}$$

$$N_1 = \frac{G}{2h}\sqrt{a^2 + b^2}$$

$$N_2 = \frac{G}{2h}\sqrt{a^2 + b^2}$$

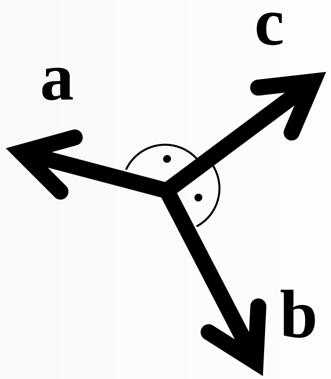
Iloczyn wektorowy



$$\mathbf{c}_x = \mathbf{i} \cdot c_x$$

$$\mathbf{c}_y = \mathbf{j} \cdot c_y$$

$$\mathbf{c}_z = \mathbf{k} \cdot c_z$$



$$\mathbf{c} = \mathbf{a} \times \mathbf{b} = \mathbf{i} \cdot c_x + \mathbf{j} \cdot c_y + \mathbf{k} \cdot c_z =$$

$$= \mathbf{i} \cdot \left(a_y b_z - a_z b_y\right) + \mathbf{j} \cdot \left(a_z b_x - a_x b_z\right) + \mathbf{k} \cdot \left(a_x b_y - a_y b_x\right) =$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

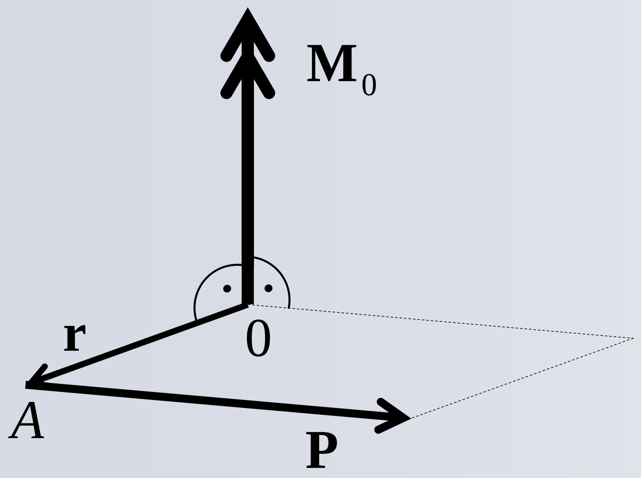
$$\mathbf{c} = \mathbf{a} \times \mathbf{b}$$

$$c_x = a_y b_z - a_z b_y$$

$$c_y = a_z b_x - a_x b_z$$

$$c_z = a_x b_y - a_y b_x$$

Moment siły względem punktu



$$\mathbf{M}_0 = \mathbf{r}\times\mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ P_x & P_y & P_z \end{vmatrix}$$

$$0(0,0,0) \qquad A(x,y,z)$$

$$r_x = x \qquad r_y = y \qquad r_z = z$$

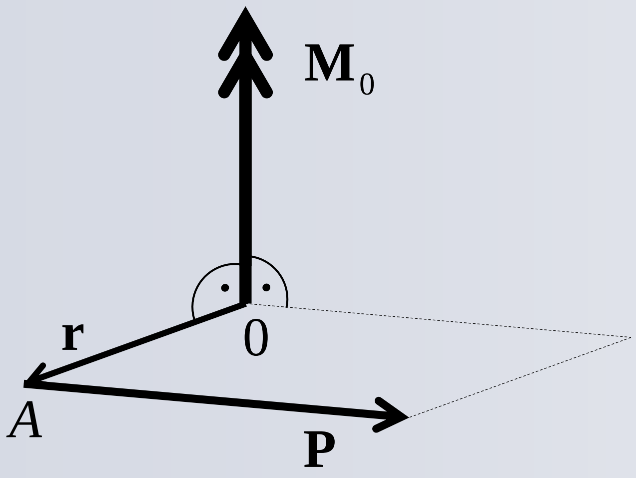
$$\mathbf{M}_0 = \mathbf{r}\times\mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix}$$

$$M_{0x} = P_z\cdot y - P_y\cdot z$$

$$M_{0y} = P_x\cdot z - P_z\cdot x$$

$$M_{0z} = P_y\cdot x - P_x\cdot y$$

Moment siły względem punktu



$$\mathbf{M}_0 = \mathbf{r} \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ P_x & P_y & P_z \end{vmatrix}$$

$$0(x_0, y_0, z_0) \qquad A(x, y, z)$$

$$r_x = x - x_0 \qquad r_y = y - y_0 \qquad r_z = z - z_0$$

$$\mathbf{M}_0 = \mathbf{r} \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x - x_0 & y - y_0 & z - z_0 \\ P_x & P_y & P_z \end{vmatrix}$$

$$M_{0x} = P_z \cdot (y - y_0) - P_y \cdot (z - z_0)$$

$$M_{0y} = P_x \cdot (z - z_0) - P_z \cdot (x - x_0)$$

$$M_{0z} = P_y \cdot (x - x_0) - P_x \cdot (y - y_0)$$

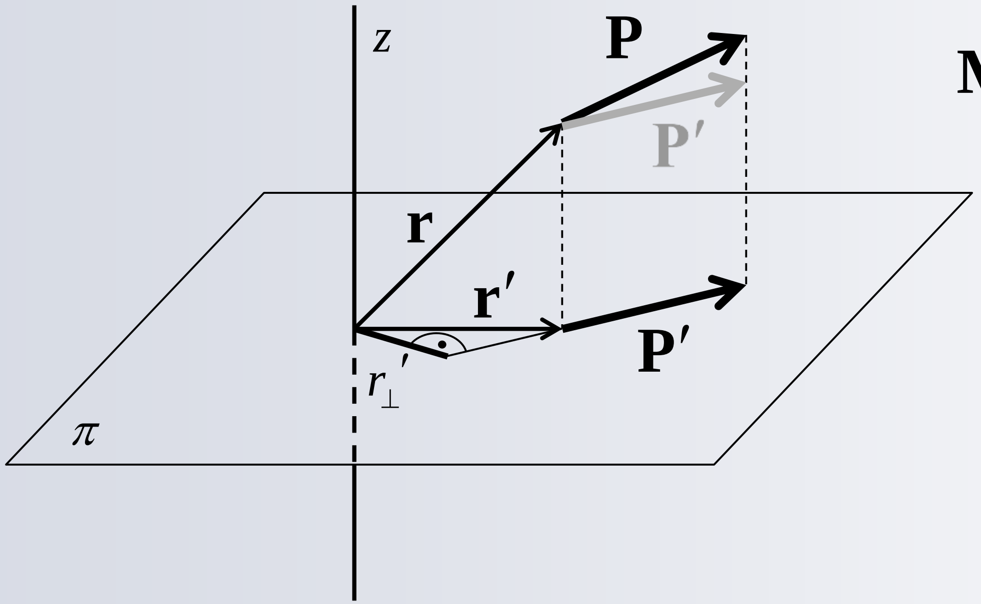
Moment wypadkowej względem punktu

$$\mathbf{W} = \sum_{i=1}^{n} \mathbf{P}_i$$

$$\mathbf{M}_0 = \mathbf{r} \times \mathbf{W} = \mathbf{r} \times \sum_{i=1}^{n} \mathbf{P}_i = \sum_{i=1}^{n} \mathbf{r} \times \mathbf{P}_i = \sum_{i=1}^{n} \mathbf{M}_{i0}$$

- Moment wypadkowej układu sił względem punktu równy jest sumie momentów od sił składowych tego układu względem tego punktu.

Moment siły względem osi



$$\mathbf{M}_z = \mathbf{r} \times \mathbf{P} = \mathbf{r}' \times \mathbf{P}'$$

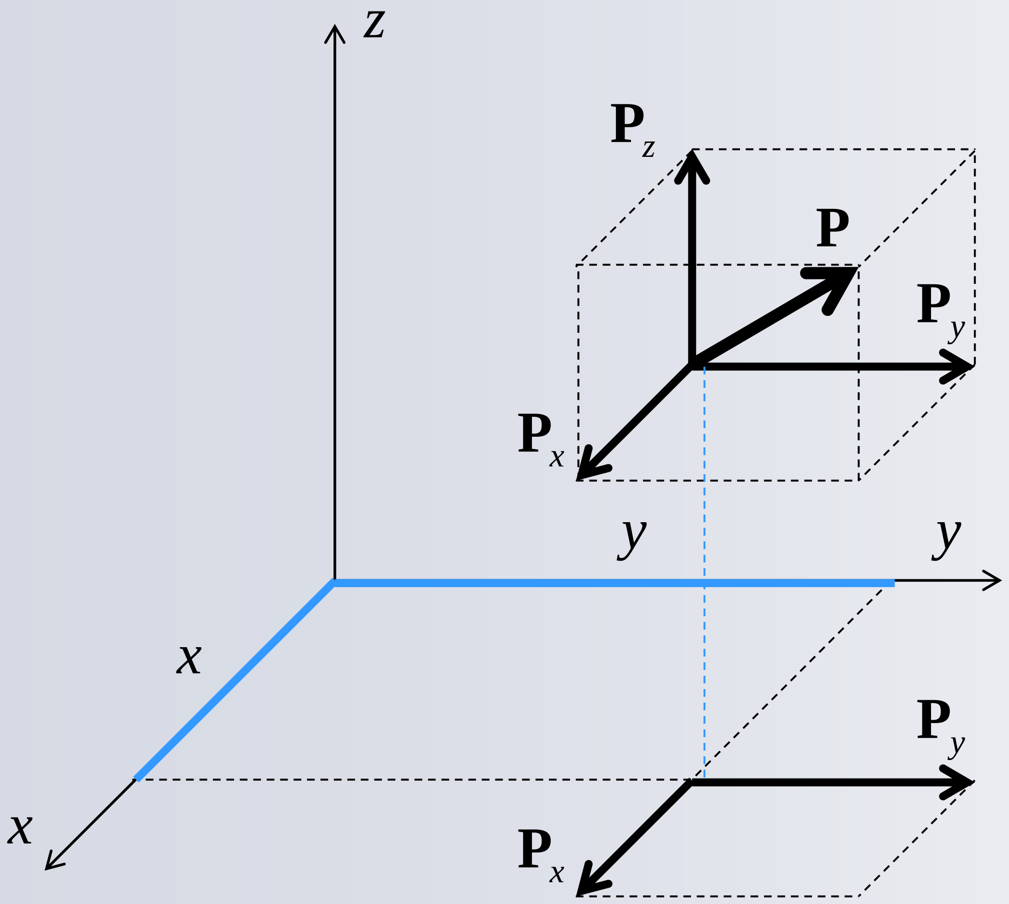
$$M_z = P' \cdot r_\perp'$$

- Moment siły względem osi równy jest momentowi rzutu siły na płaszczyznę prostopadłą do osi względem punktu, w którym oś przebija płaszczyznę.

Moment siły względem osi

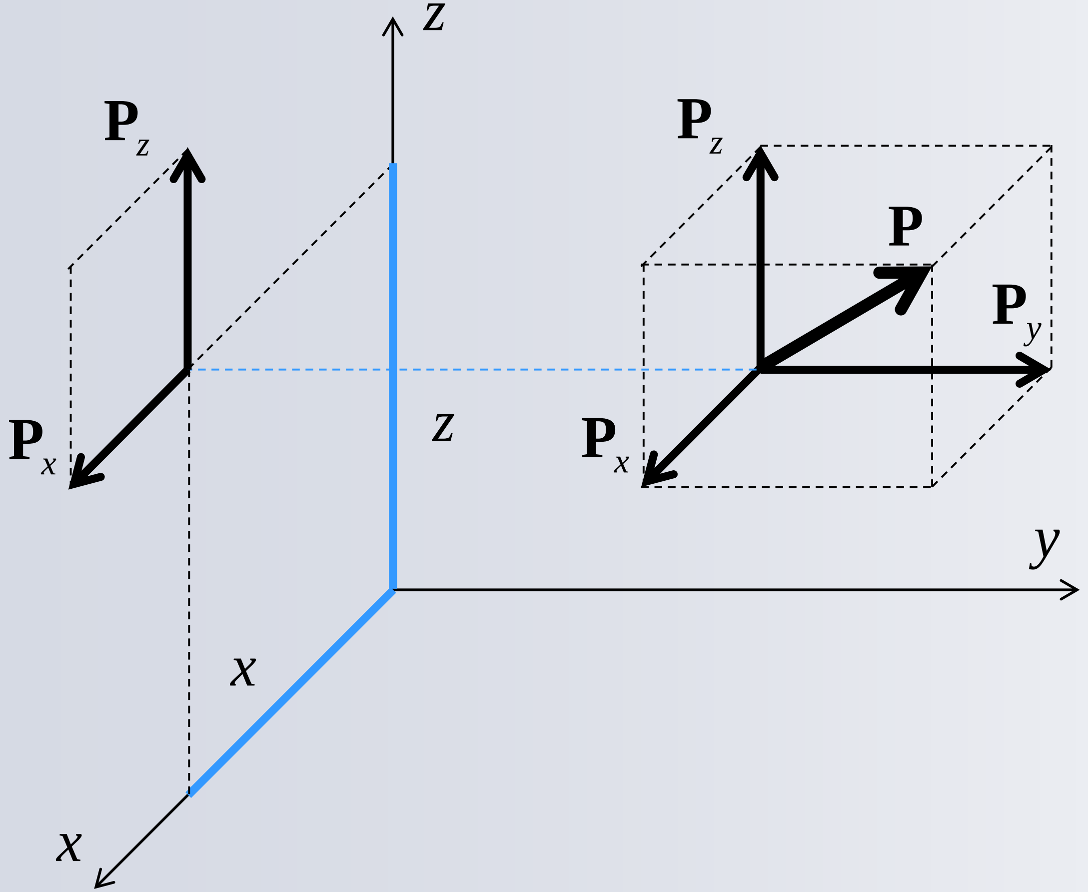
- Moment siły względem osi jest równy jest równy 0, gdy:
  - Rzut $\mathbf{P}'$ na płaszczyznę π prostopadłą do osi z jest równy 0 (siła równoległa do osi);
  - Długość ramienia $r_\perp{}'$ jest równa 0 (linia działania siły przecina oś).
- Moment siły względem osi równy jest rzutowi na oś momentu siły względem dowolnego punktu leżącego na osi.

Moment siły względem osi układu współrzędnych



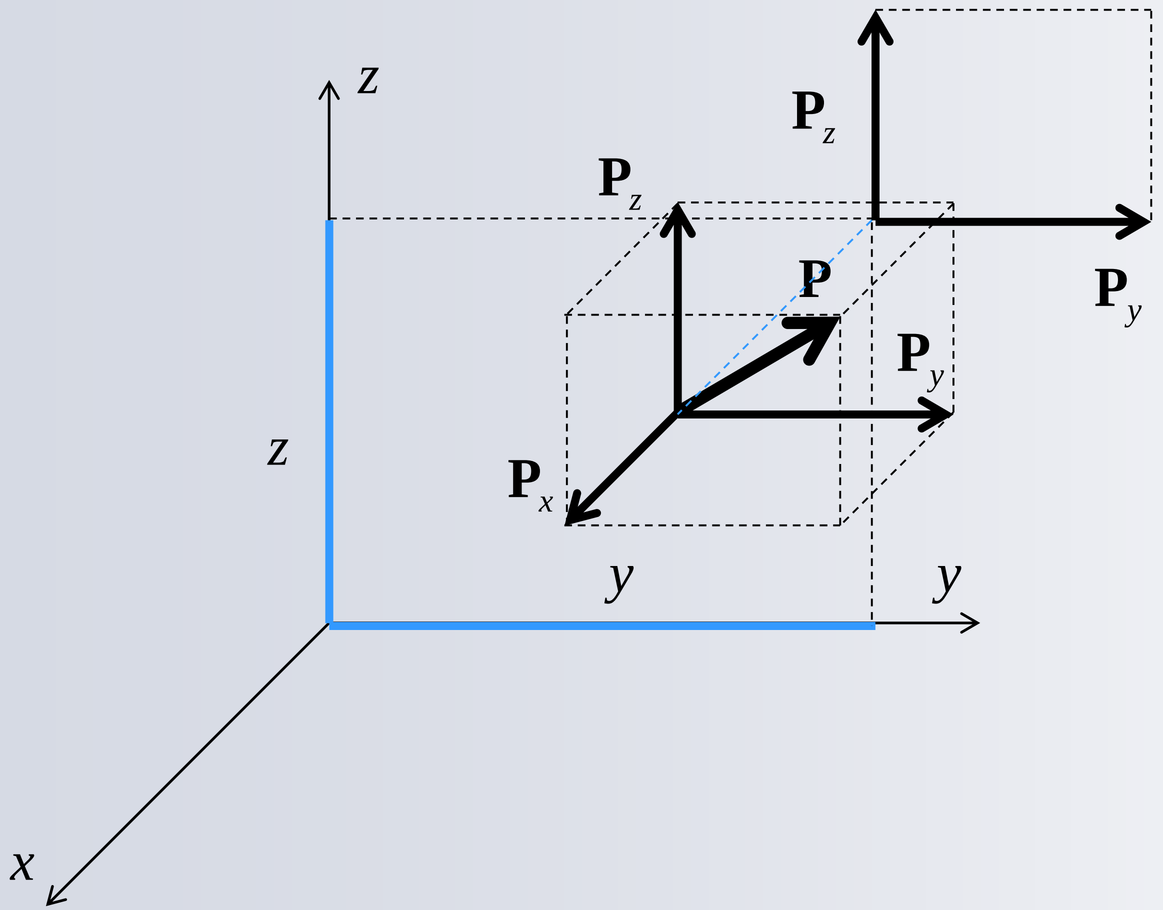
$$M_z = -P_x \cdot y + P_y \cdot x$$

Moment siły względem osi układu współrzędnych



$$M_y = -P_z \cdot x + P_x \cdot z$$

Moment siły względem osi układu współrzędnych



$$M_x = -P_y \cdot z + P_z \cdot y$$

Dowolny przestrzenny układ sił

- Dowolny przestrzenny układ sił można zastąpić siłą wypadkową przyłożoną do dowolnego bieguna redukcji 0 i parą sił o momencie równym sumie momentów od sił składowych względem tego bieguna redukcji.

$$\mathbf{W} = \sum_{i=1}^{n} \mathbf{P}_i$$ - wektor główny

$$\mathbf{M}_0 = \sum_{i=1}^{n} \mathbf{M}_{i0}$$ - moment główny

Dowolny przestrzenny układ sił

$$W_x = \sum_{i=1}^{n} P_{ix} \qquad \cos\alpha = \frac{W_x}{W}$$

$$W_y = \sum_{i=1}^{n} P_{iy} \qquad \cos\beta = \frac{W_y}{W}$$

$$W_z = \sum_{i=1}^{n} P_{iz} \qquad \cos\gamma = \frac{W_z}{W}$$

$$W = \sqrt{W_x^{\,2} + W_y^{\,2} + W_z^{\,2}}$$

$$M_{0x} = \sum_{i=1}^{n} M_{ix} = \sum_{i=1}^{n}\left(P_{iz}\cdot\left(y_i - y_0\right) - P_{iy}\cdot\left(z_i - z_0\right)\right)$$

$$M_{0y} = \sum_{i=1}^{n} M_{iy} = \sum_{i=1}^{n}\left(P_{ix}\cdot\left(z_i - z_0\right) - P_{iz}\cdot\left(x_i - x_0\right)\right)$$

$$M_{0z} = \sum_{i=1}^{n} M_{iz} = \sum_{i=1}^{n}\left(P_{iy}\cdot\left(x_i - x_0\right) - P_{ix}\cdot\left(y_i - y_0\right)\right)$$

Redukcja do początku układu współrzędnych

$$W_x = \sum_{i=1}^{n} P_{ix} \qquad \cos\alpha = \frac{W_x}{W}$$

$$W_y = \sum_{i=1}^{n} P_{iy} \qquad \cos\beta = \frac{W_y}{W} \qquad W = \sqrt{W_x^{\,2} + W_y^{\,2} + W_z^{\,2}}$$

$$W_z = \sum_{i=1}^{n} P_{iz} \qquad \cos\gamma = \frac{W_z}{W}$$

$$M_{0x} = \sum_{i=1}^{n} M_{ix} = \sum_{i=1}^{n} \left(P_{iz} \cdot y_i - P_{iy} \cdot z_i \right)$$

$$M_{0y} = \sum_{i=1}^{n} M_{iy} = \sum_{i=1}^{n} \left(P_{ix} \cdot z_i - P_{iz} \cdot x_i \right)$$

$$M_{0z} = \sum_{i=1}^{n} M_{iz} = \sum_{i=1}^{n} \left(P_{iy} \cdot x_i - P_{ix} \cdot y_i \right)$$

Warunki równowagi dowolnego przestrzennego układu sił

$$\mathbf{W} = \sum_{i=1}^{n} \mathbf{P}_i = \mathbf{0} \qquad \mathbf{M}_0 = \sum_{i=1}^{n} \mathbf{M}_{i0} = \mathbf{0}$$

$$\mathbf{W} = \mathbf{0} \qquad \sum_{i=1}^{n} P_{ix} = \sum X = 0 \qquad \mathbf{M}_0 = \mathbf{0} \qquad \sum_{i=1}^{n} M_{ix} = \sum M_x = 0$$

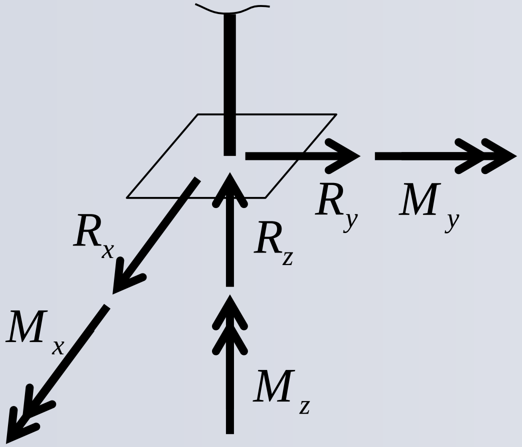
$$\sum_{i=1}^{n} P_{iy} = \sum Y = 0 \qquad \sum_{i=1}^{n} M_{iy} = \sum M_y = 0$$

$$\sum_{i=1}^{n} P_{iz} = \sum Z = 0 \qquad \sum_{i=1}^{n} M_{iz} = \sum M_z = 0$$

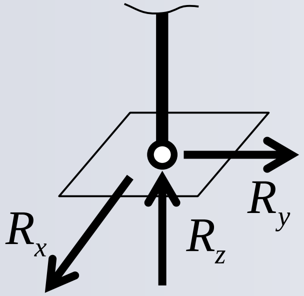
Układy prętowe

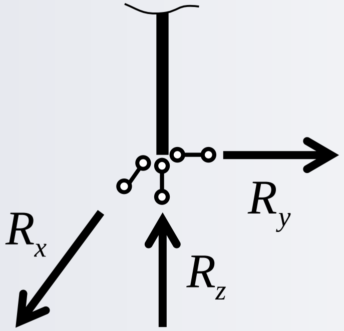
- Kratownice przestrzenne
- Ruszty
  - O węzłach przegubowych
  - O węzłach sztywnych
- Ramy przestrzenne
  - Na siatce prostopadłościanu
  - Z prętami ukośnymi
- Pręty zakrzywione w przestrzeni
- Układy prętów różnego typu

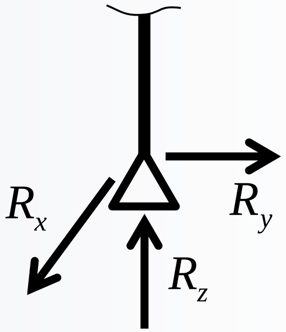
Podpory



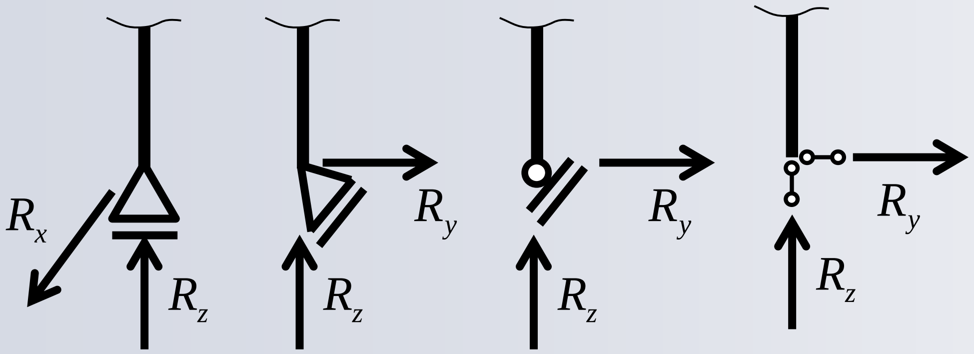
- Sztywne zamocowanie – 6 reakcji (3 siły, 3 momenty)
- Podpora przegubowa nieprzesuwna – 3 reakcje (3 siły)



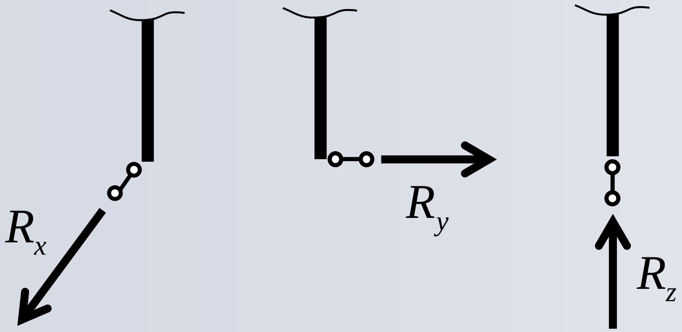




Podpory



- Podpora przegubowa przesuwna wzdłuż prostej – 2 reakcje (2 siły)



- Podpora przegubowa przesuwna po płaszczyźnie – 1 reakcja (1 siła)

Przeguby

- Przegub Cardana (możliwe zginanie, brak możliwości skręcania – wzajemnego obrotu prętów względem osi)
- Przegub walcowy (możliwe tylko zginanie w jednym kierunku)
- Przegub kulisty (całkowita swoboda wzajemnego obrotu)

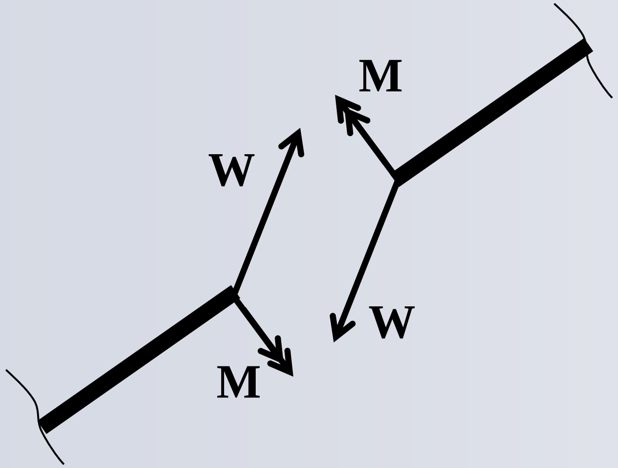
Siły wewnętrzne

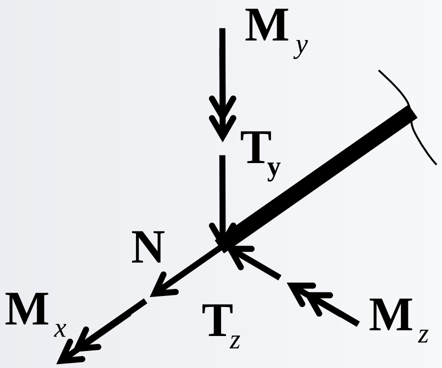
- Wypadkowa siła i wypadkowy moment wzajemnego oddziaływania
- Wypadkowa siła:
  - Siła normalna (osiowa)
  - Dwie składowe siły tnącej (poprzecznej)
- Wypadkowy moment:
  - Moment skręcający
  - Dwie składowe momentu zginającego

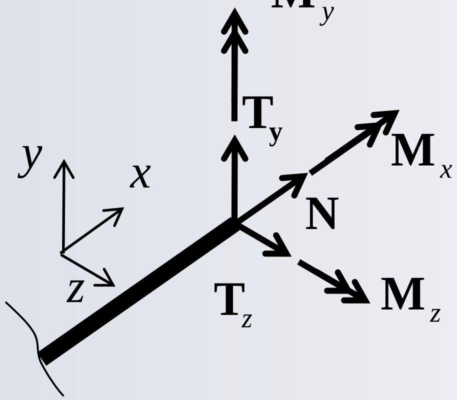
Siły wewnętrzne

$$\mathbf{W} = \mathbf{W}_x + \mathbf{W}_y + \mathbf{W}_z$$

$$\mathbf{M} = \mathbf{M}_x + \mathbf{M}_y + \mathbf{M}_z$$







$$\mathbf{W}_x = \mathbf{N}$$

$$\mathbf{W}_y = \mathbf{T}_y$$

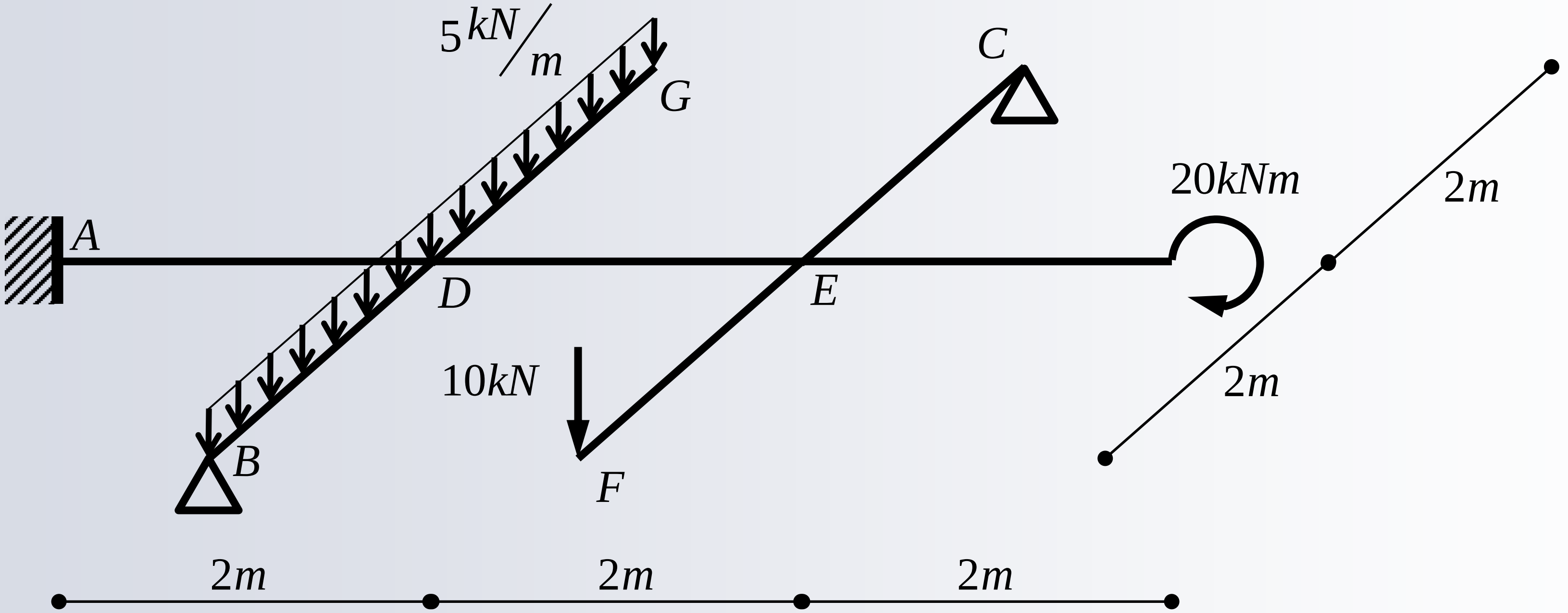
$$\mathbf{W}_z = \mathbf{T}_z$$

$$\mathbf{M}_x = \mathbf{M}_s$$

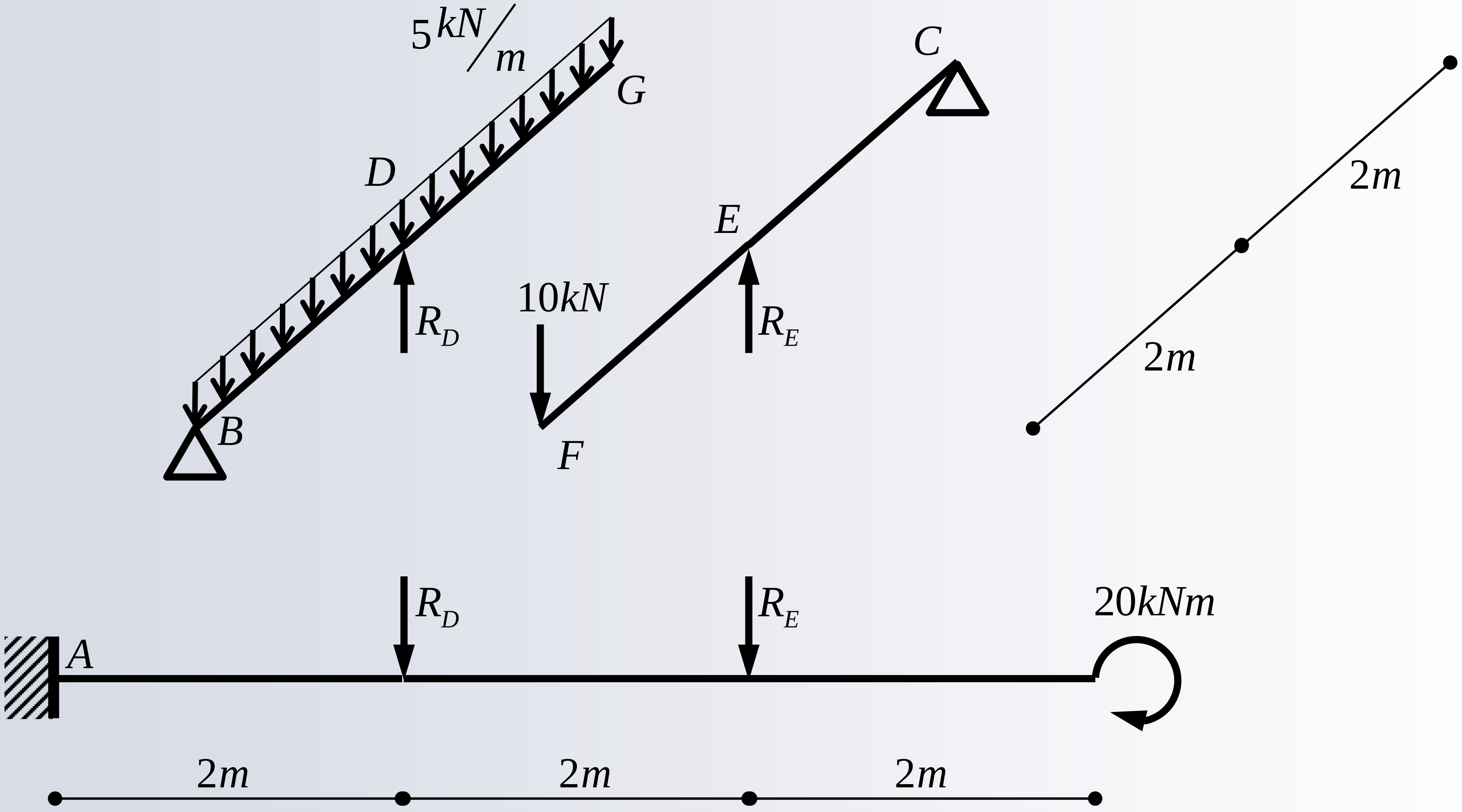
$$\mathbf{M}_y = \mathbf{M}_y^{zg} = \mathbf{M}_y^{xz}$$

$$\mathbf{M}_z = \mathbf{M}_z^{zg} = \mathbf{M}_z^{xy}$$

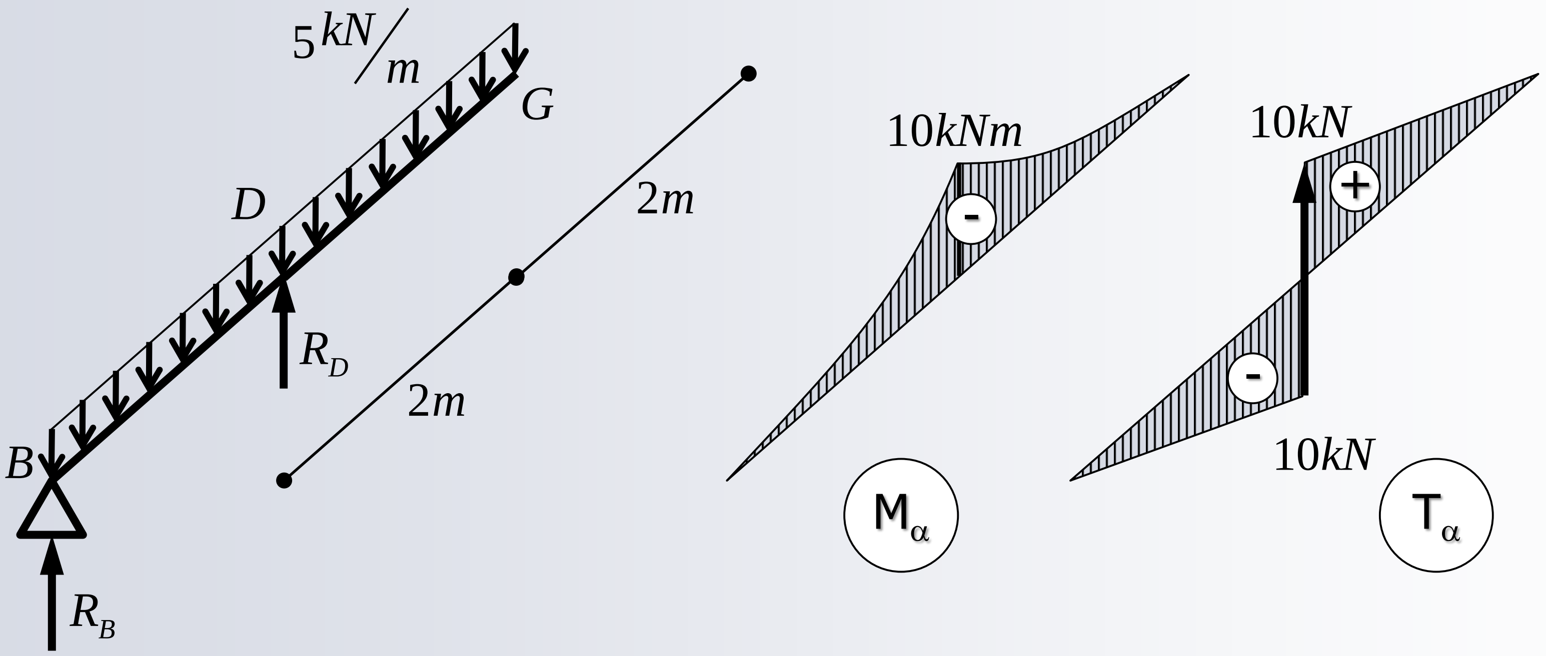
Przykład – ruszt o węzłach przegubowych



Przykład 1 – ruszt o węzłach przegubowych



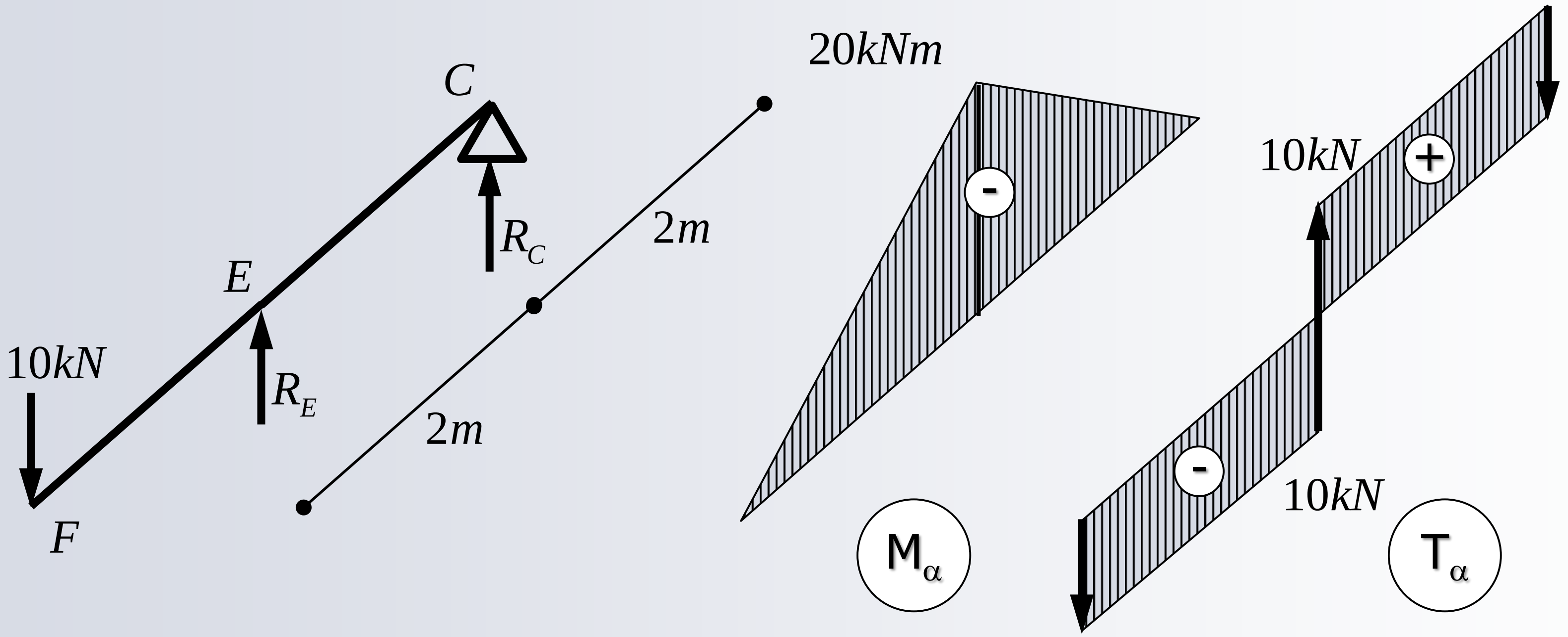
Belka B-D-G



$$\sum M_B = R_D \cdot 2m - 5\frac{kN}{m} \cdot 4m \cdot \frac{1}{2} 4m = 0 \qquad R_D = 20kN$$

$$\sum Y = R_B + R_D - 5\frac{kN}{m} \cdot 4m = 0 \qquad R_B = 0kN$$

Belka C-E-F



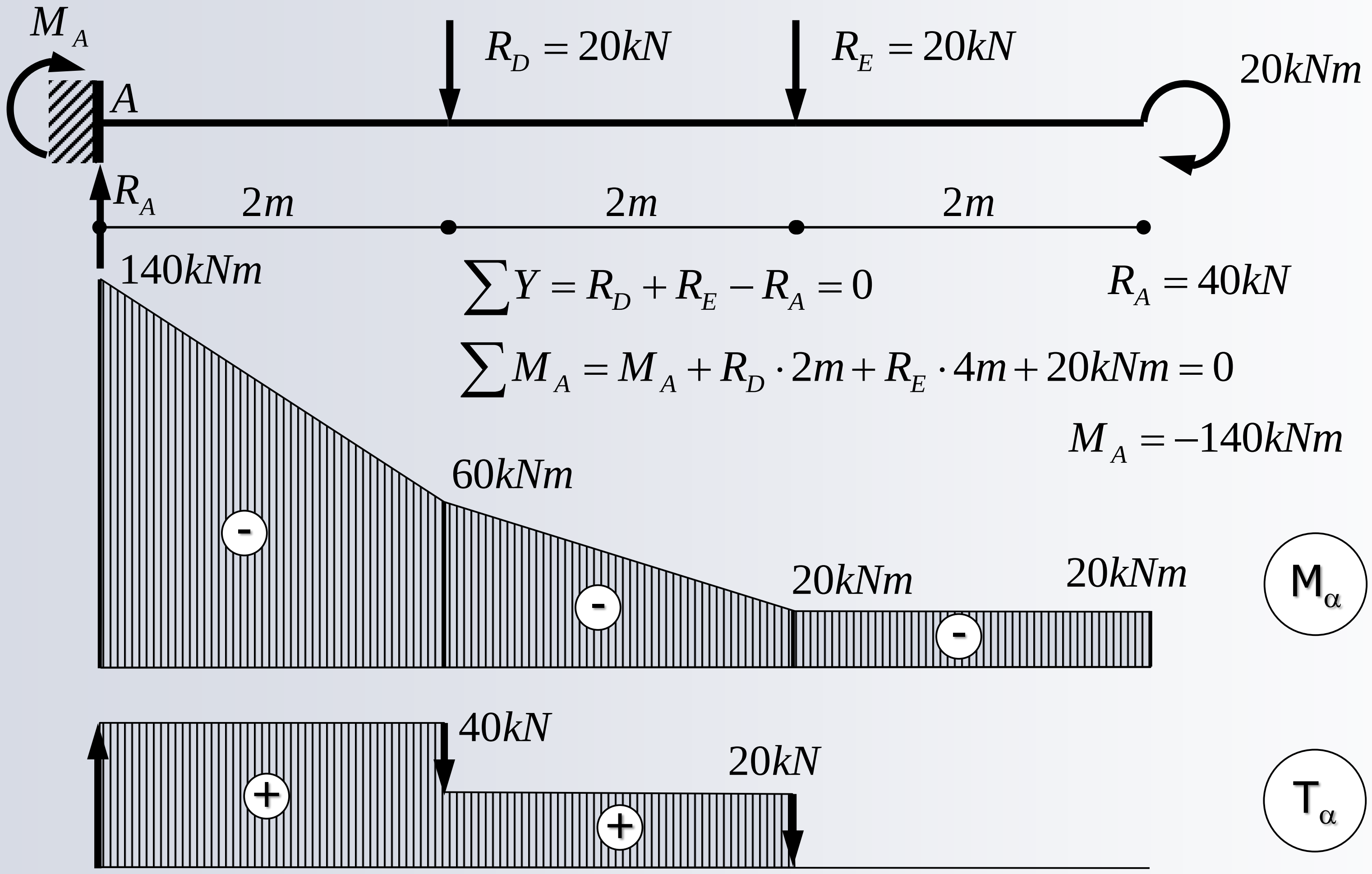
$$\sum M_C = R_E \cdot 2m - 10kN \cdot 4m = 0$$

$$R_E = 20kN$$

$$\sum Y = R_E + R_C - 10kN = 0$$

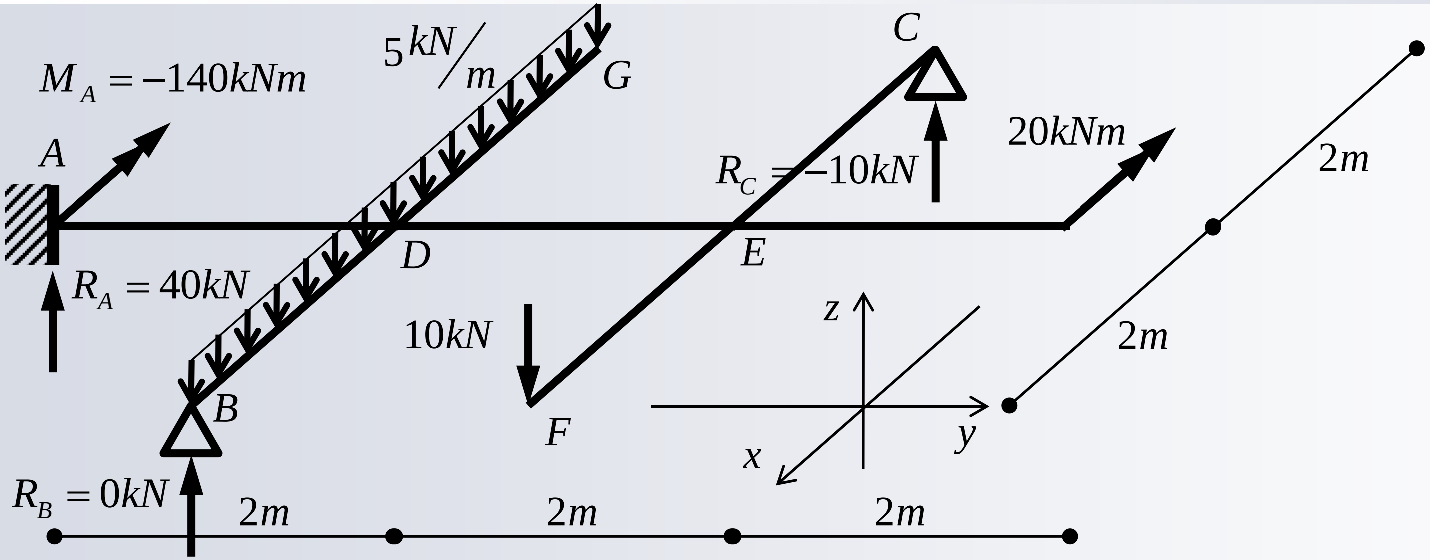
$$R_C = -10kN$$

Belka C-E-F



M_A
A
$R_D = 20kN$
$R_E = 20kN$
$20kNm$
R_A
$2m$
$2m$
$2m$
$140kNm$
$\sum Y = R_D + R_E - R_A = 0$
$R_A = 40kN$
$\sum M_A = M_A + R_D \cdot 2m + R_E \cdot 4m + 20kNm = 0$
$M_A = -140kNm$
$60kNm$
-
-
$20kNm$
$20kNm$
-
M_α
$40kN$
$20kN$
+
+
T_α

Sprawdzenie reakcji



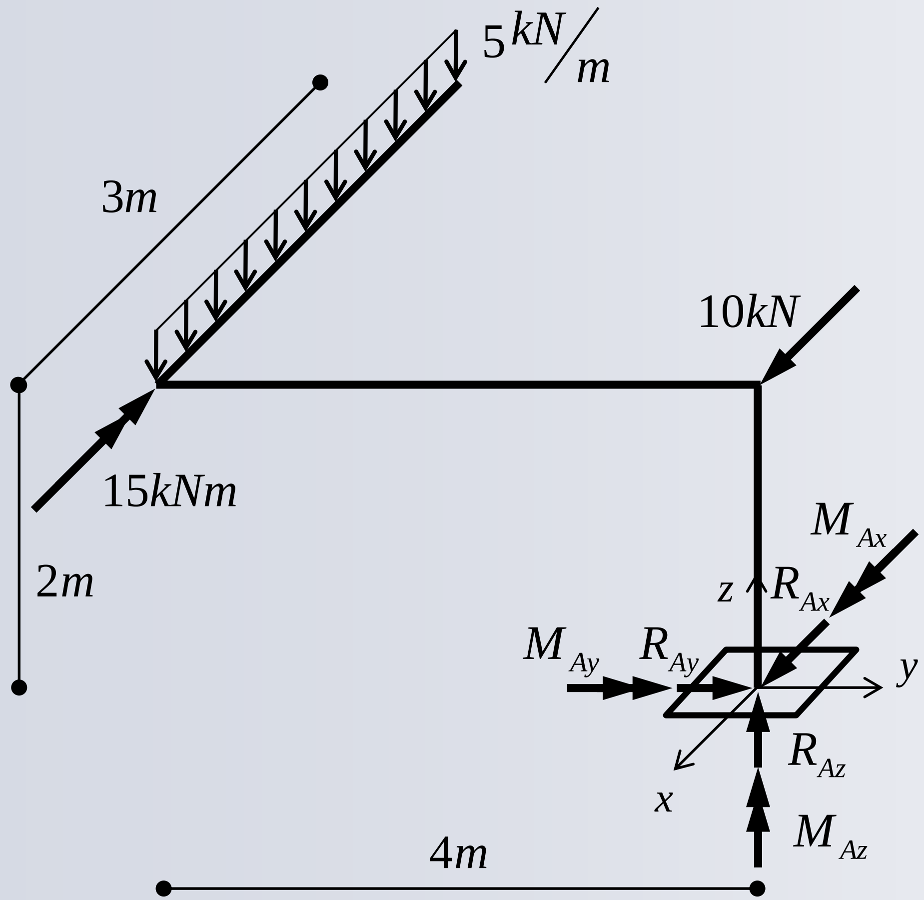
$$\sum Z = R_A + R_B + R_C - 10kN - 5\,{}^{kN}\!/_{m} \cdot 4m = 40kN + 0kN - 10kN - 10kN - 20kN = 0$$

$$\sum M_x = -M_A - 20kNm - R_A \cdot 6m - R_B \cdot 4m + 5\,{}^{kN}\!/_{m} \cdot 4m \cdot 4m - R_C \cdot 2m + 10kN \cdot 2m =$$

$$= 140kNm - 20kNm - 40kN \cdot 6m - 0kN \cdot 4m + 5\,{}^{kN}\!/_{m} \cdot 4m \cdot 4m + 10kN \cdot 2m + 10kN \cdot 2m = 0$$

$$\sum M_y = R_A \cdot 2m - 5\,{}^{kN}\!/_{m} \cdot 4m \cdot 2m + R_C \cdot 4m = 40kN \cdot 2m - 5\,{}^{kN}\!/_{m} \cdot 4m \cdot 2m - 10kN \cdot 4m = 0$$

Przykład 2 – rama przestrzenna wspornikowa



$$\sum X = R_{Ax} + 10kN = 0 \qquad R_{Ax} = -10kN$$

$$\sum Y = R_{Ay} = 0 \qquad R_{Ay} = 0$$

$$\sum Z = R_{Az} - 5 \, kN/m \cdot 3m = 0 \qquad R_{Az} = 15kN$$

$$\sum M_x = M_{Ax} - 15kNm + 5 \, kN/m \cdot 3m \cdot 4m = 0$$

$$M_{Ax} = -45kNm$$

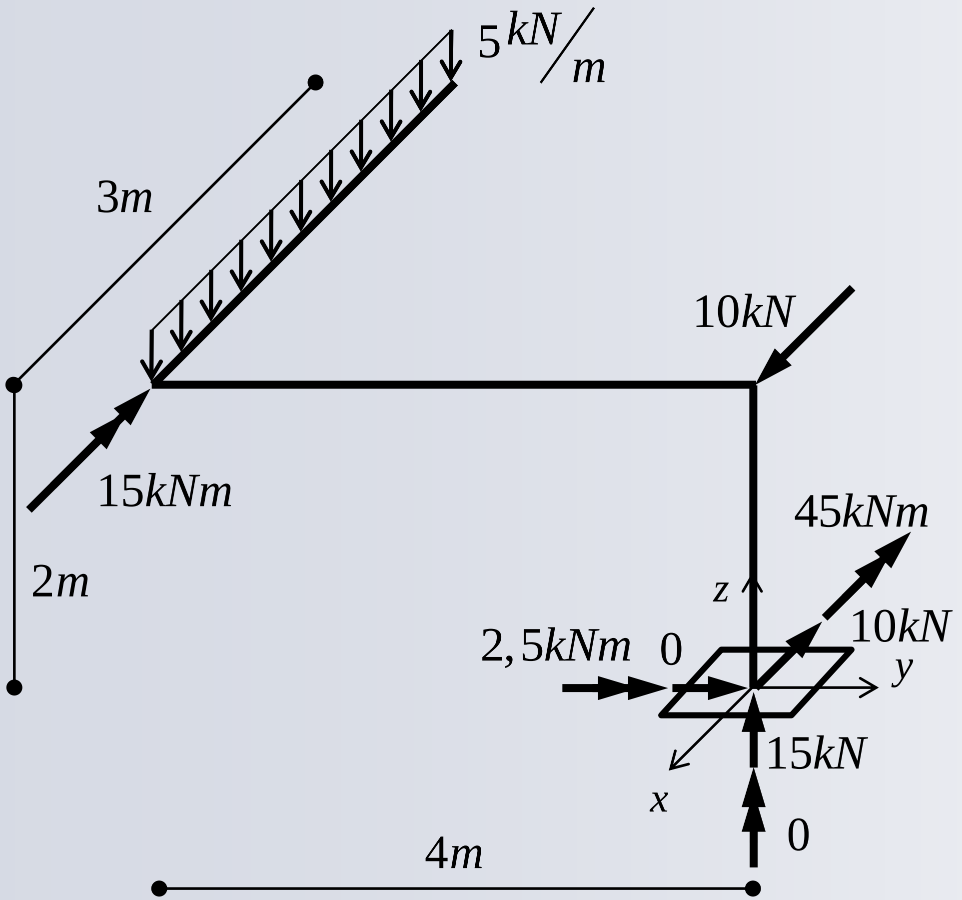
$$\sum M_y = M_{Ay} - 5 \, kN/m \cdot 3m \cdot \frac{1}{2} 3m + 10kN \cdot 2m = 0$$

$$M_{Ay} = 2{,}5kNm$$

$$\sum M_z = M_{Az} = 0$$

$$M_{Az} = 0$$

Przykład 2 – rama przestrzenna wspornikowa



$$\sum X = R_{Ax} + 10kN = 0 \qquad R_{Ax} = -10kN$$

$$\sum Y = R_{Ay} = 0 \qquad R_{Ay} = 0$$

$$\sum Z = R_{Az} - 5\,{}^{kN}\!/_{m} \cdot 3m = 0 \qquad R_{Az} = 15kN$$

$$\sum M_x = M_{Ax} - 15kNm + 5\,{}^{kN}\!/_{m} \cdot 3m \cdot 4m = 0$$

$$M_{Ax} = -45kNm$$

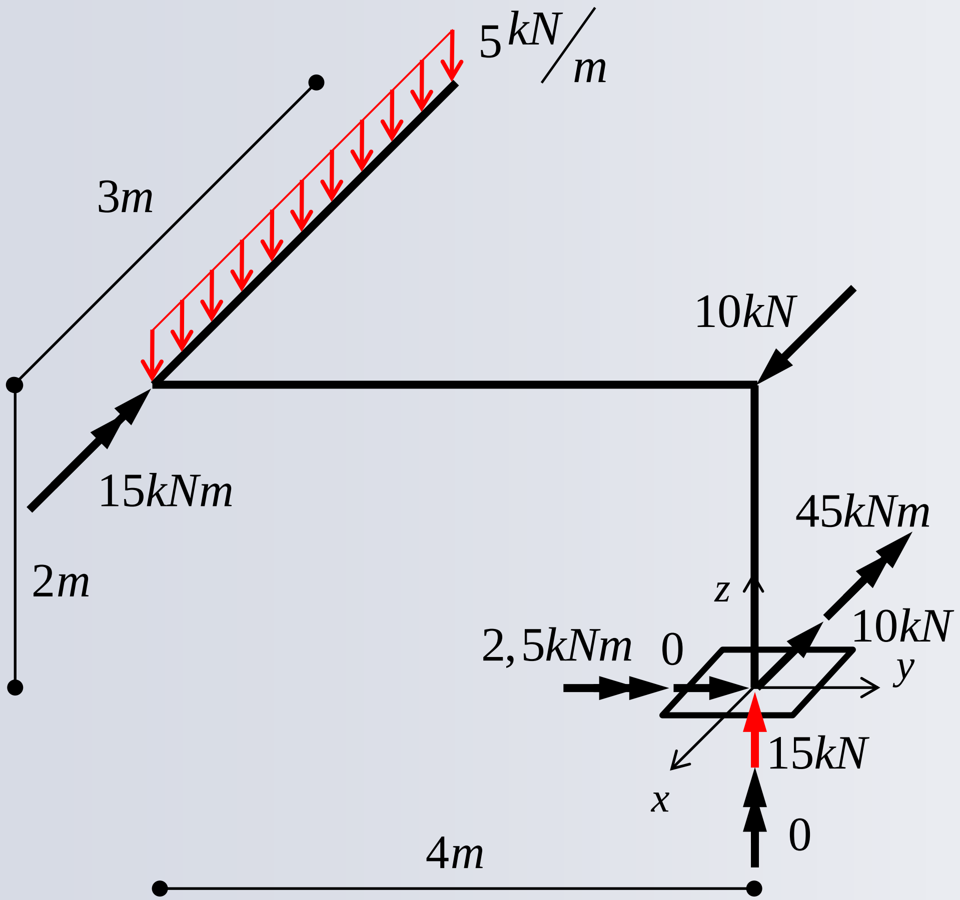
$$\sum M_y = M_{Ay} - 5\,{}^{kN}\!/_{m} \cdot 3m \cdot \frac{1}{2} 3m + 10kN \cdot 2m = 0$$

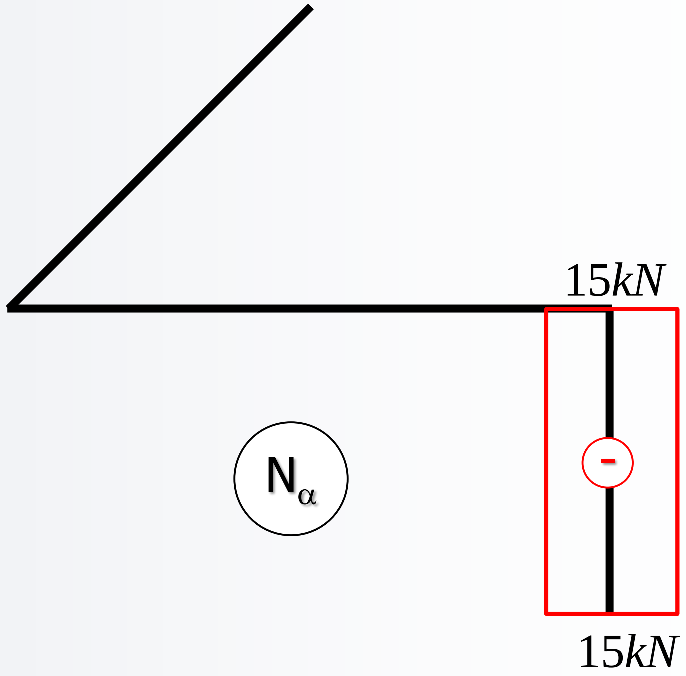
$$M_{Ay} = 2{,}5kNm$$

$$\sum M_z = M_{Az} = 0$$

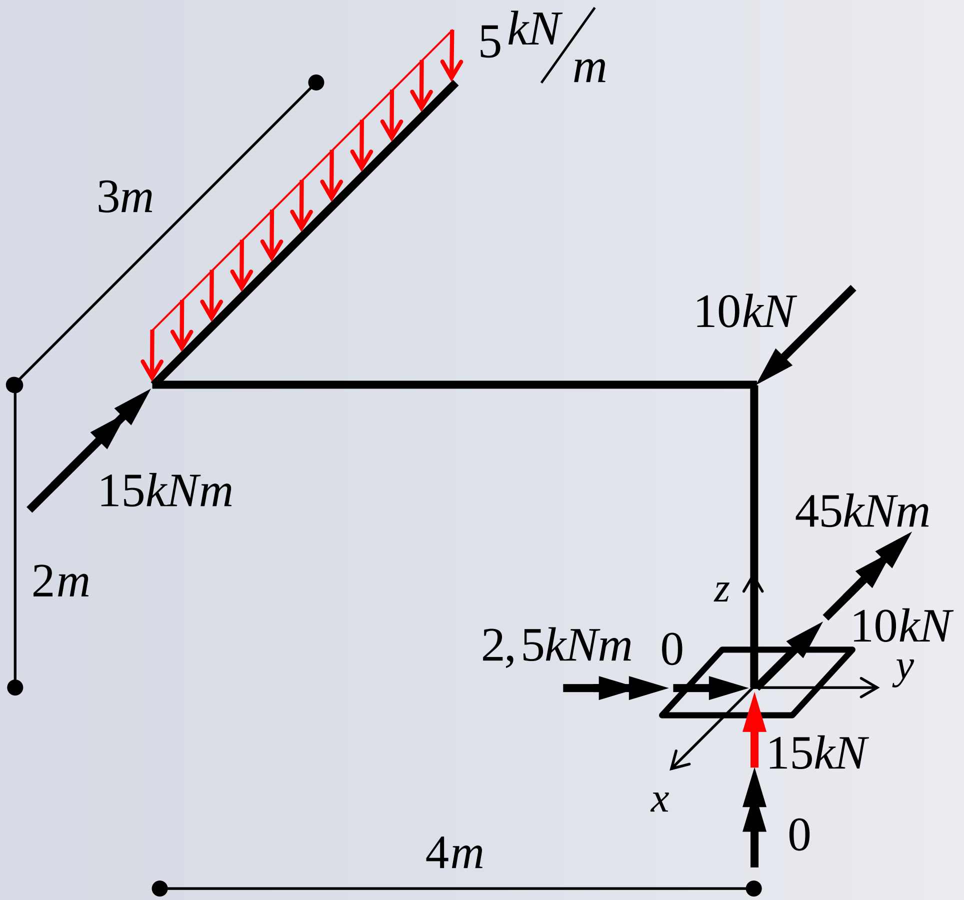
$$M_{Az} = 0$$

Siły normalne

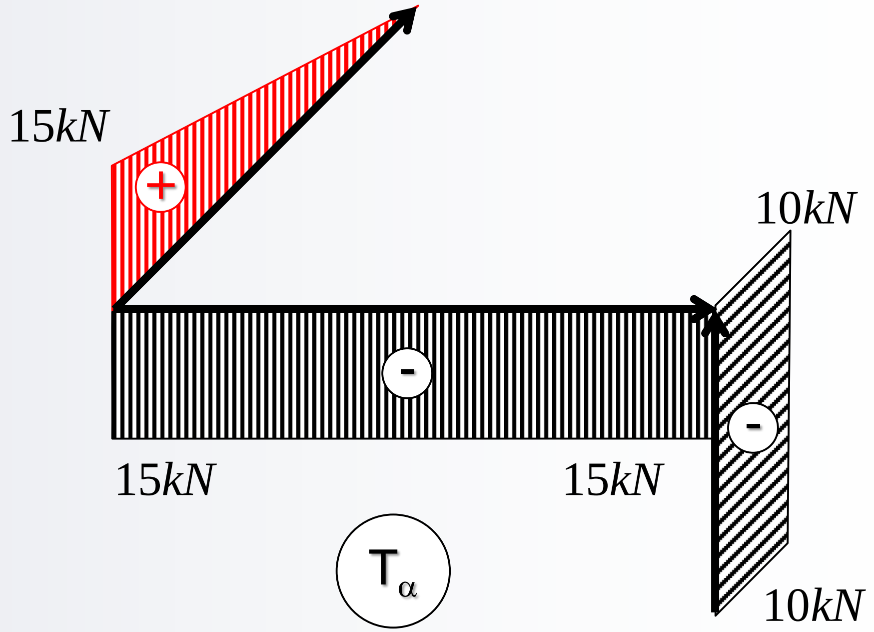




Siły tnące

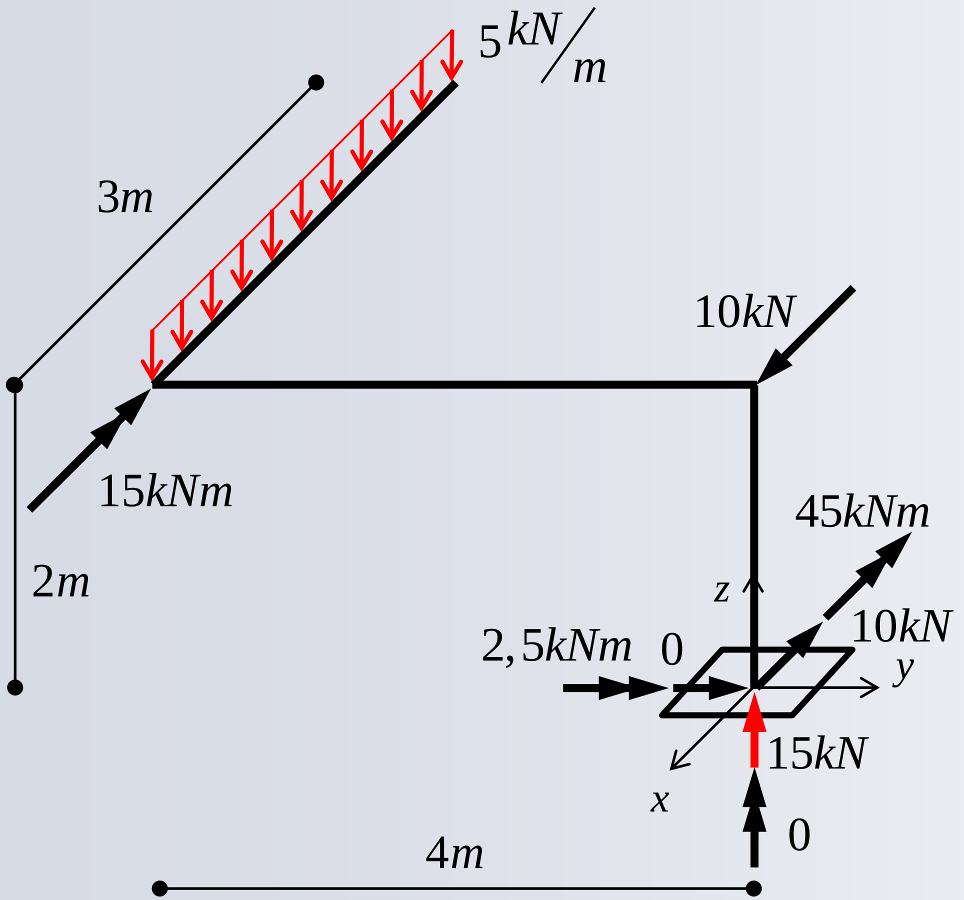


$5\,kN/m$
3m
10kN
15kNm
45kNm
2m
z
2,5kNm 0
10kN
y
15kN
x
0
4m

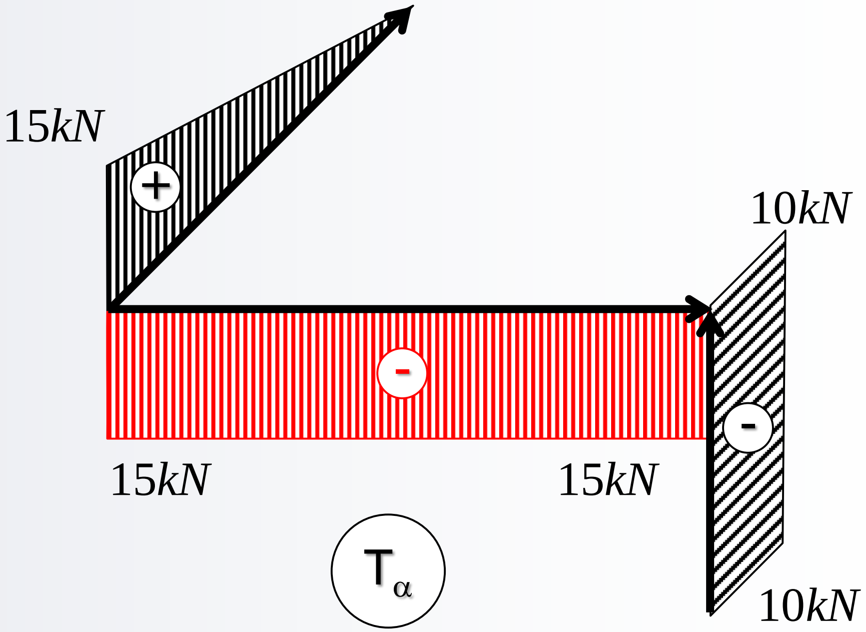


15kN
+
10kN
-
-
15kN
15kN
T_α
10kN

Siły tnące

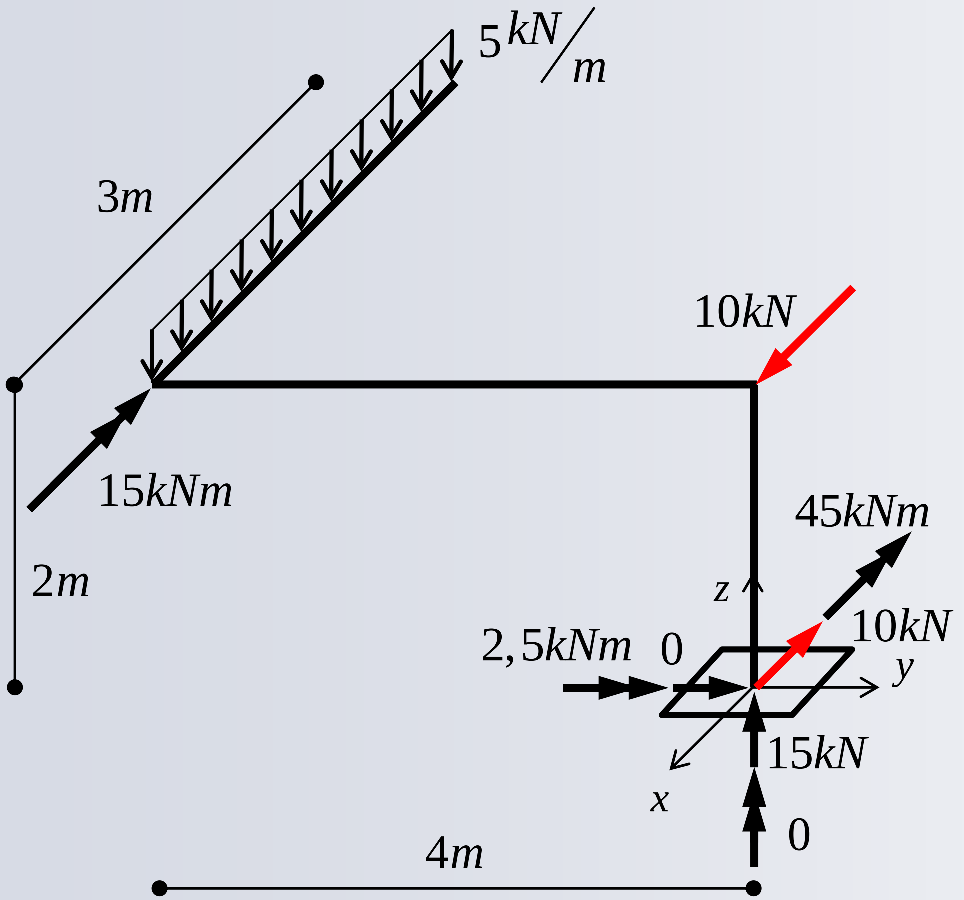


$5\,{}^{kN}\!/_{m}$
3m
10kN
15kNm
2m
45kNm
z
2,5kNm
0
10kN
y
15kN
x
0
4m

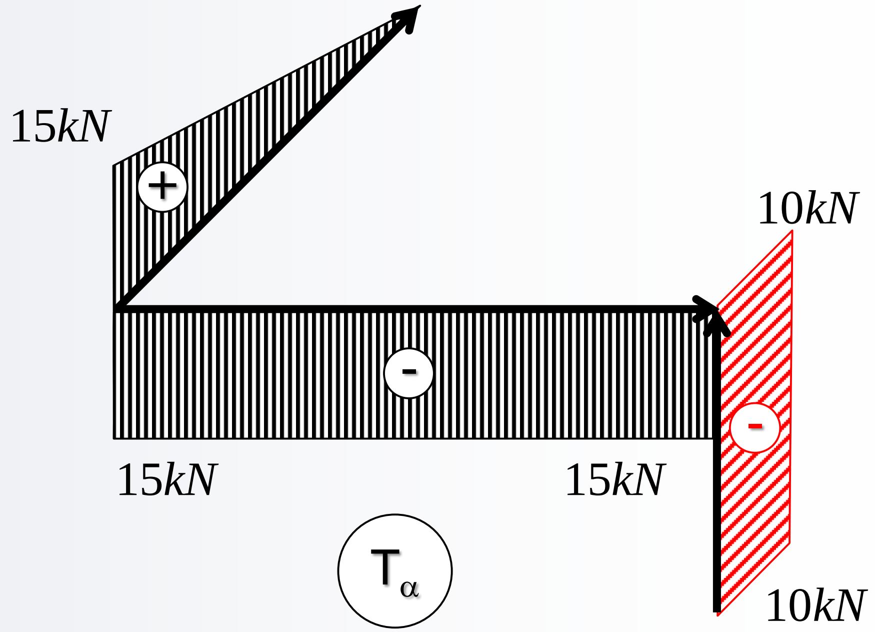


15kN
+
10kN
-
-
15kN
15kN
T_α
10kN

Siły tnące

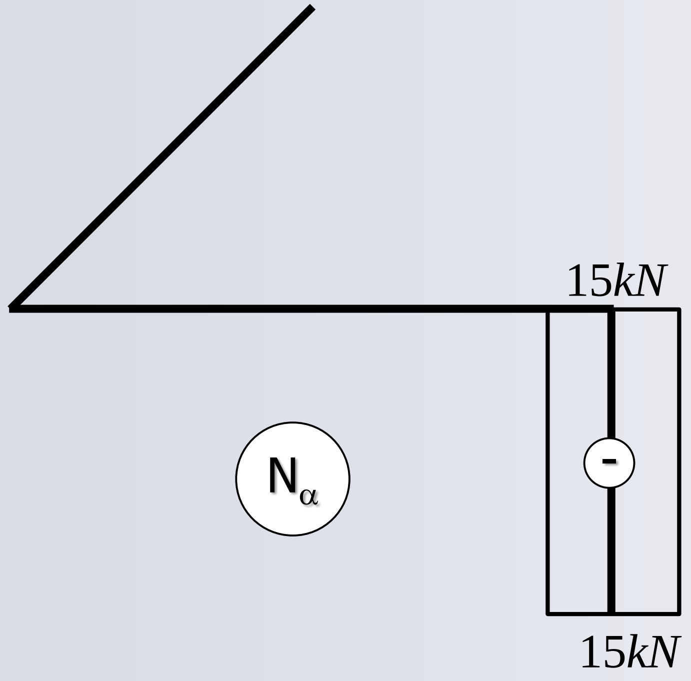


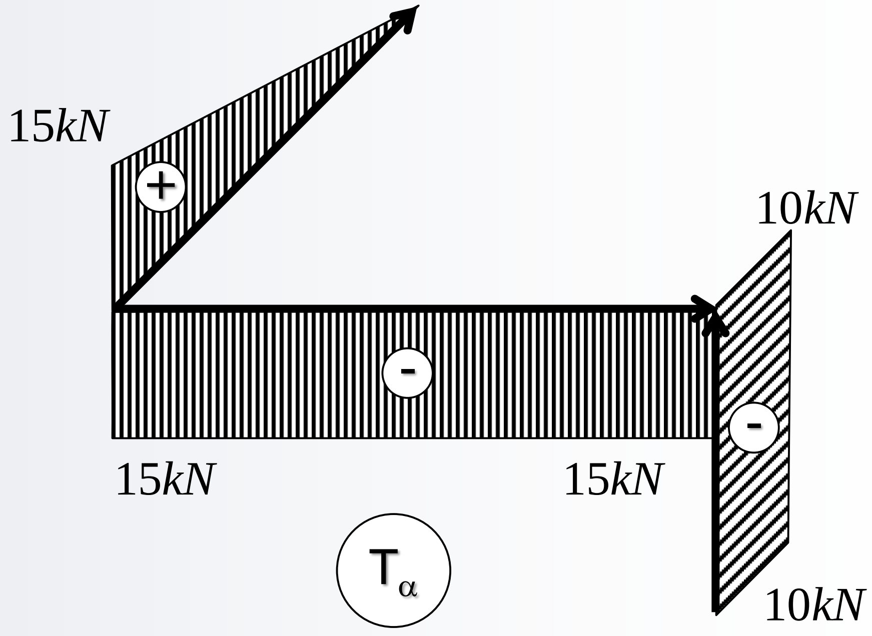
5 kN/m
3m
10kN
15kNm
45kNm
2m
z
2,5kNm 0
10kN
y
15kN
x
0
4m



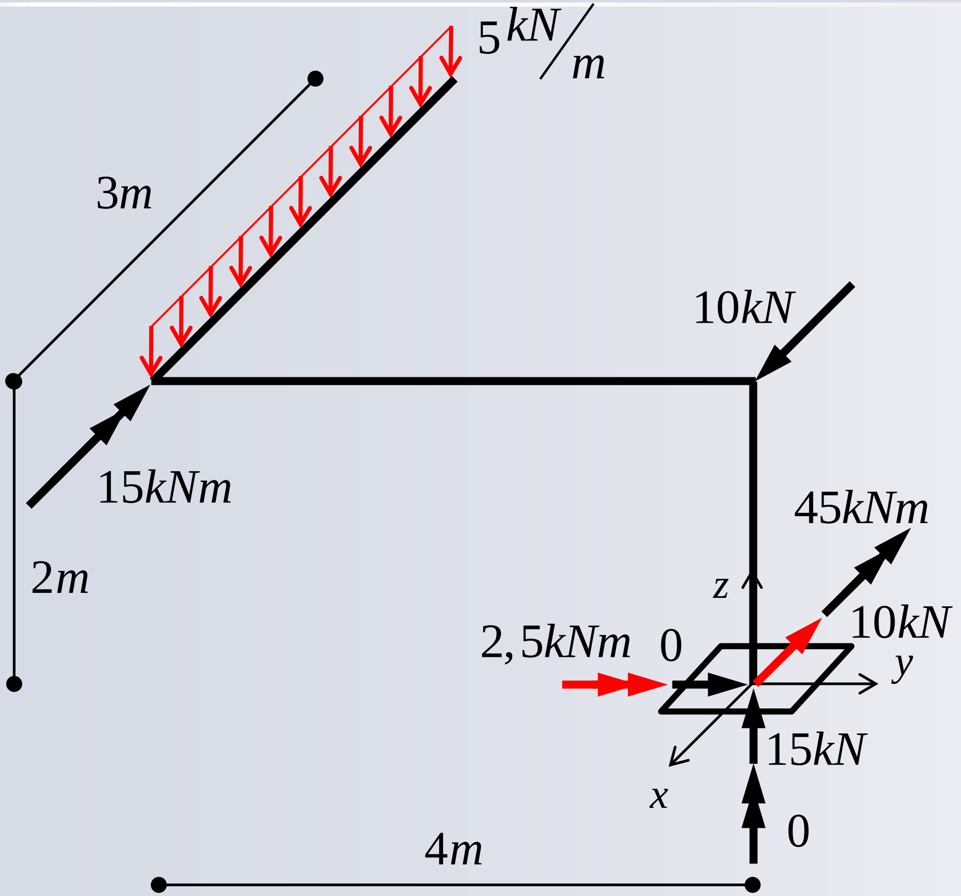
15kN
+
10kN
-
-
15kN
15kN
Tα
10kN

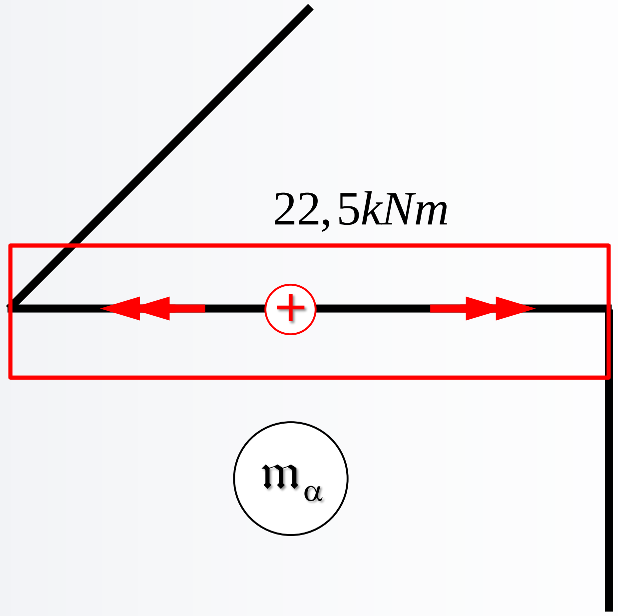
Siły normalne i tnące



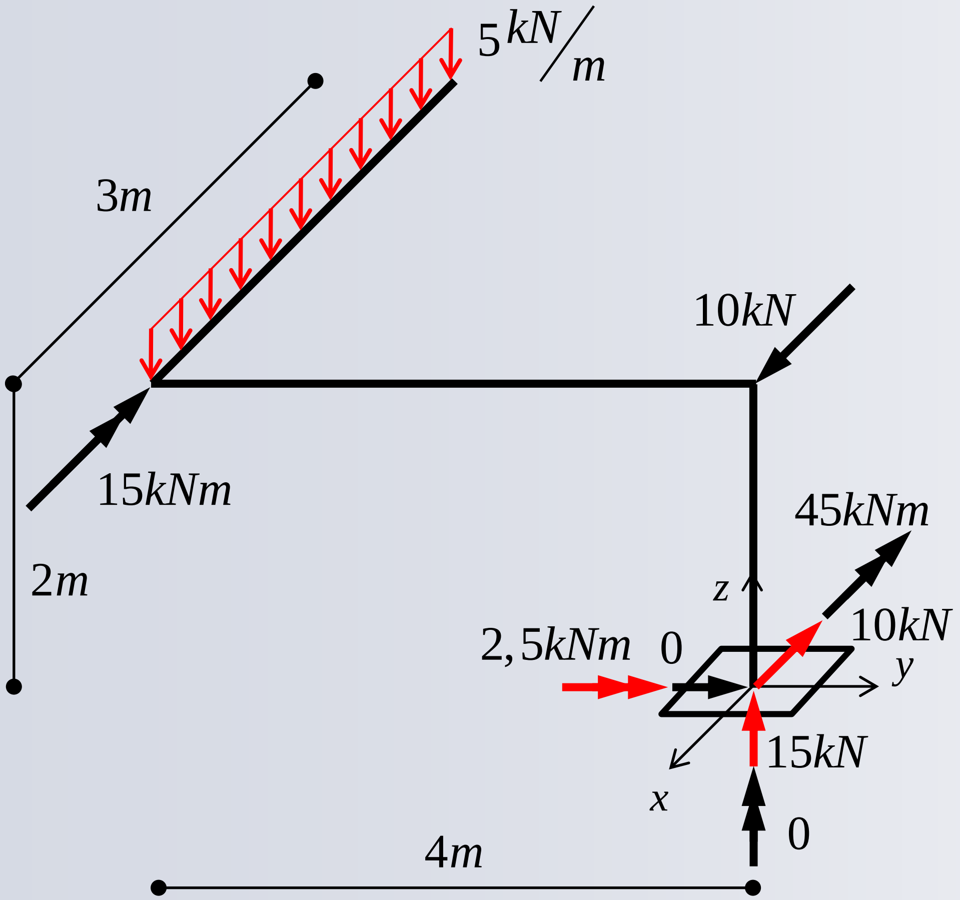


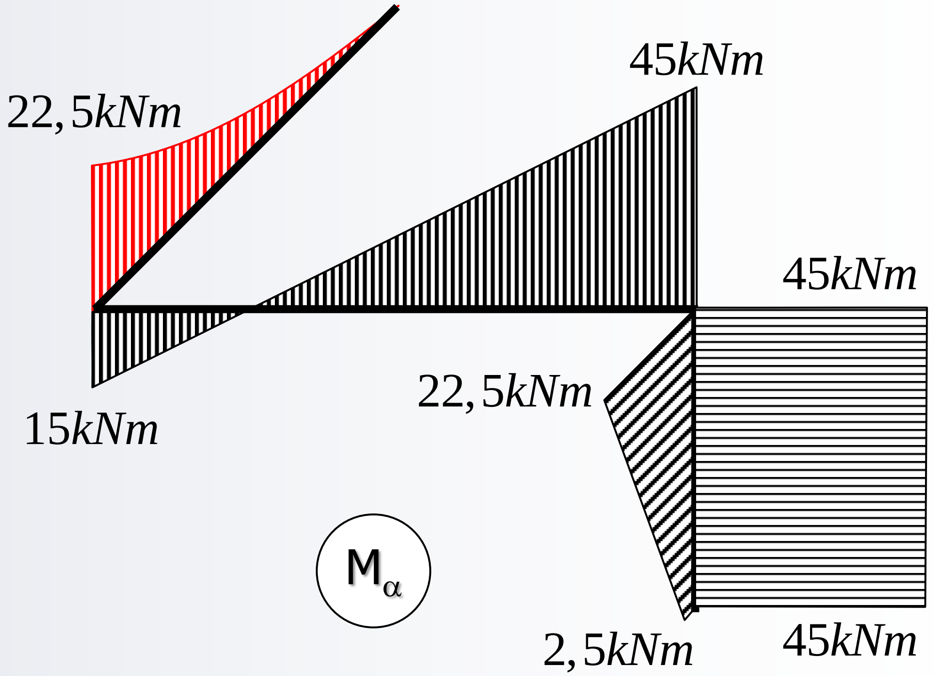
Momenty skręcające



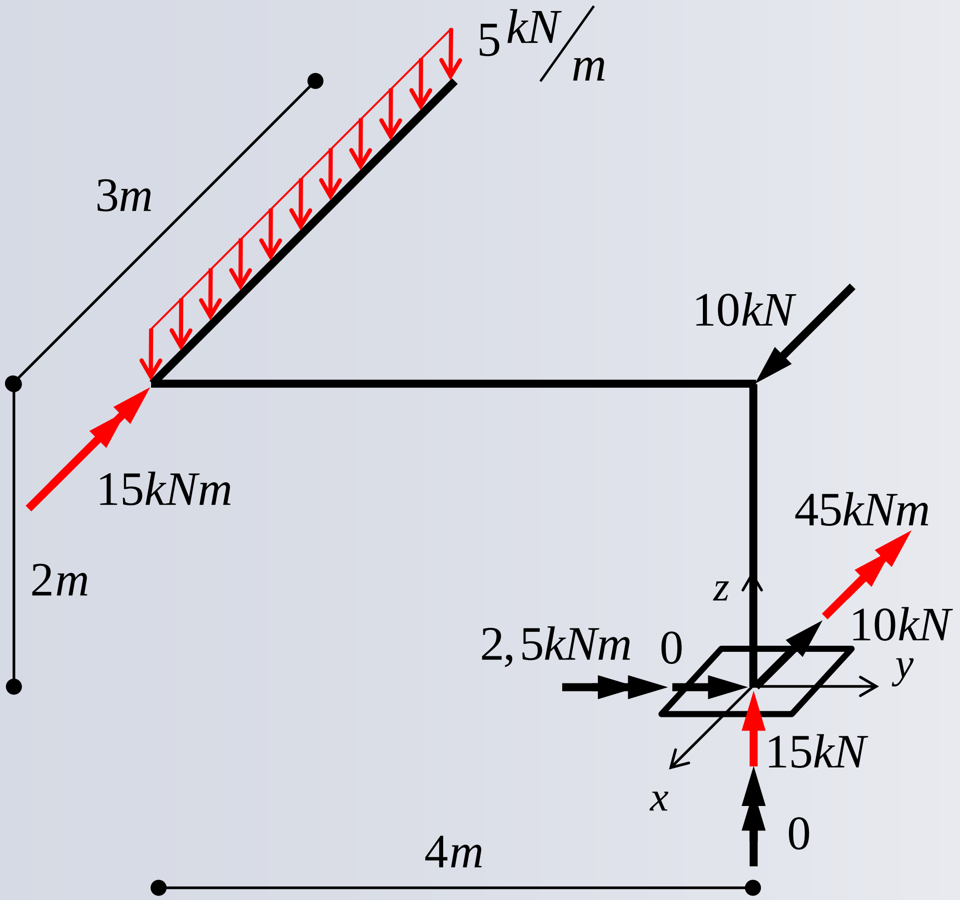


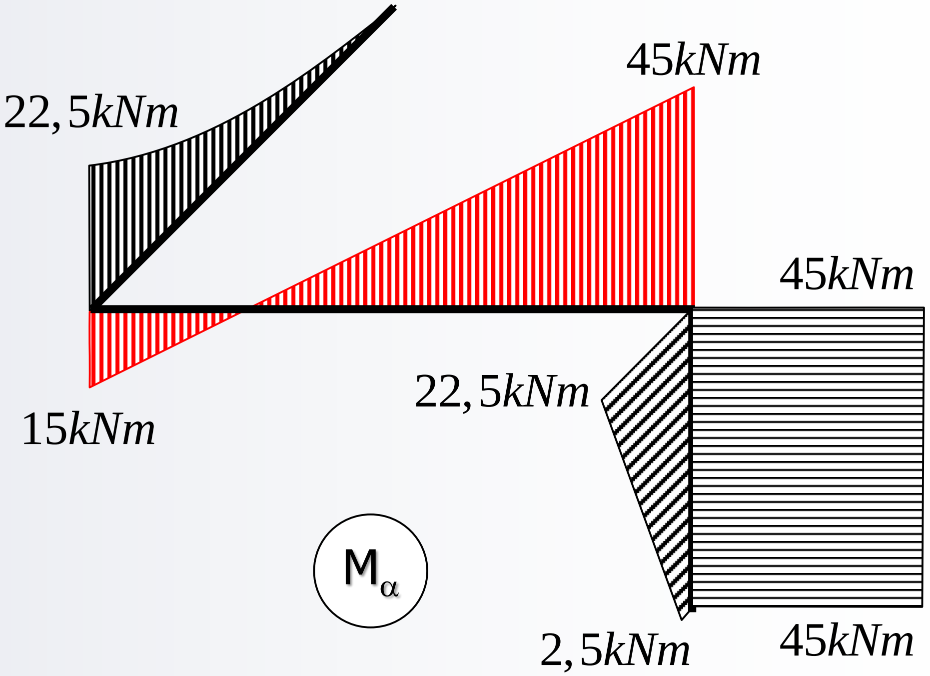
Momenty zginające



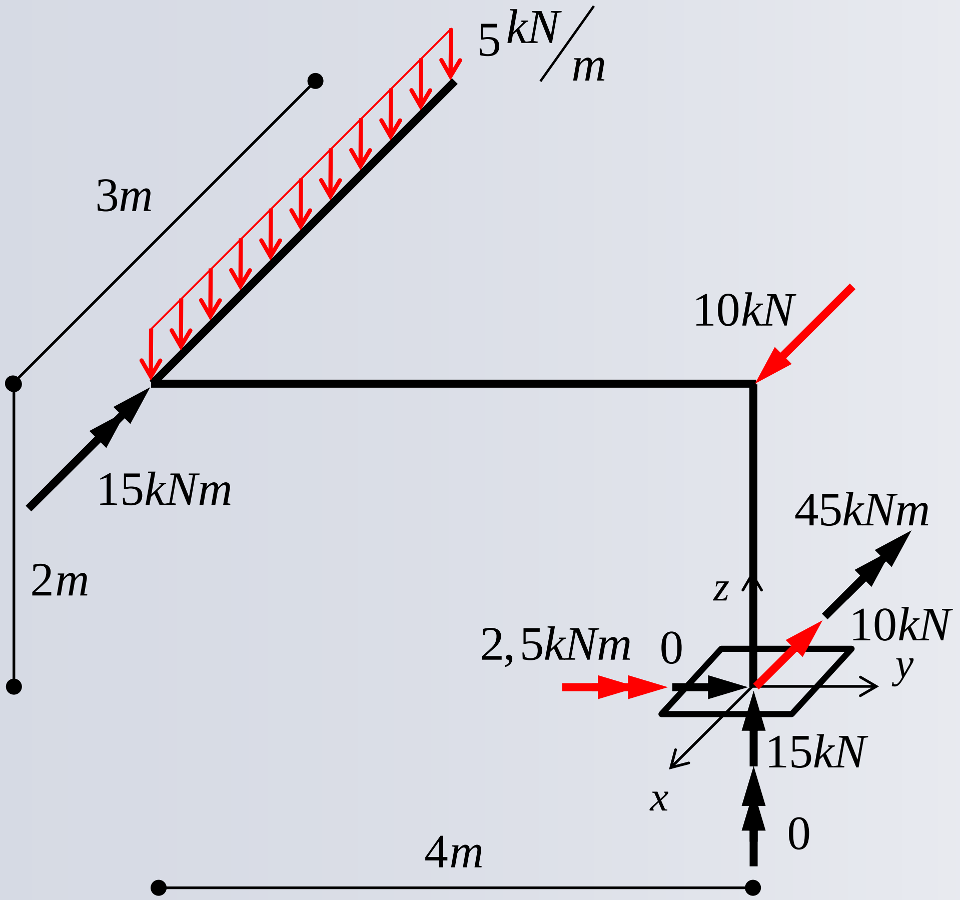


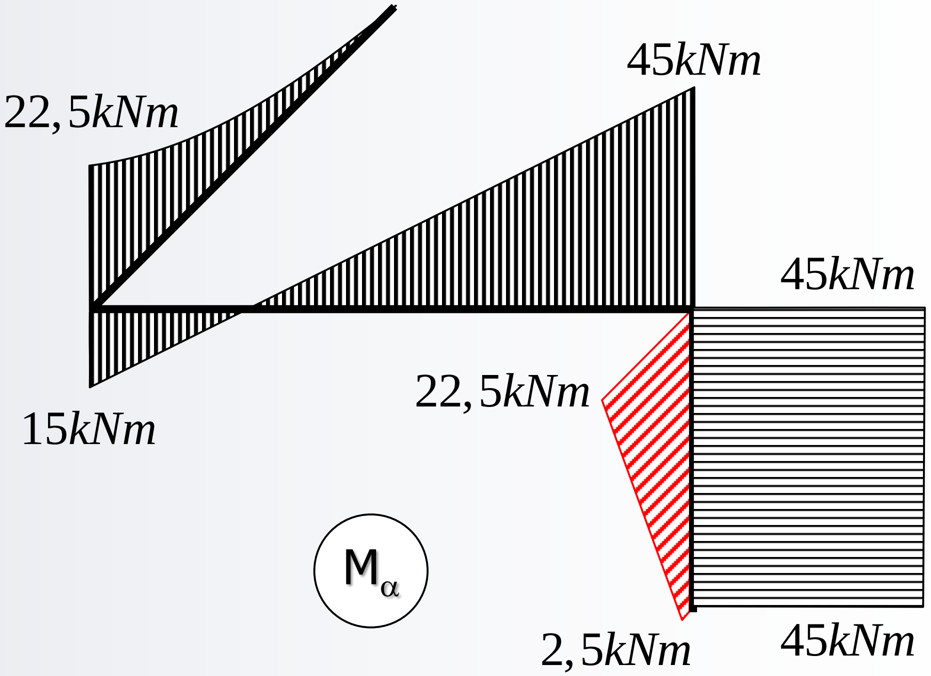
Momenty zginające



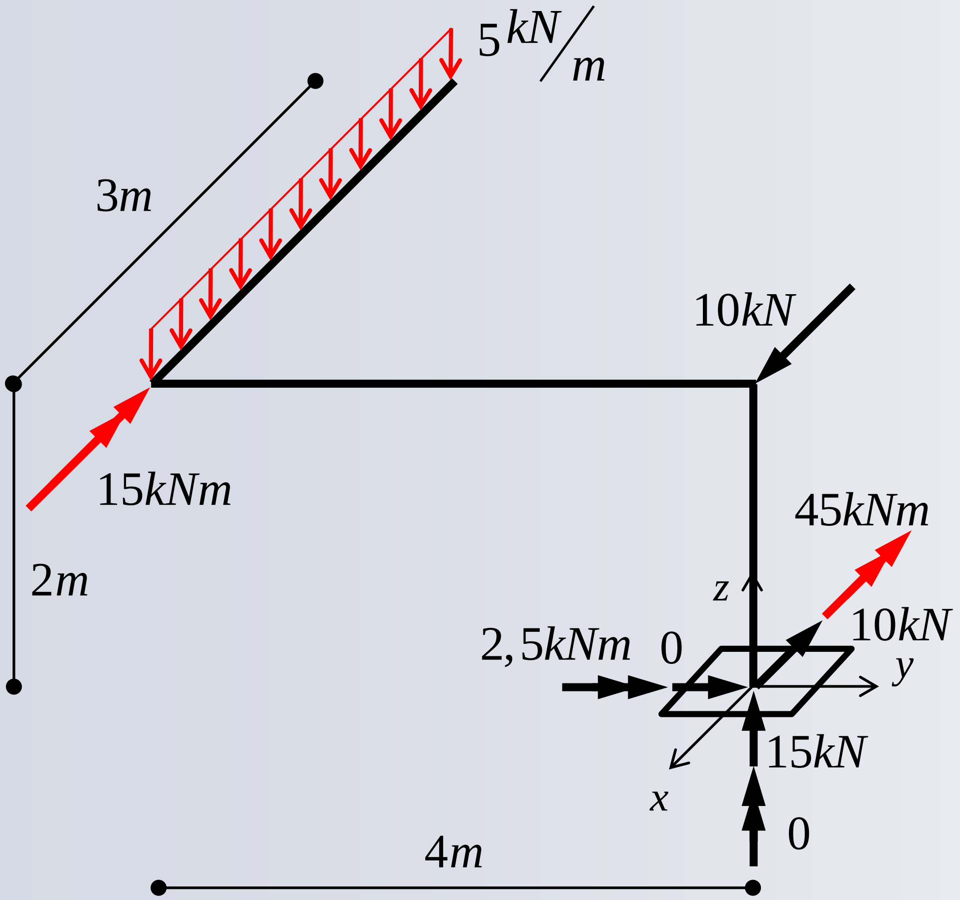


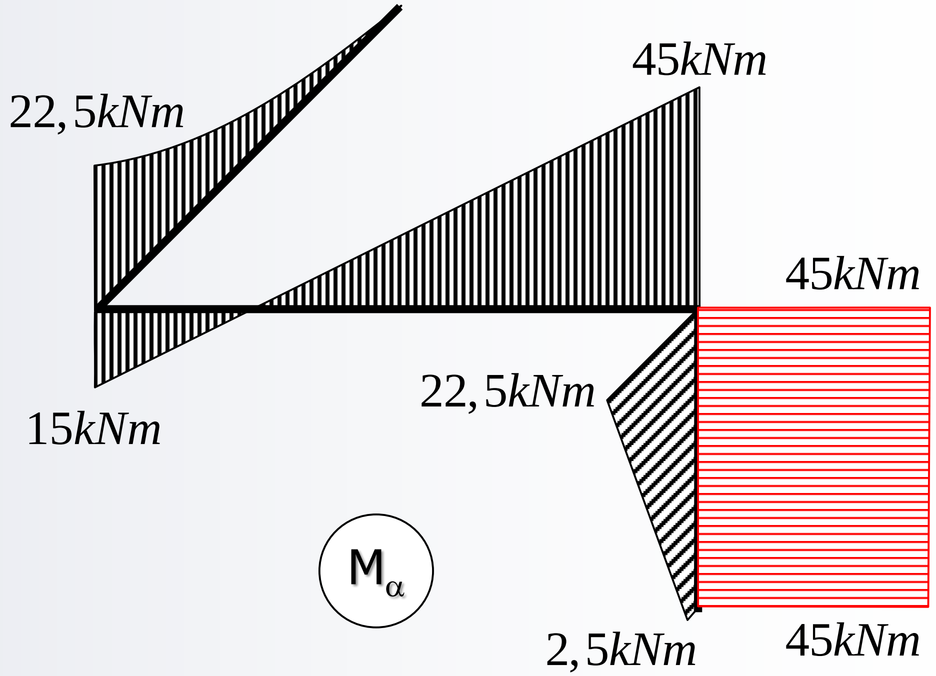
Momenty zginające



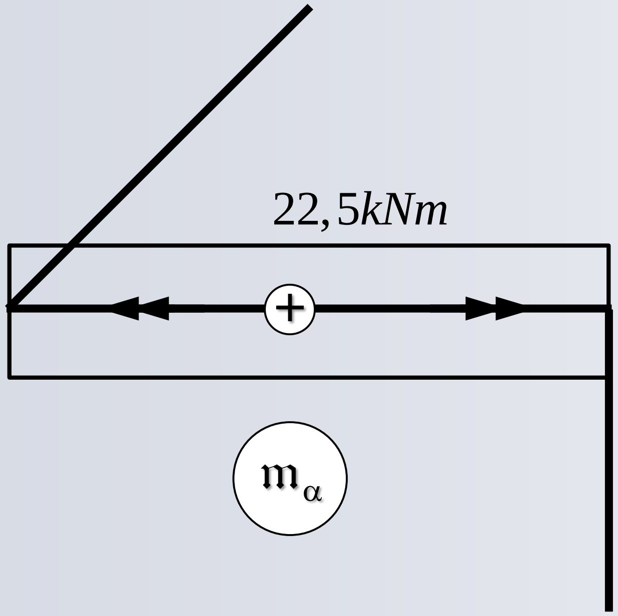


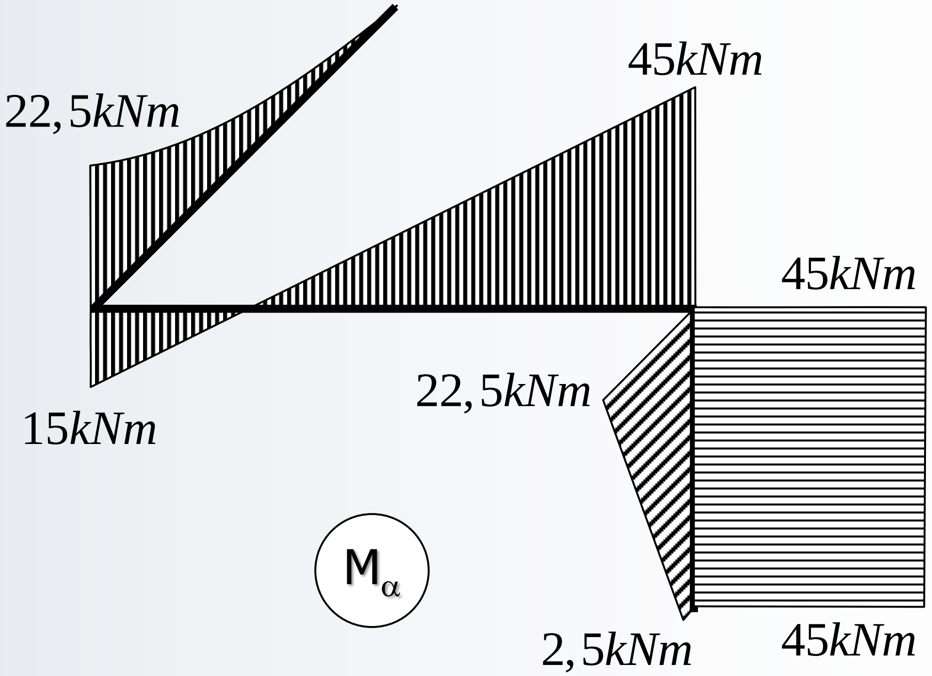
Momenty zginające



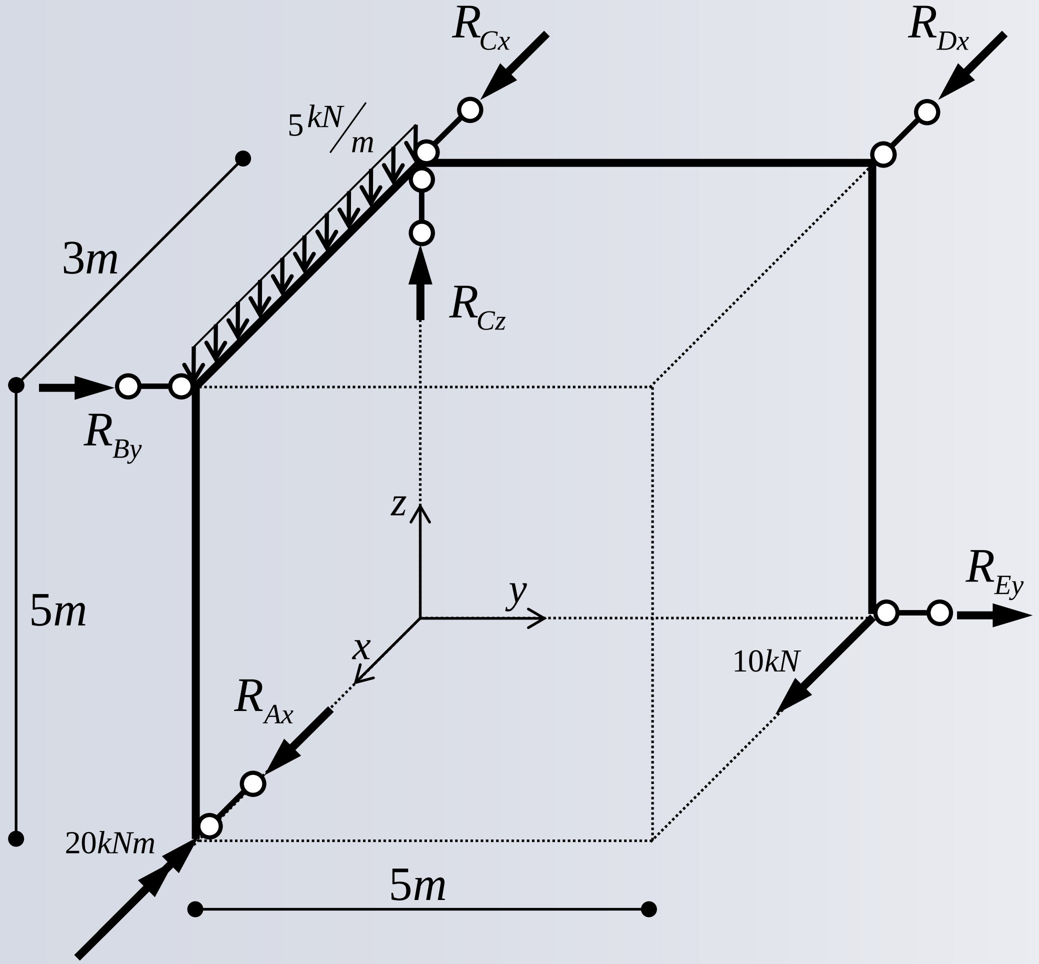


Momenty skręcające i zginające





Przykład 3 – rama przestrzenna



$$\sum X = R_{Ax} + R_{Cx} + R_{Dx} + 10kN = 0$$

$$\sum Y = R_{By} + R_{Ey} = 0$$

$$\sum Z = R_{Cz} - 5\,{}^{kN}\!/_{m} \cdot 3m = 0$$

$$\sum M_x = -R_{By} \cdot 5m - 20kNm = 0$$

$$\sum M_y = 5\,{}^{kN}\!/_{m} \cdot 3m \cdot \frac{1}{2} 3m + R_{Cx} \cdot 5m + R_{Dx} \cdot 5m = 0$$

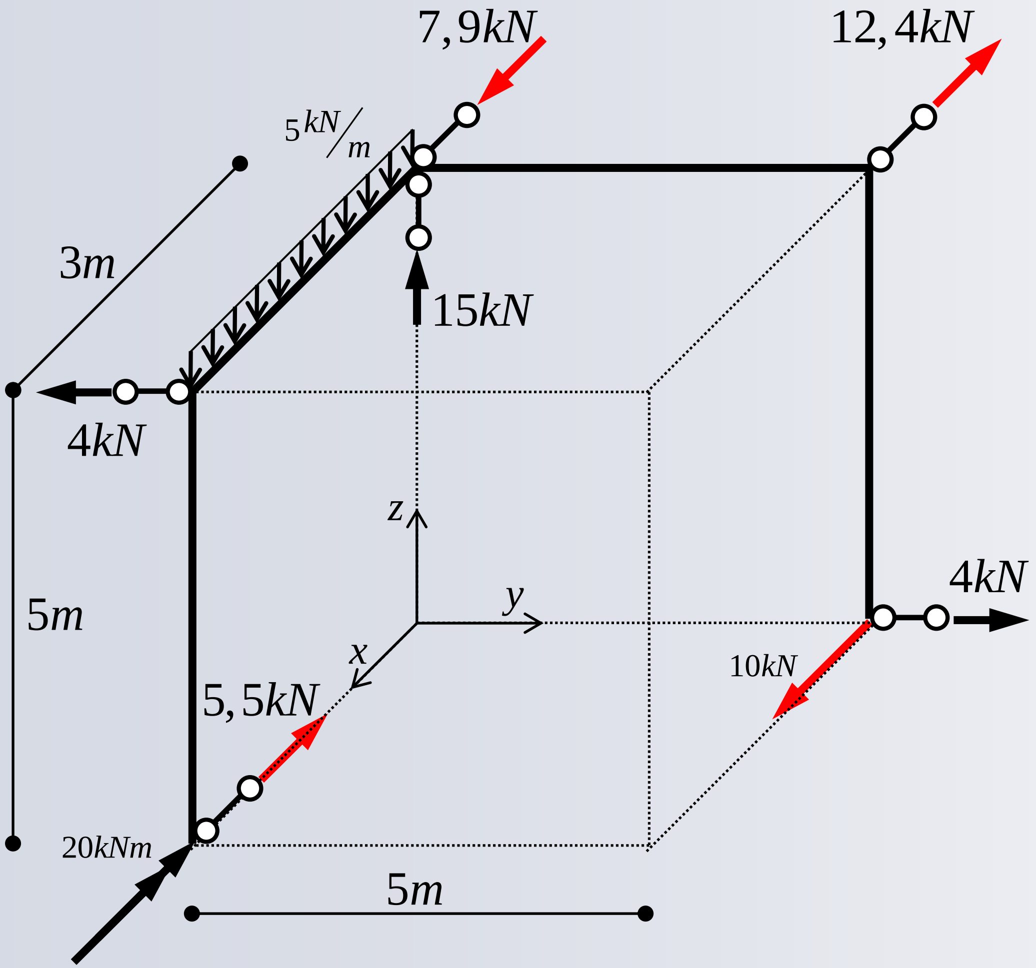
$$\sum M_z = R_{By} \cdot 3m - R_{Dx} \cdot 5m - 10kN \cdot 5m = 0$$

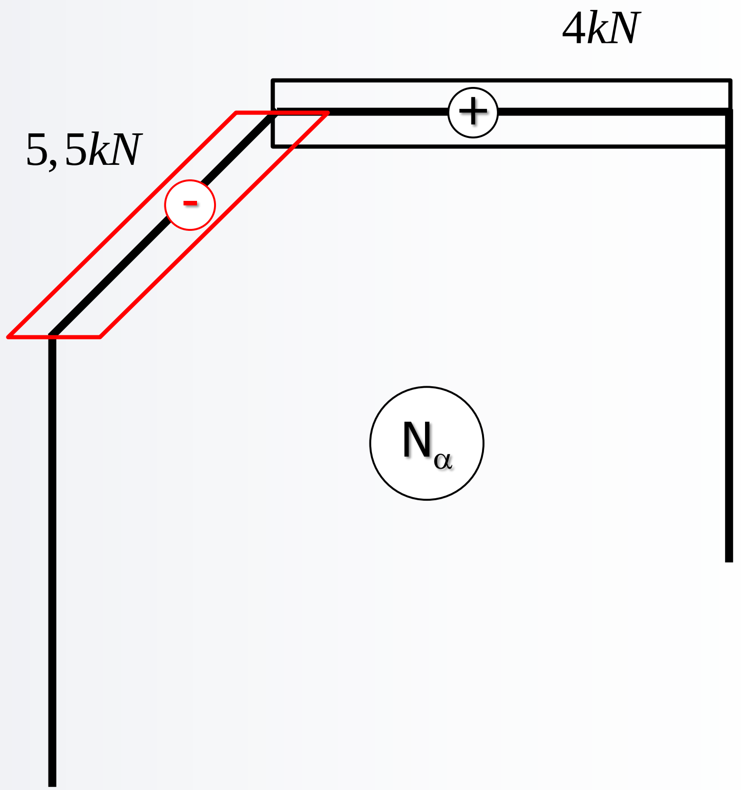
$R_{Ax} = -5{,}5kN$ $\qquad R_{By} = -4kN$

$R_{Dx} = -12{,}4kN$ $\qquad R_{Ey} = 4kN$

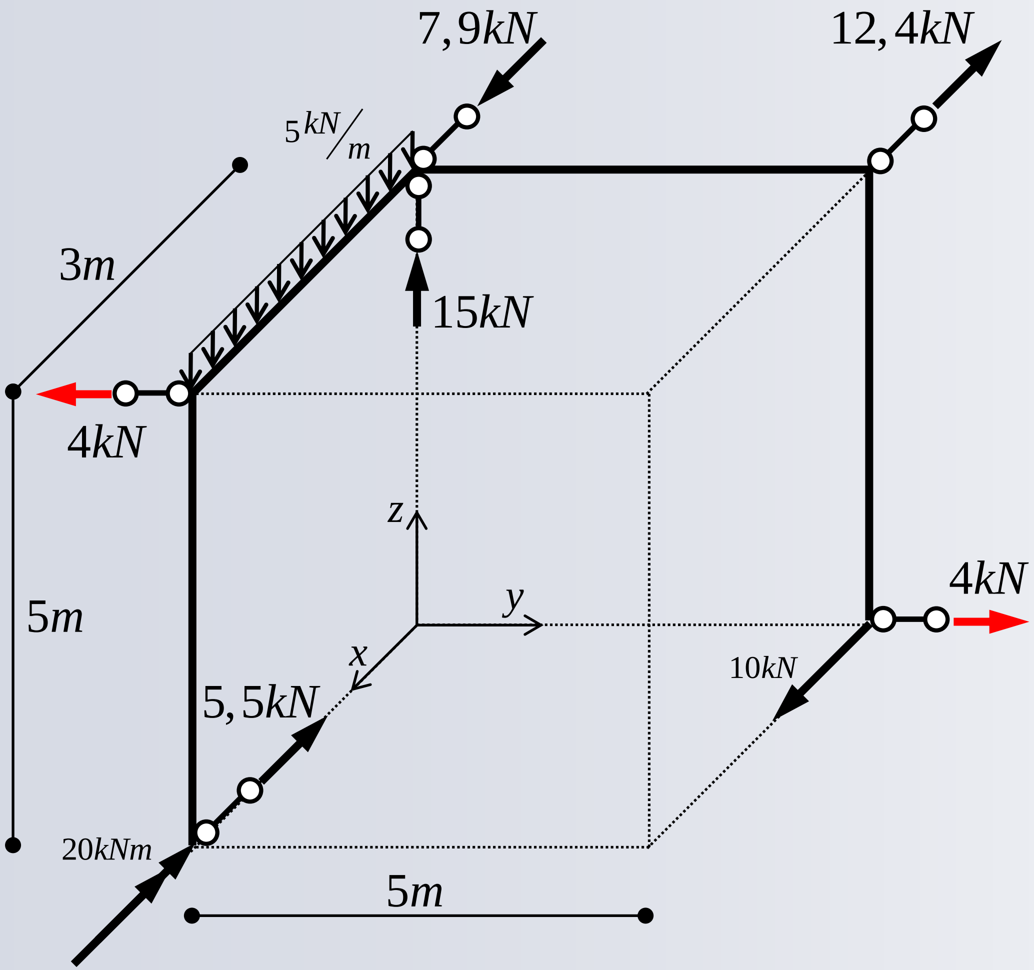
$R_{Cx} = 7{,}9kN$ $\qquad R_{Cz} = 15kN$

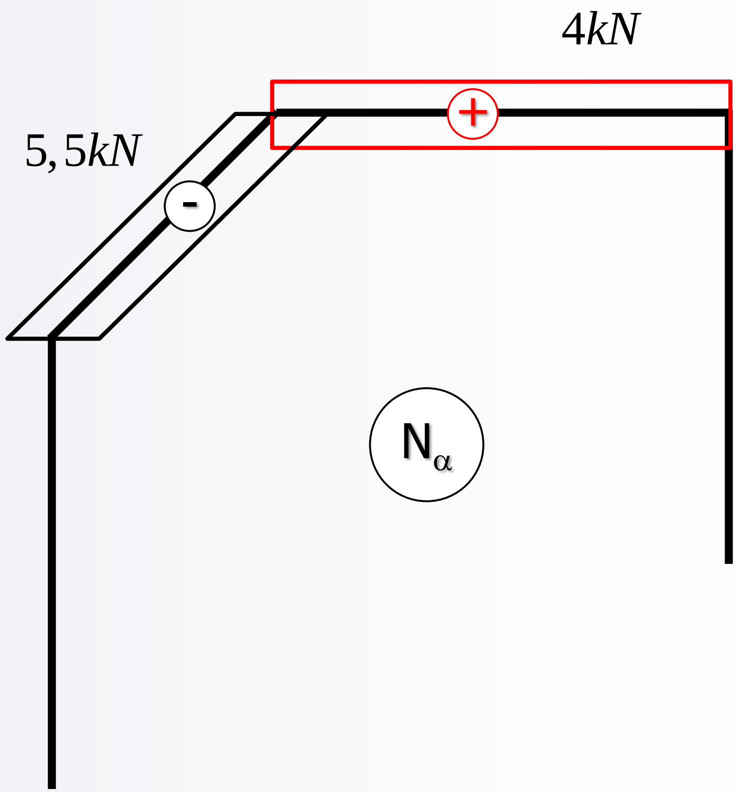
Siły normalne



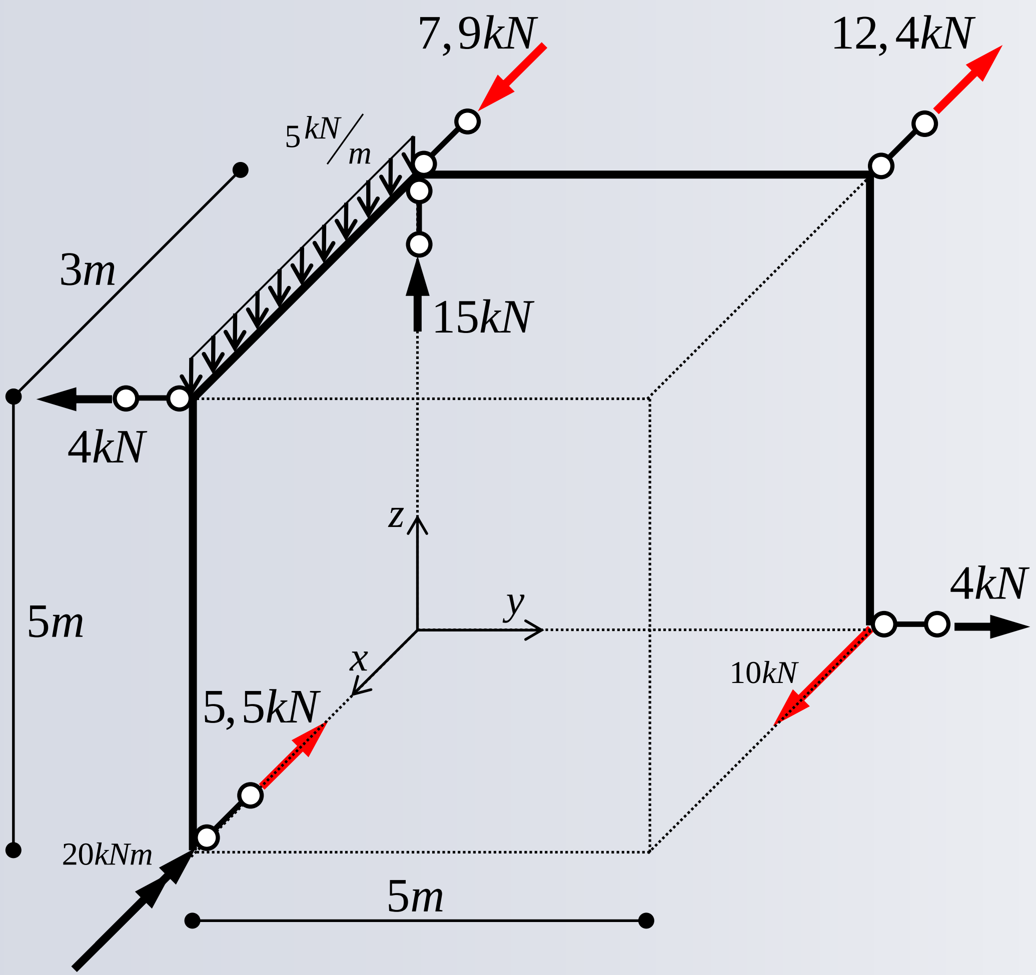


Siły normalne

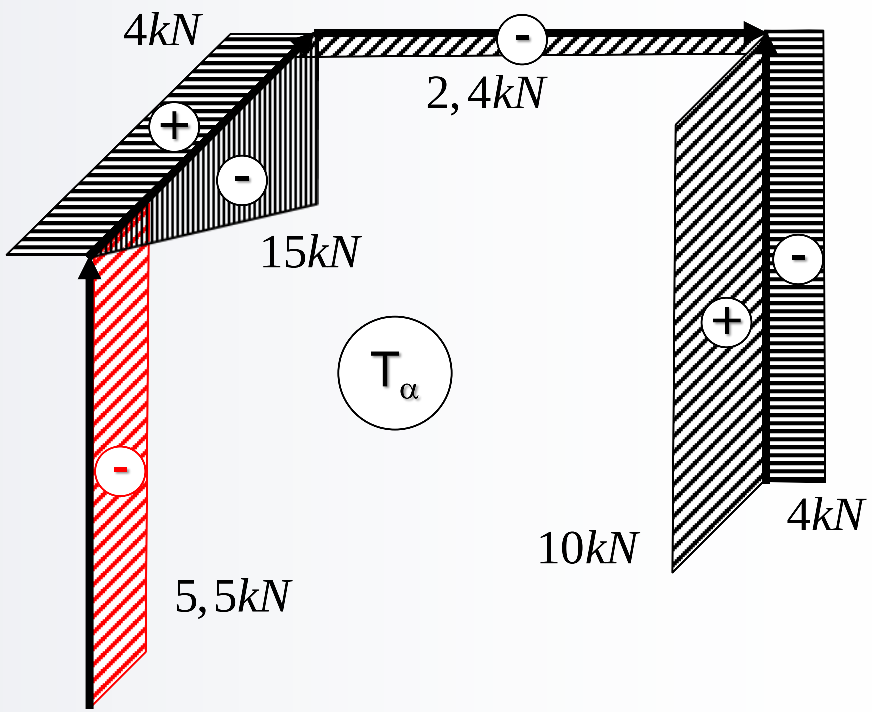




Siły tnące

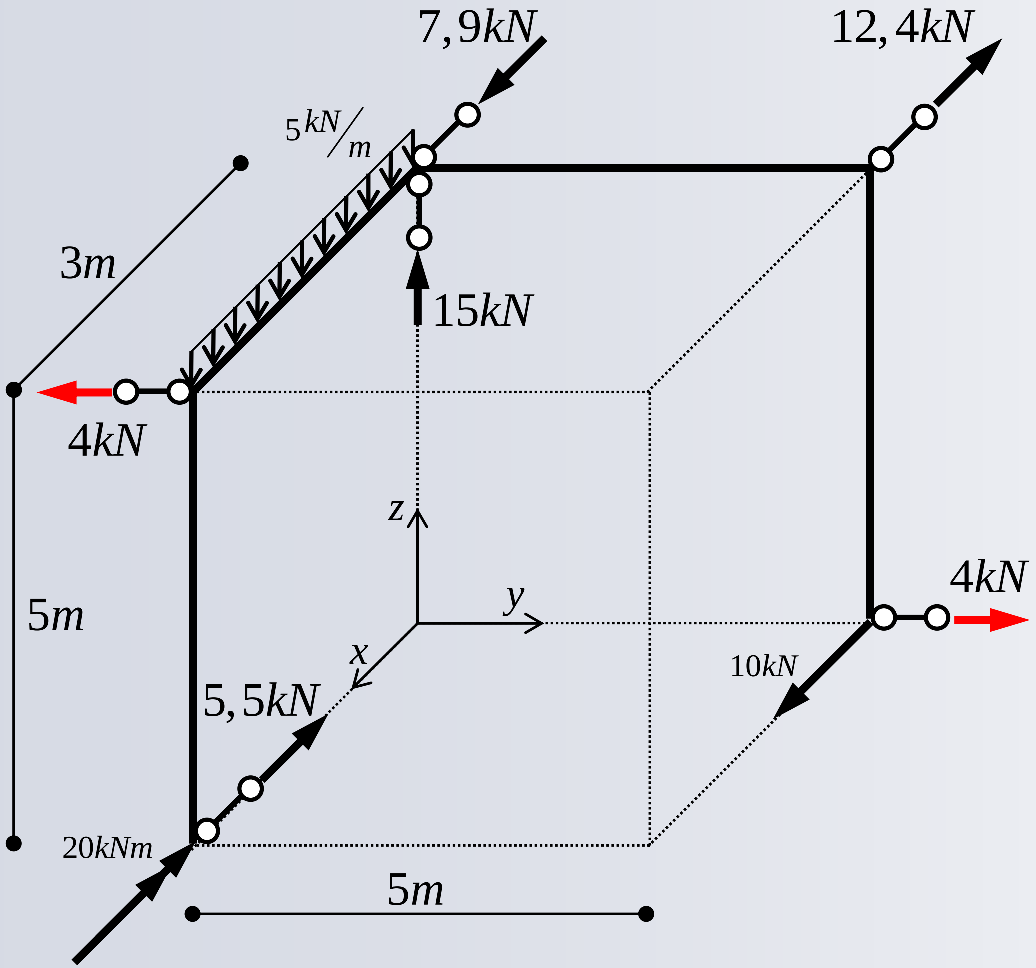


$7,9kN$
$12,4kN$
$5\, {kN}/{m}$
$3m$
$15kN$
$4kN$
z
y
x
$5m$
$4kN$
$10kN$
$5,5kN$
$20kNm$
$5m$

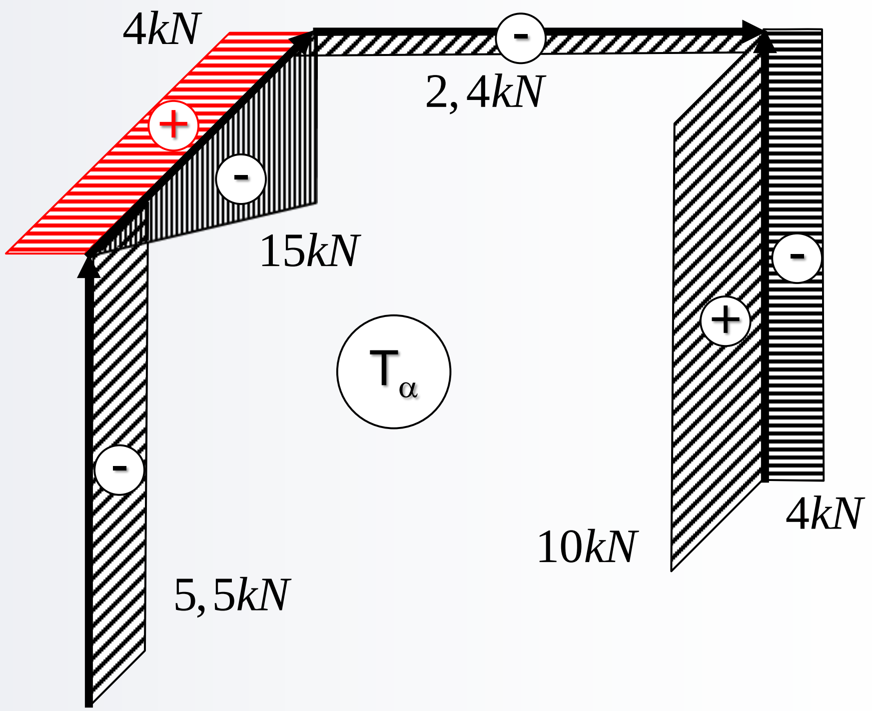


$4kN$
-
$2,4kN$
+
-
$15kN$
-
+
T_α
-
$4kN$
$10kN$
$5,5kN$

Siły tnące

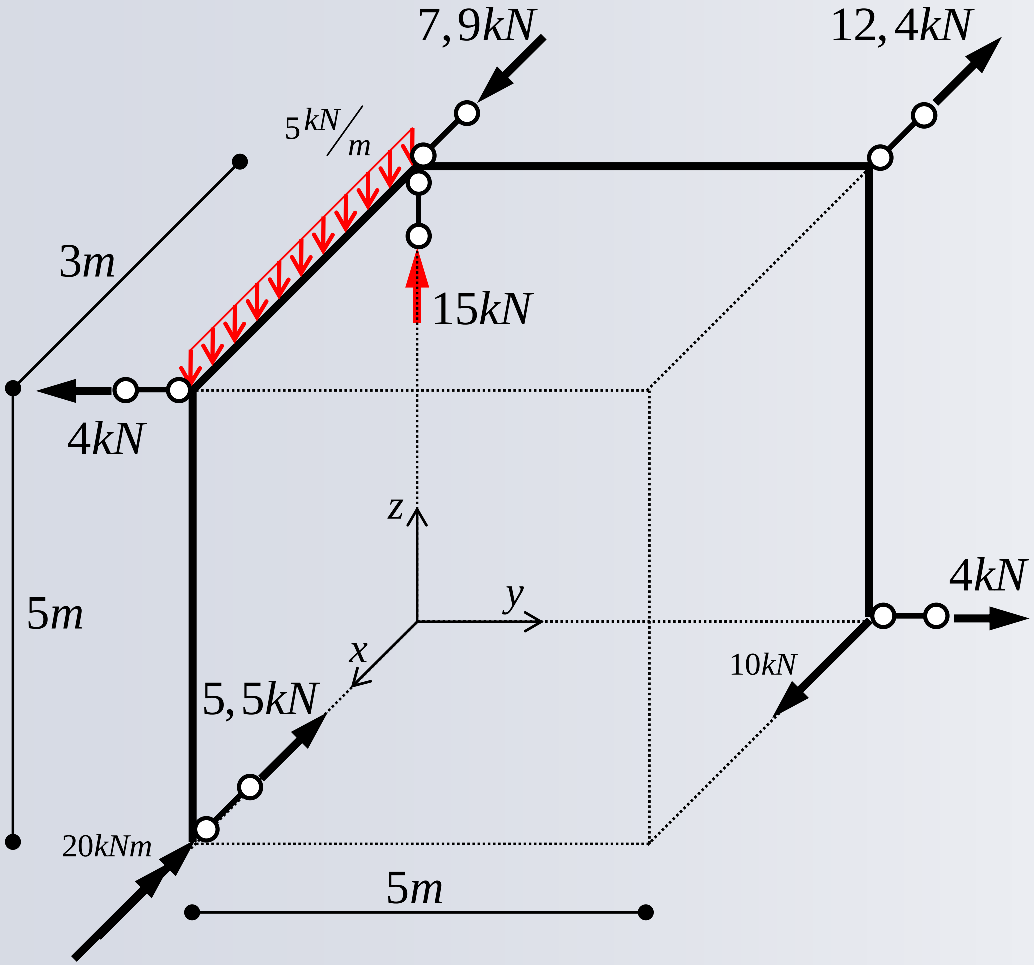


7,9kN
12,4kN
5 kN/m
3m
15kN
4kN
z
y
x
5m
5,5kN
10kN
4kN
20kNm
5m

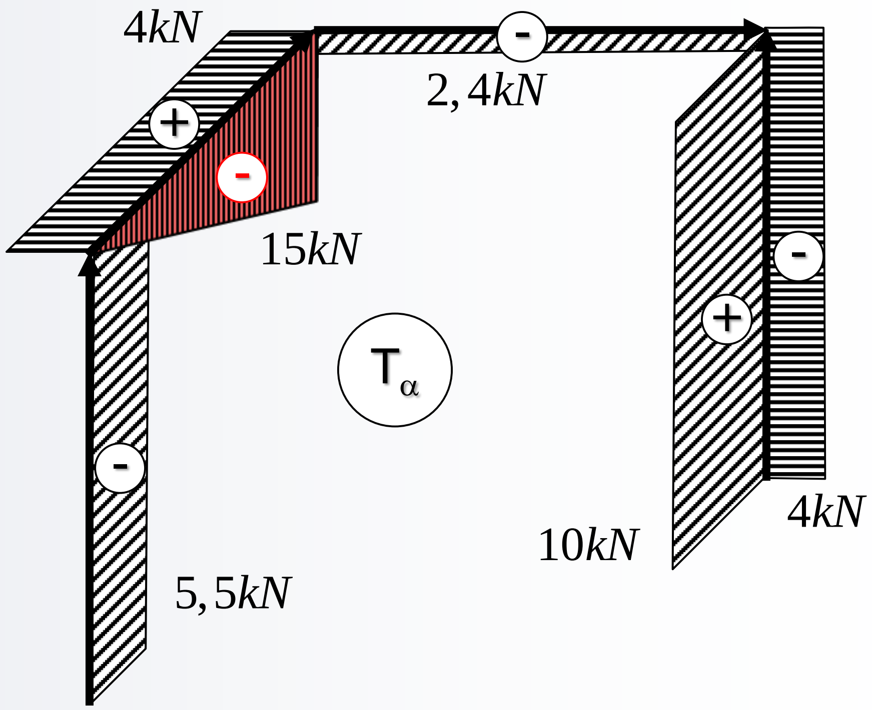


4kN
-
2,4kN
+
-
15kN
-
+
Tα
-
10kN
4kN
5,5kN

Siły tnące

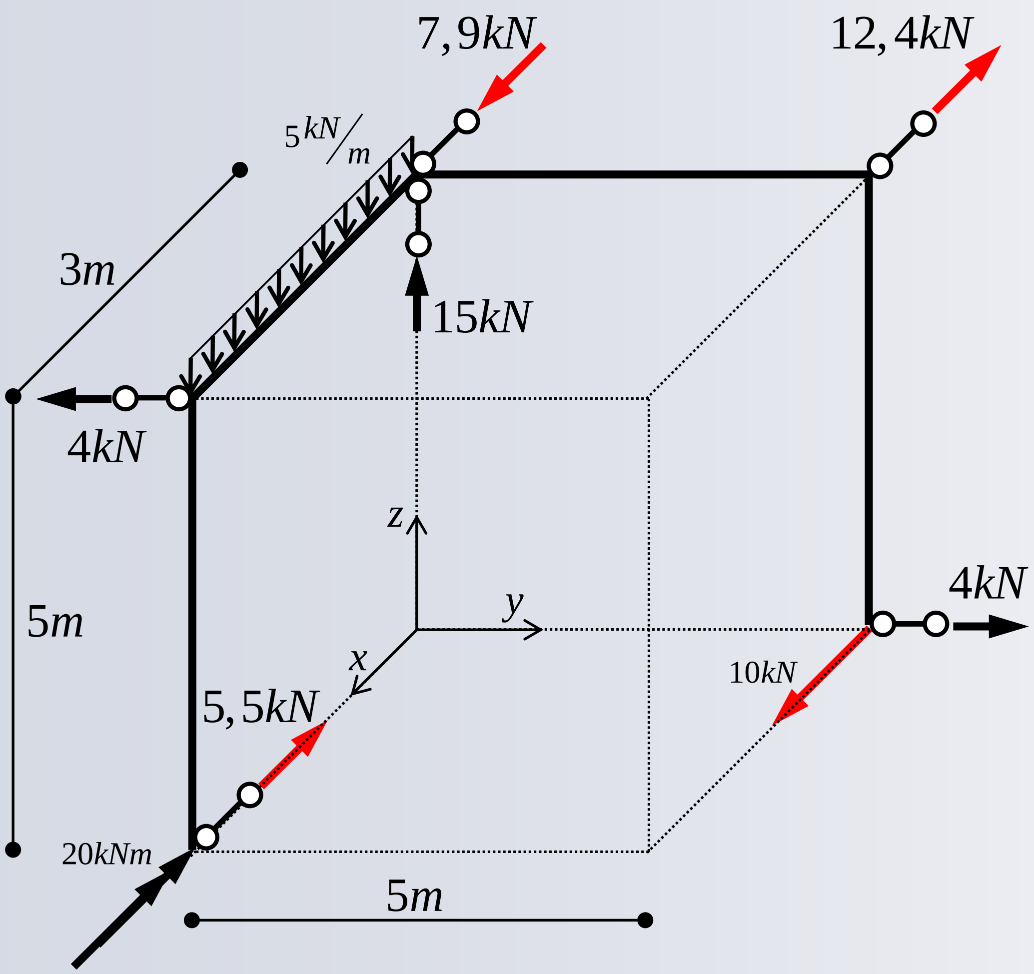


$7,9kN$
$12,4kN$
$5\,{}^{kN}\!/_{m}$
$3m$
$15kN$
$4kN$
z
y
x
$5m$
$4kN$
$10kN$
$5,5kN$
$20kNm$
$5m$

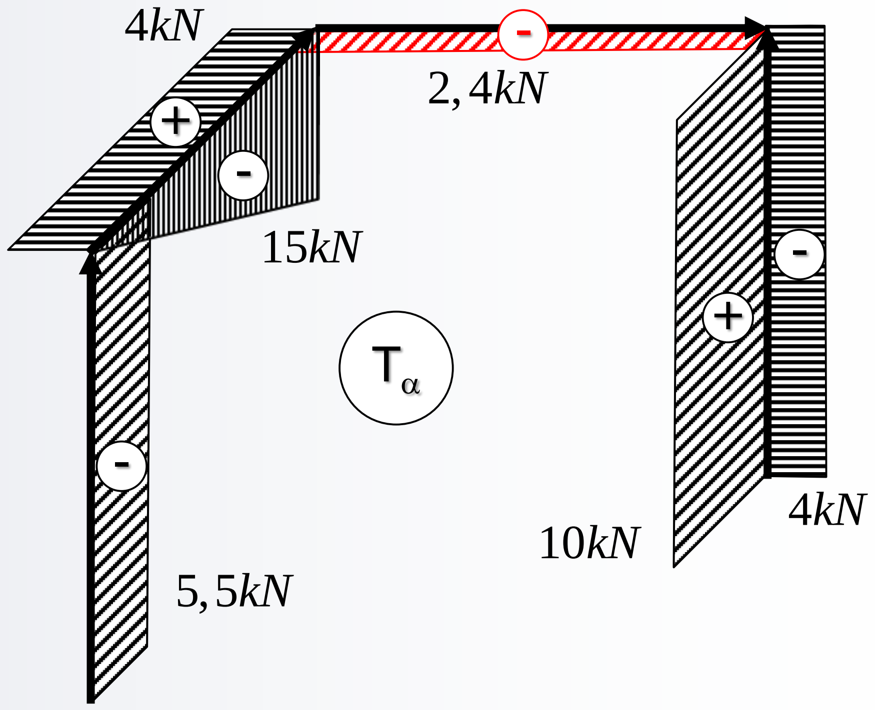


$4kN$
$2,4kN$
$15kN$
T_α
$10kN$
$4kN$
$5,5kN$

Siły tnące

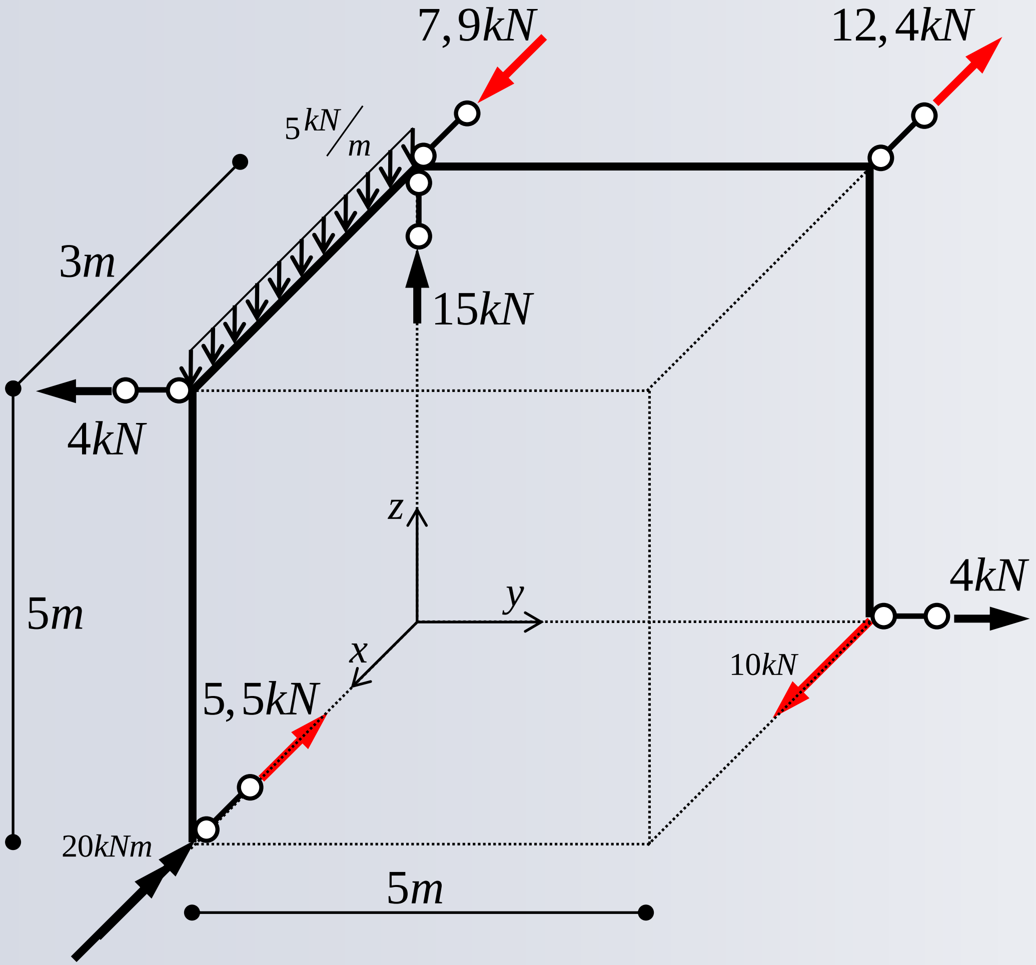


7,9kN
12,4kN
5kN/m
3m
15kN
4kN
z
y
x
5m
4kN
10kN
5,5kN
20kNm
5m

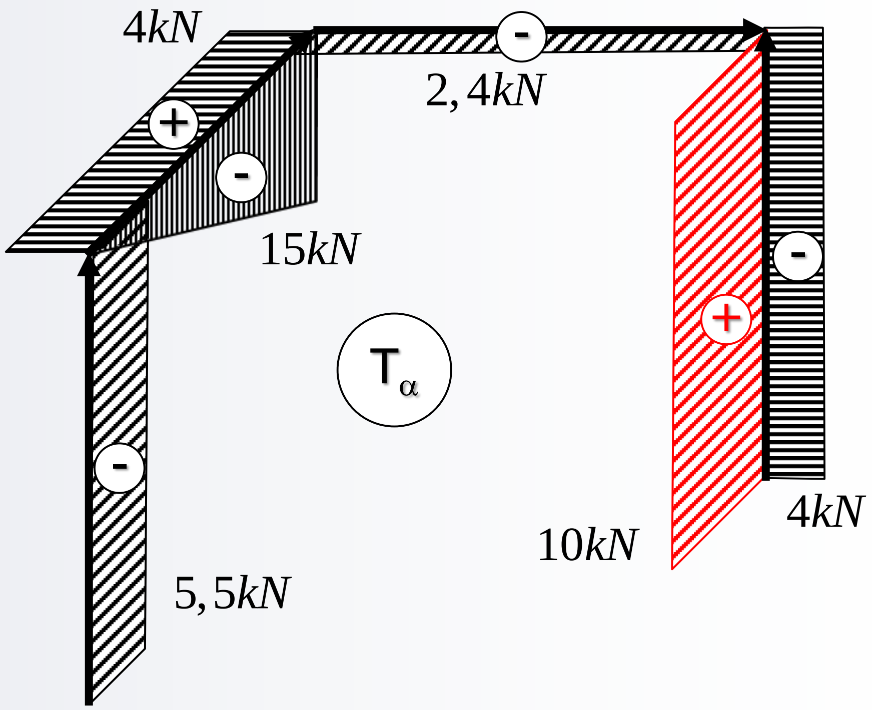


4kN
2,4kN
15kN
T_α
5,5kN
10kN
4kN

Siły tnące

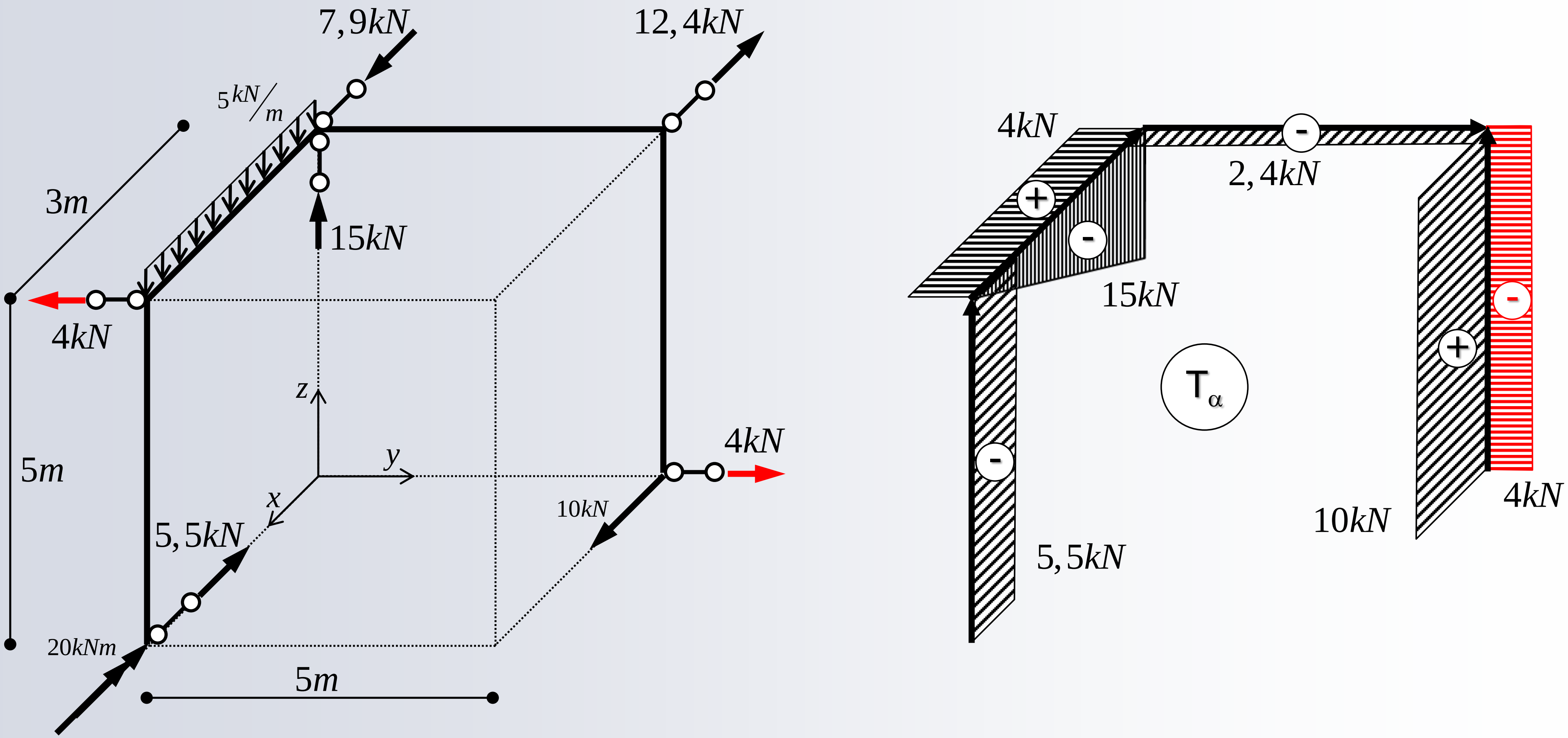


7,9kN
12,4kN
5 kN/m
3m
15kN
4kN
z
y
x
5m
5,5kN
10kN
4kN
20kNm
5m



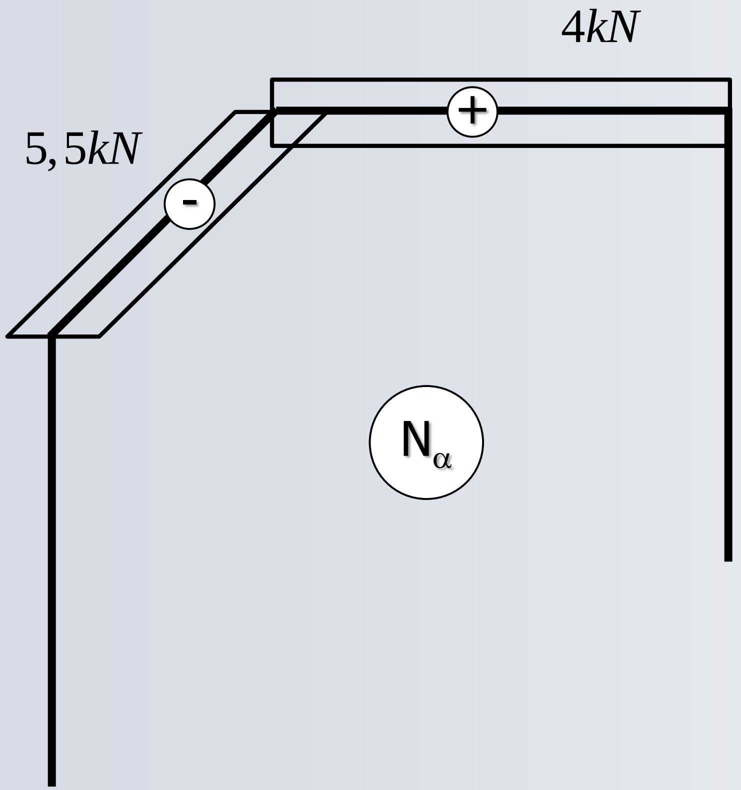
4kN
2,4kN
15kN
T_α
10kN
4kN
5,5kN

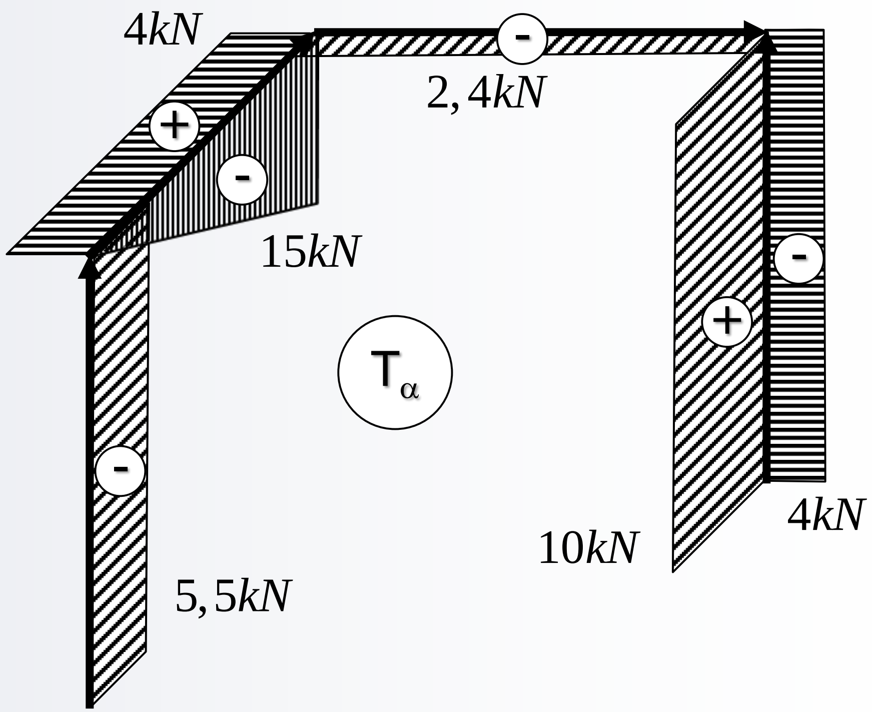
Siły tnące



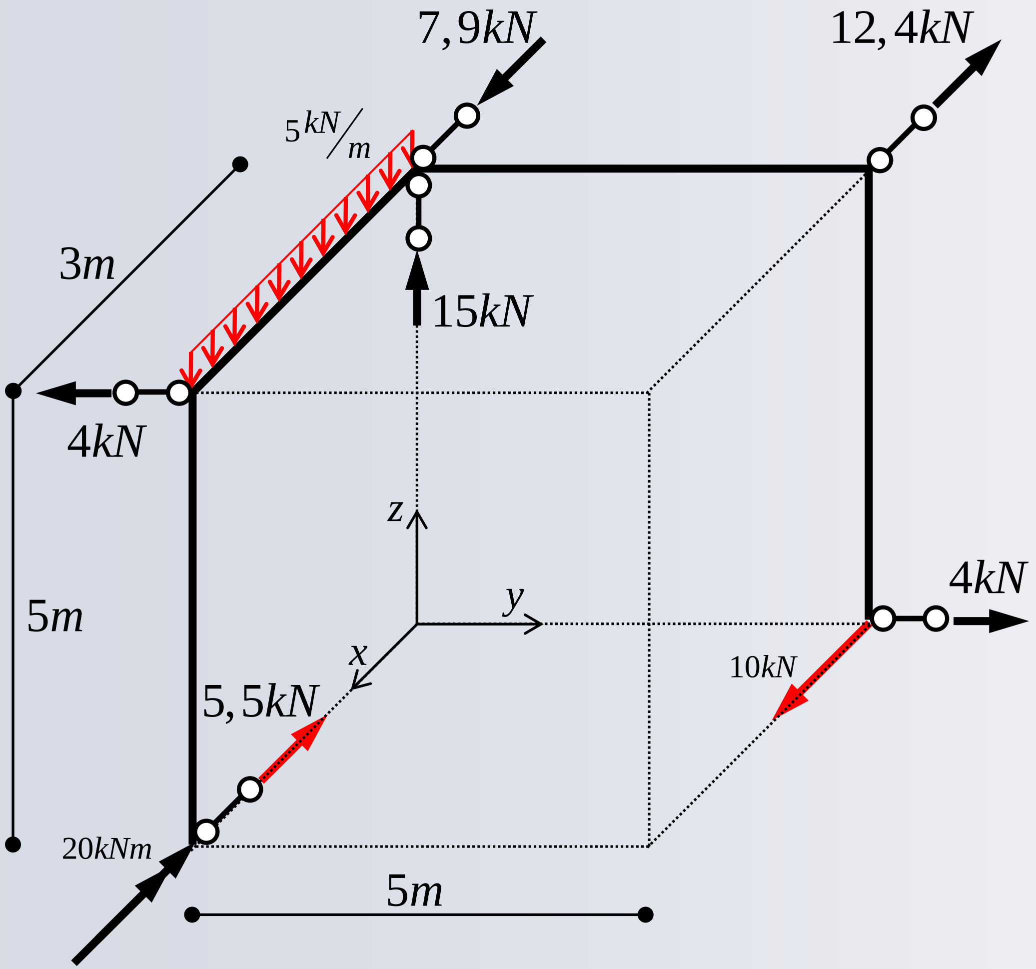
7,9kN
12,4kN
5 kN/m
3m
15kN
4kN
z
y
x
5m
4kN
10kN
5,5kN
20kNm
5m
4kN
2,4kN
+
-
15kN
Tα
-
+
-
5,5kN
10kN
4kN

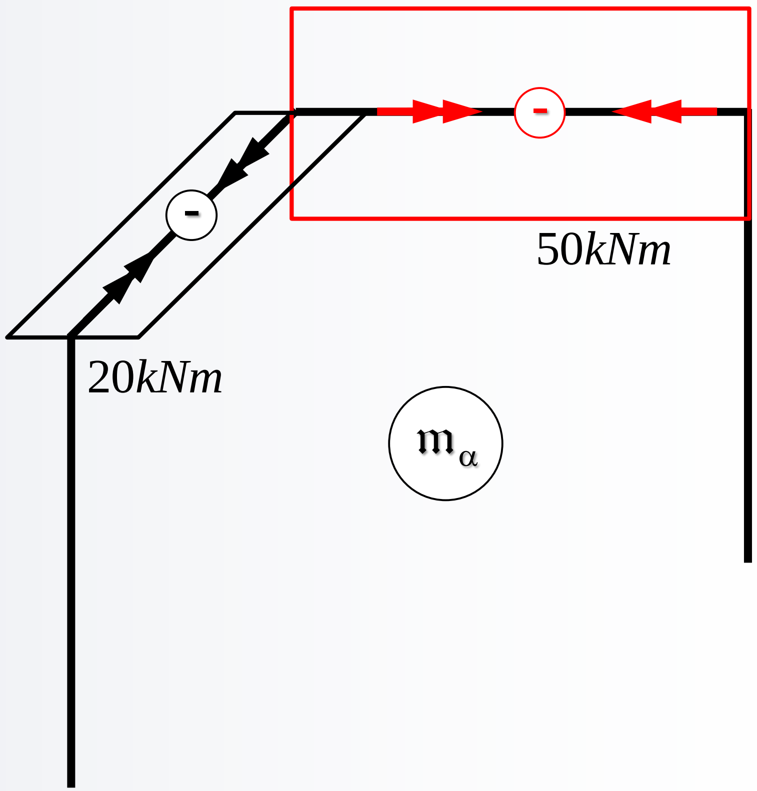
Siły normalne i tnące



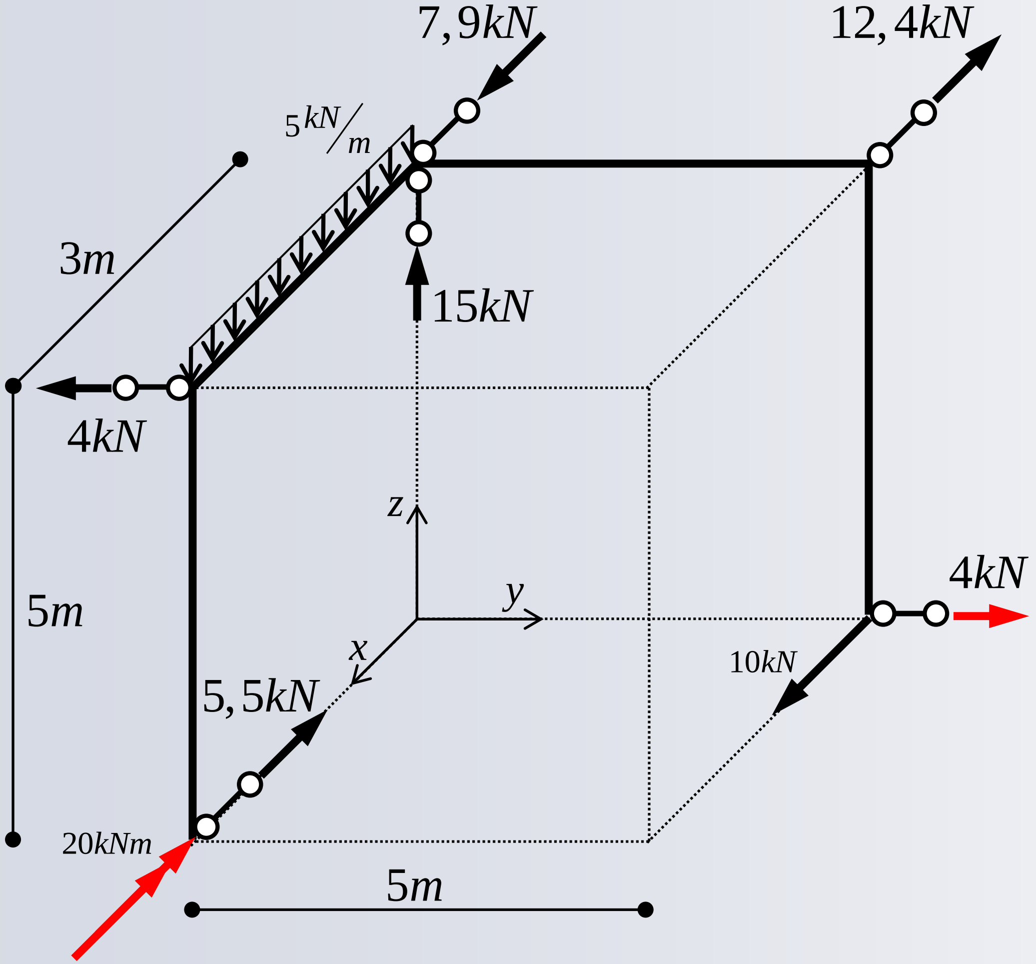


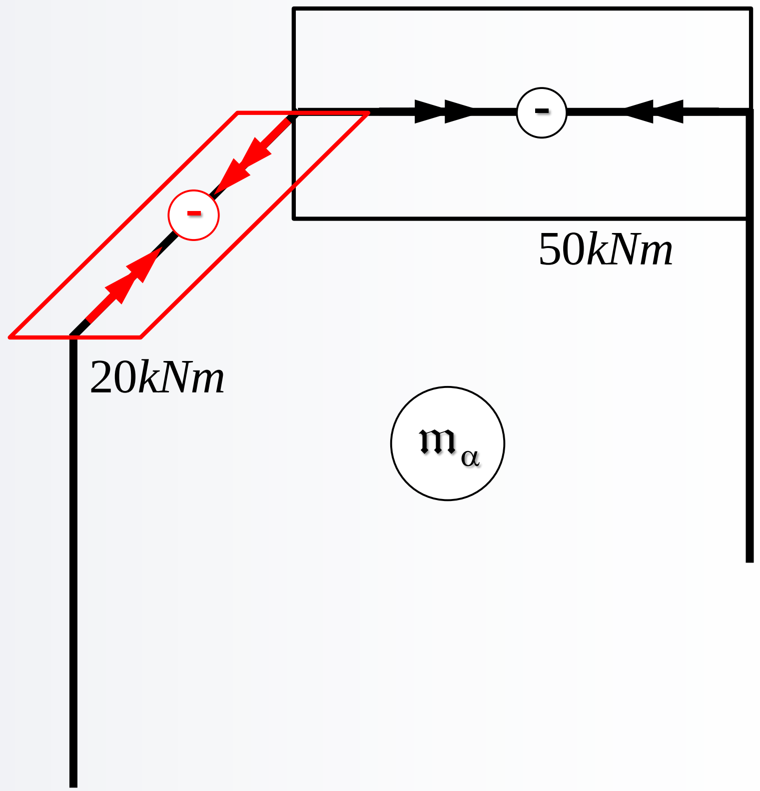
Momenty skręcające



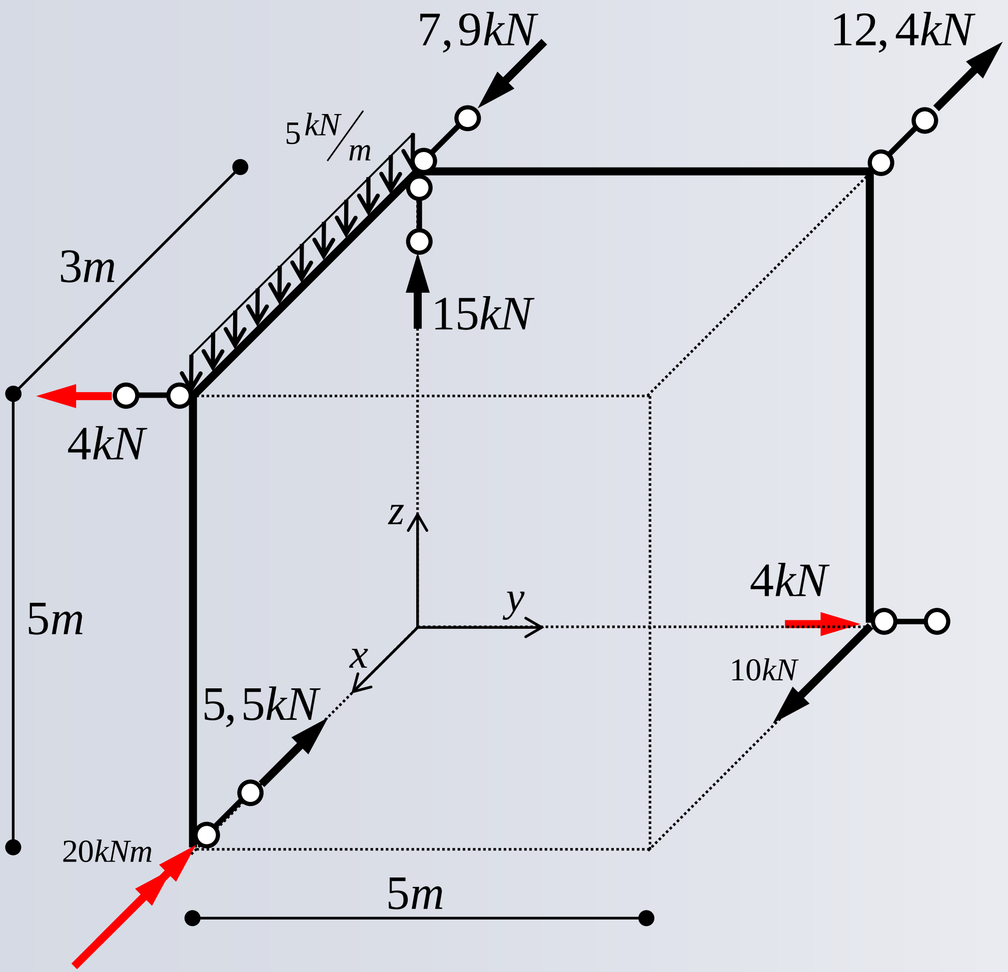


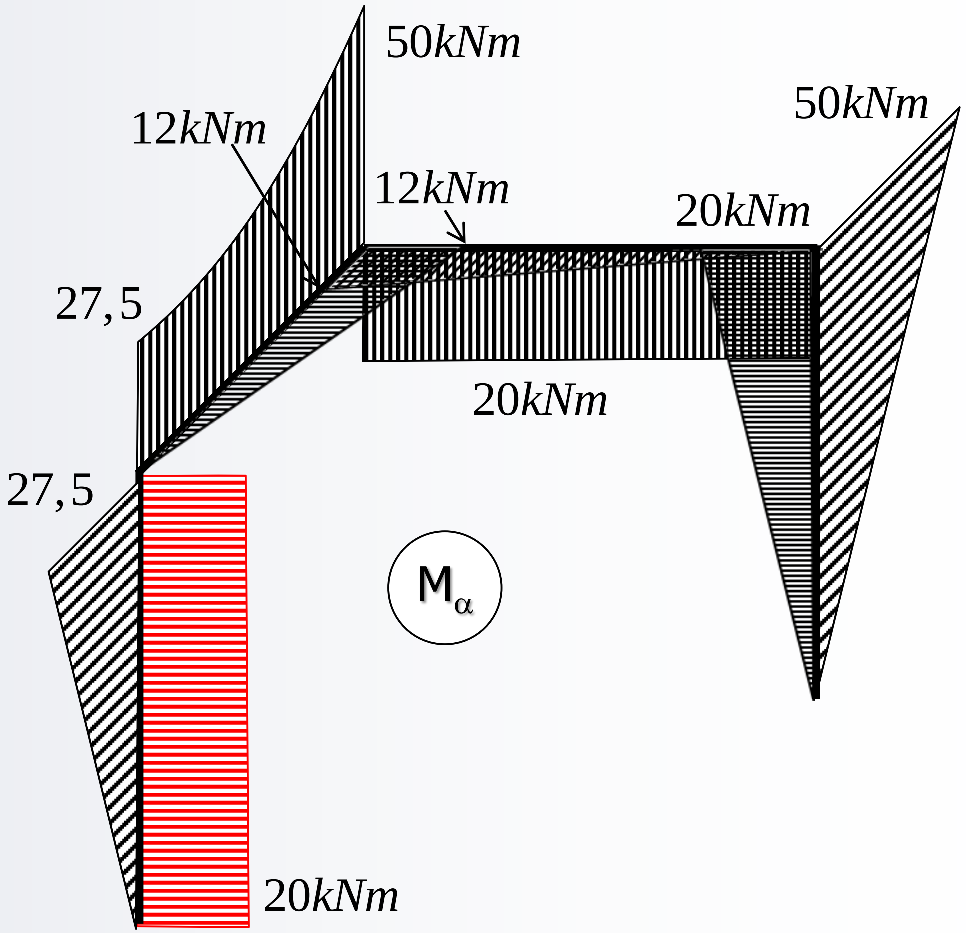
Momenty skręcające



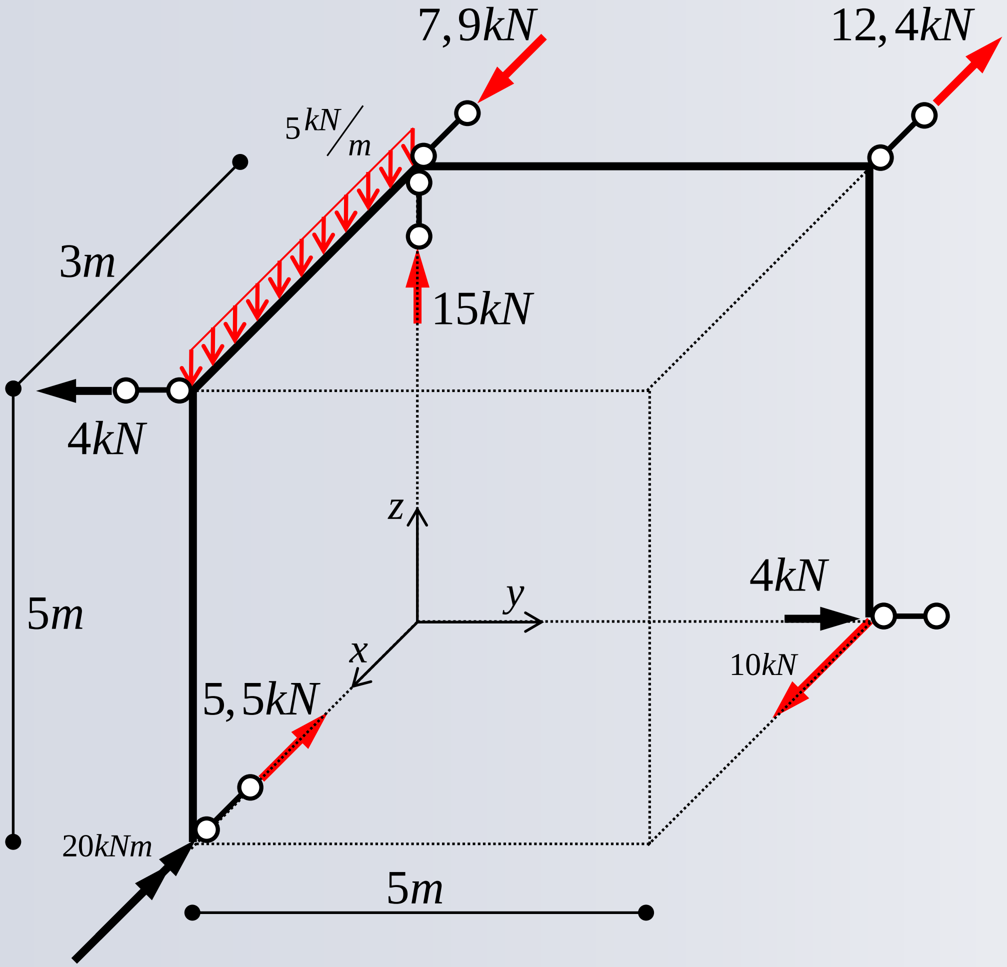


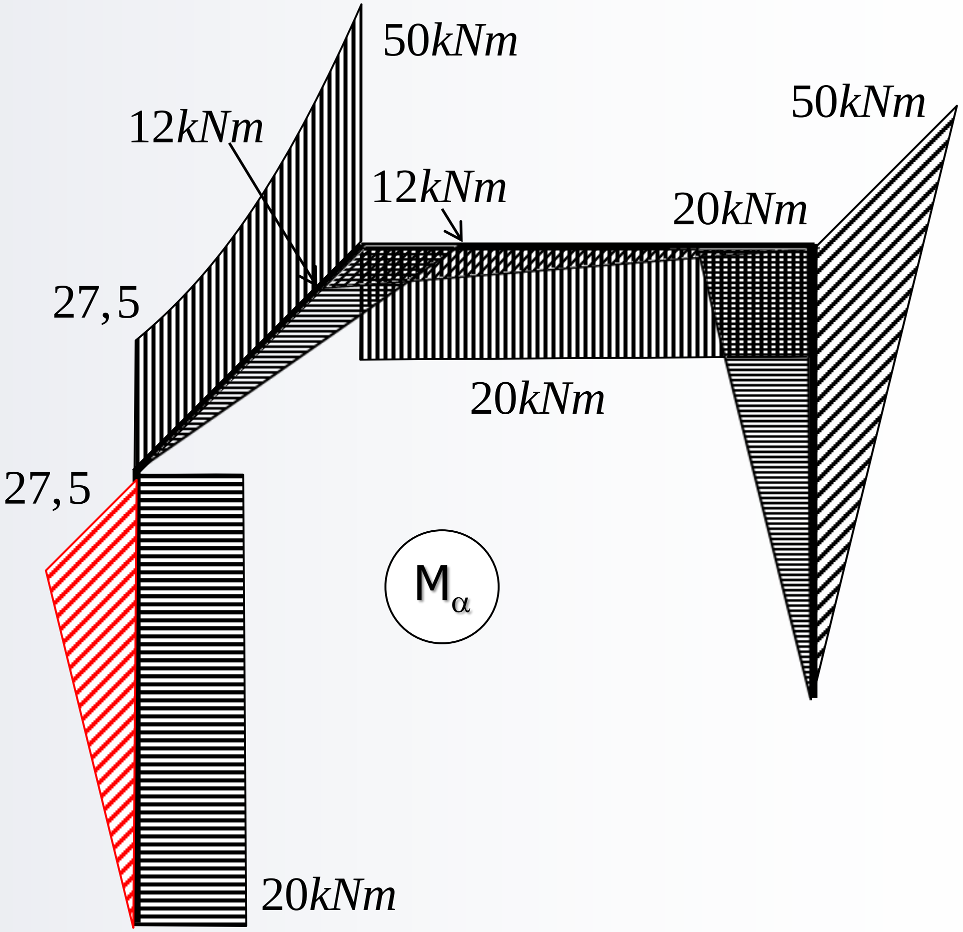
Momenty zginające



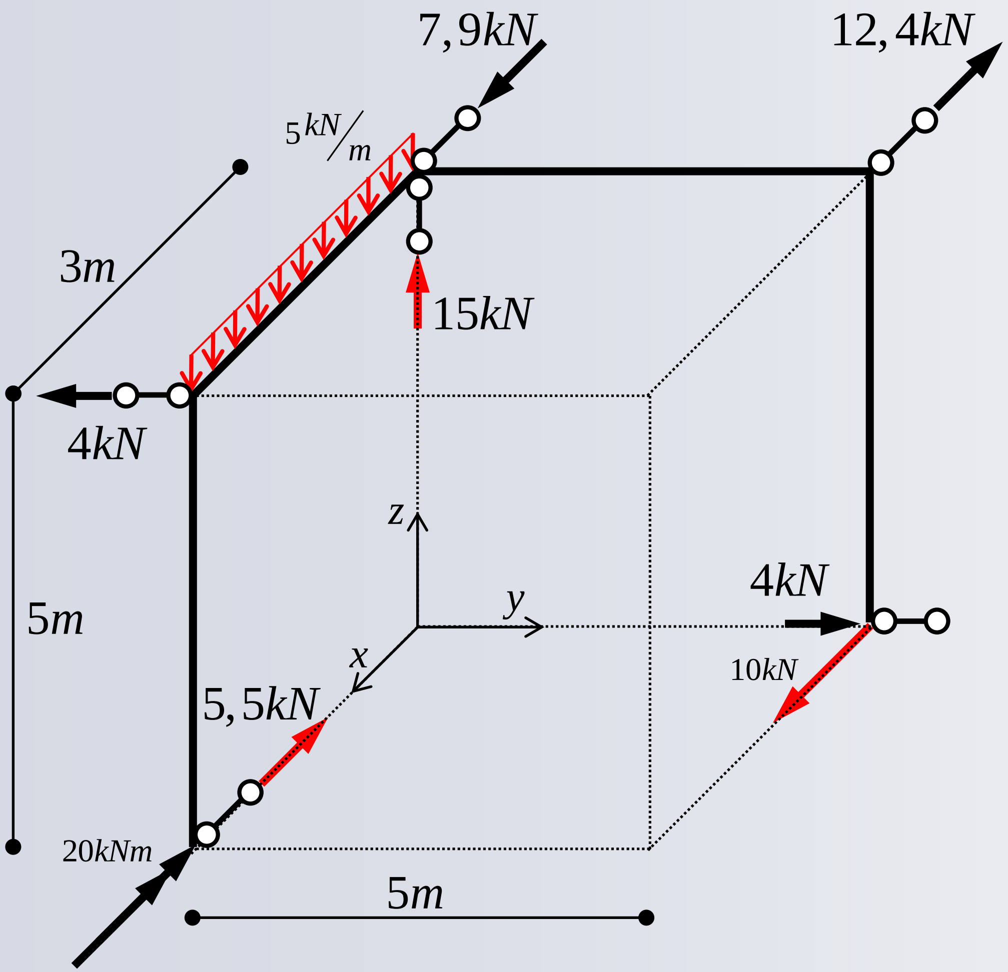


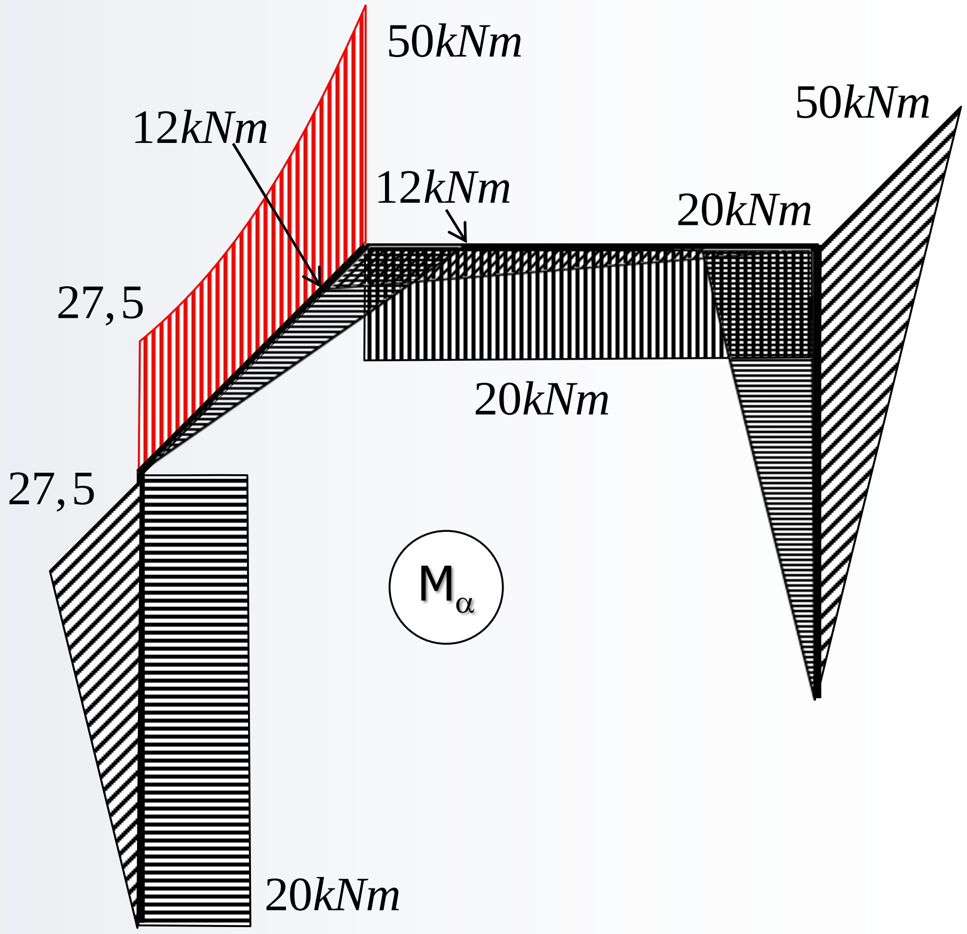
Momenty zginające



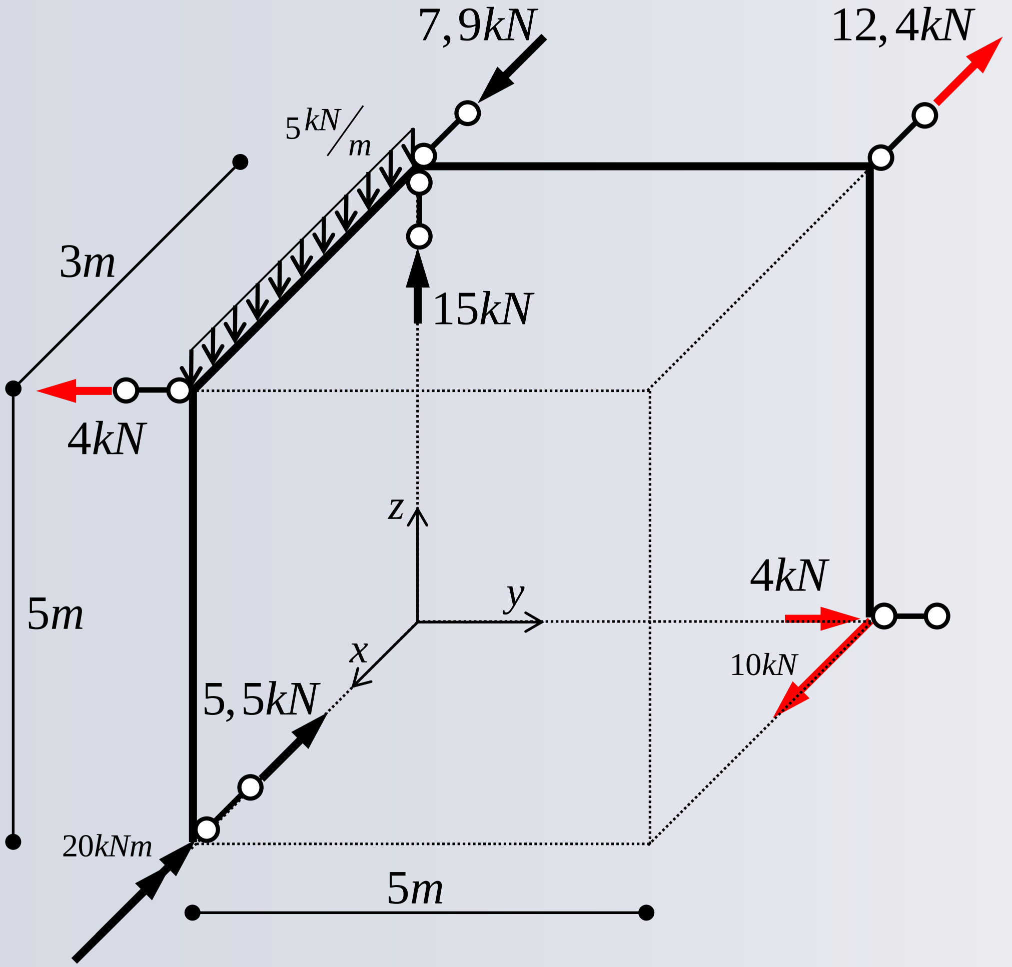


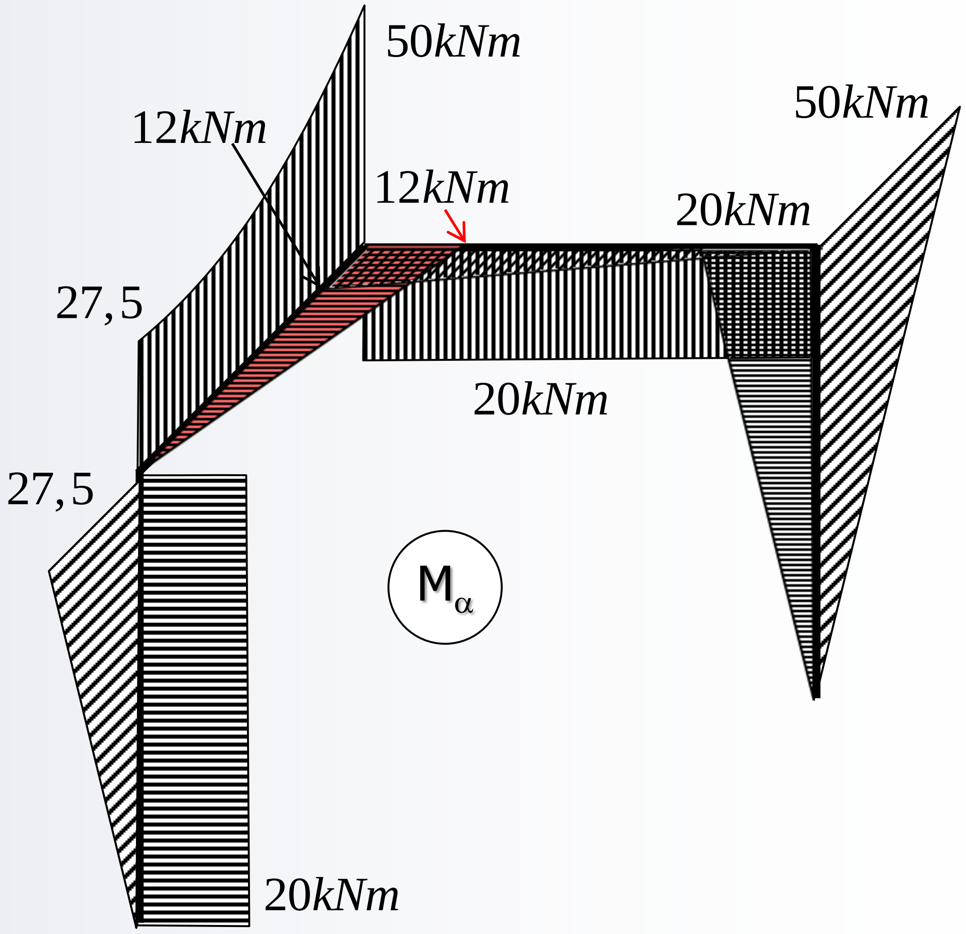
Momenty zginające



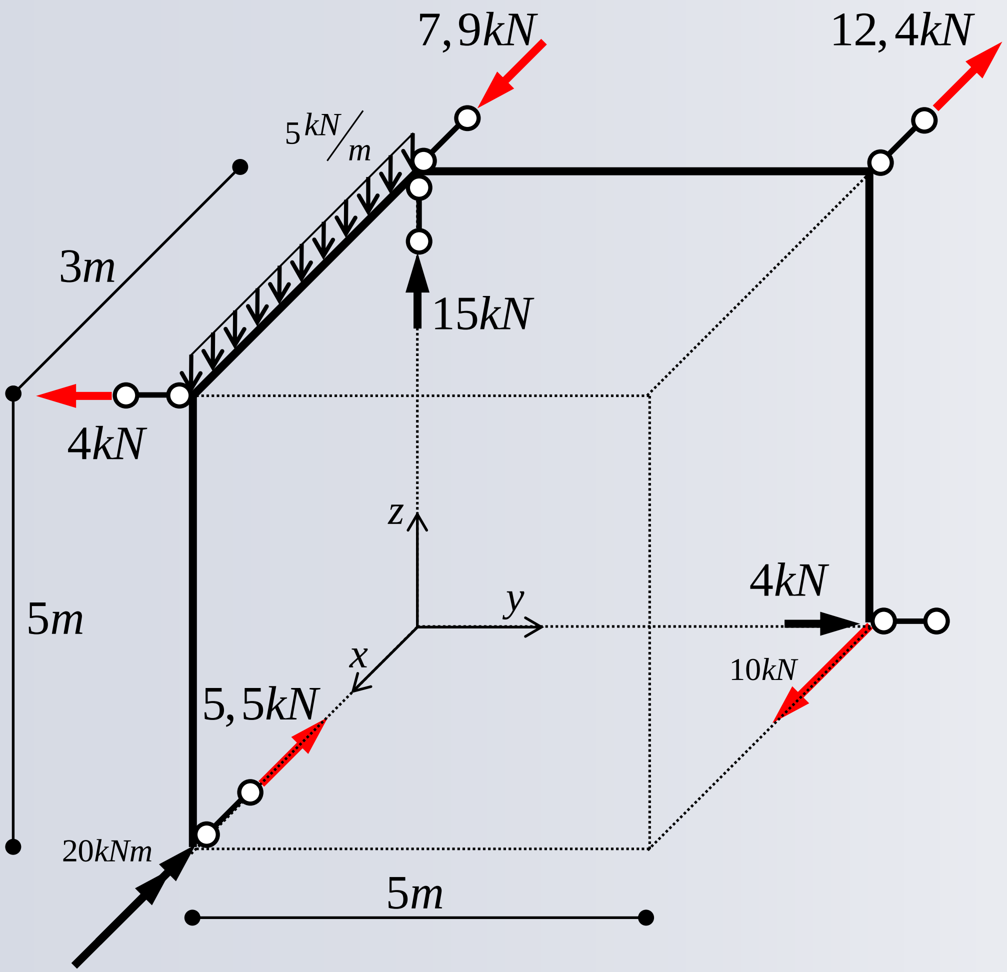


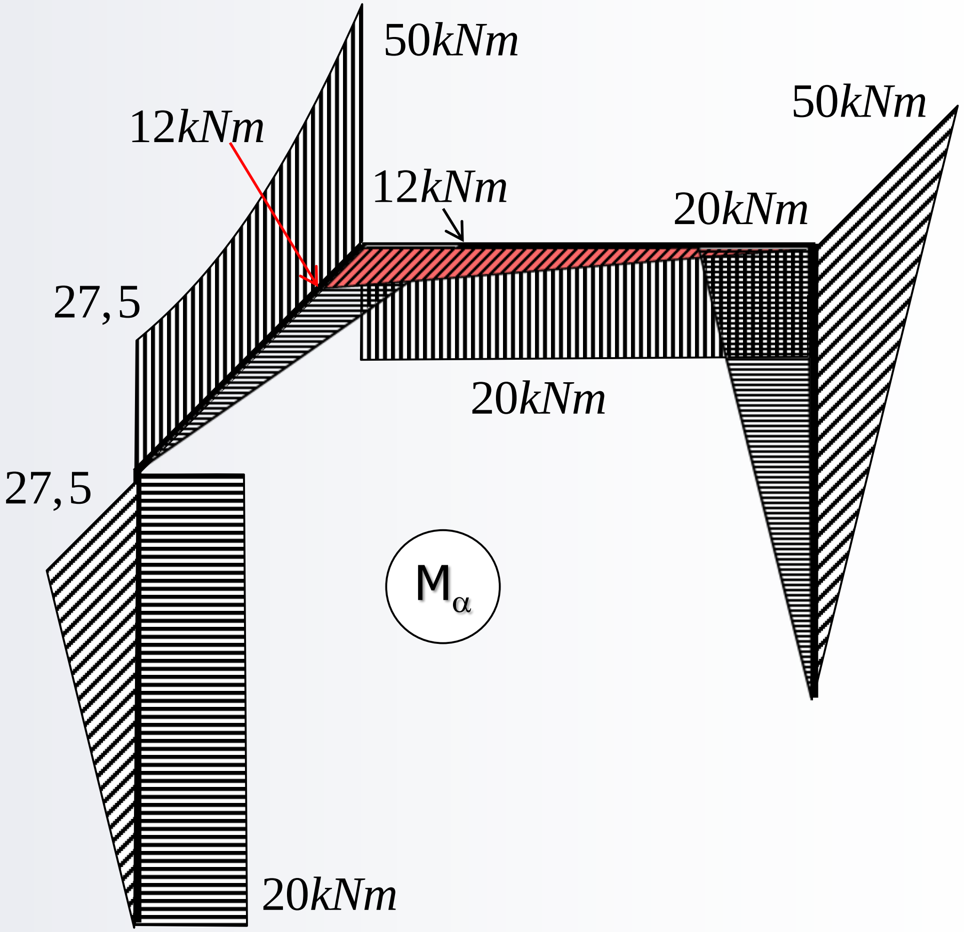
Momenty zginające



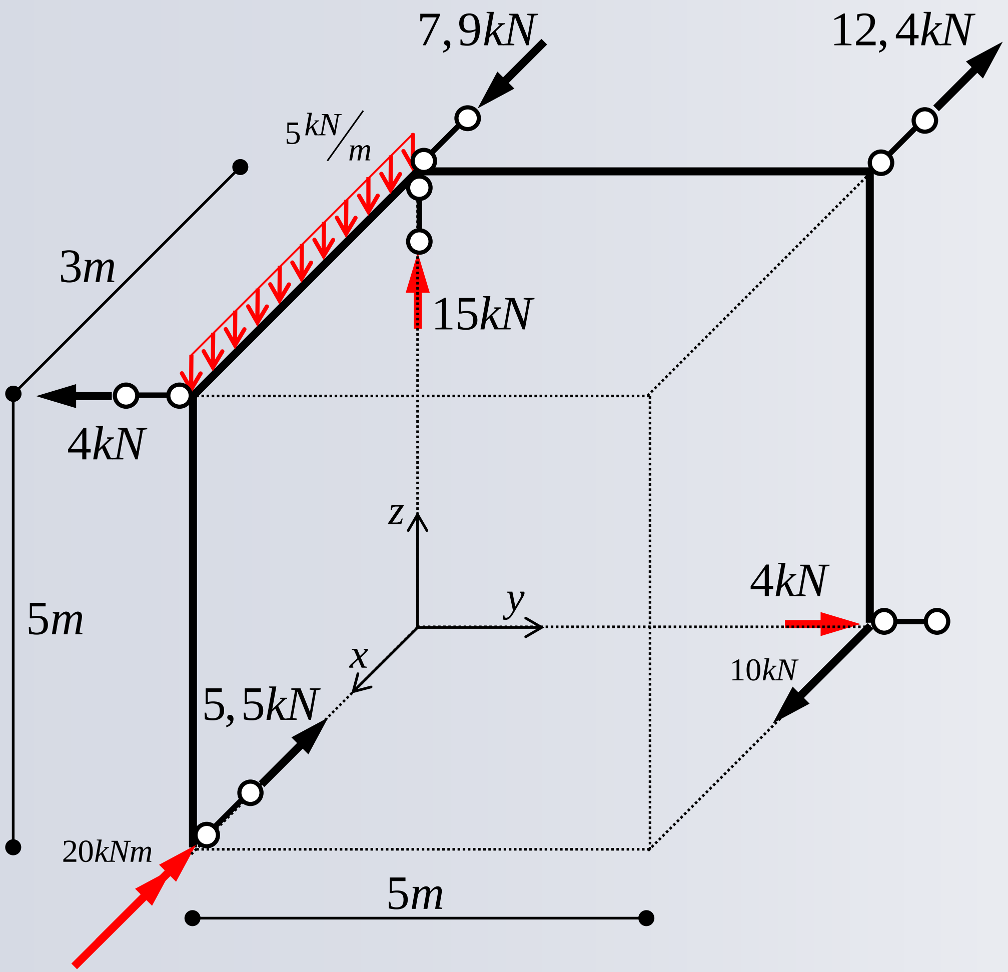


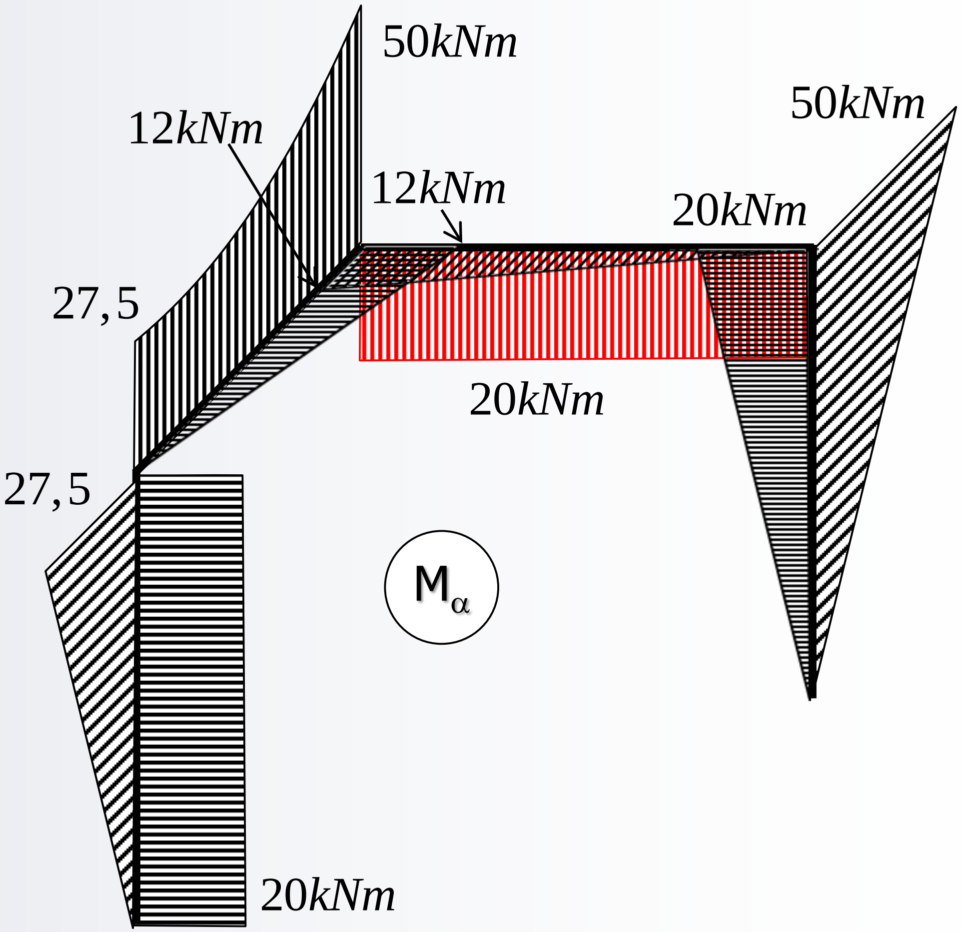
Momenty zginające





Momenty zginające





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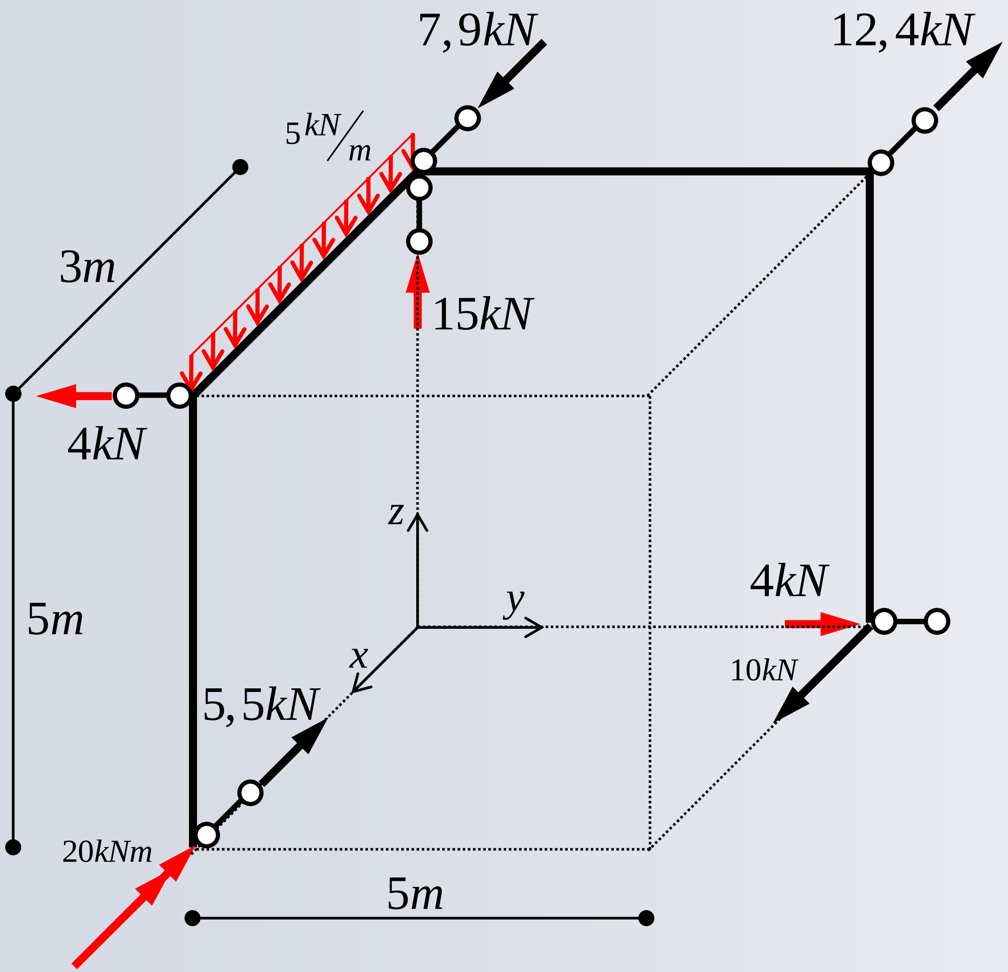

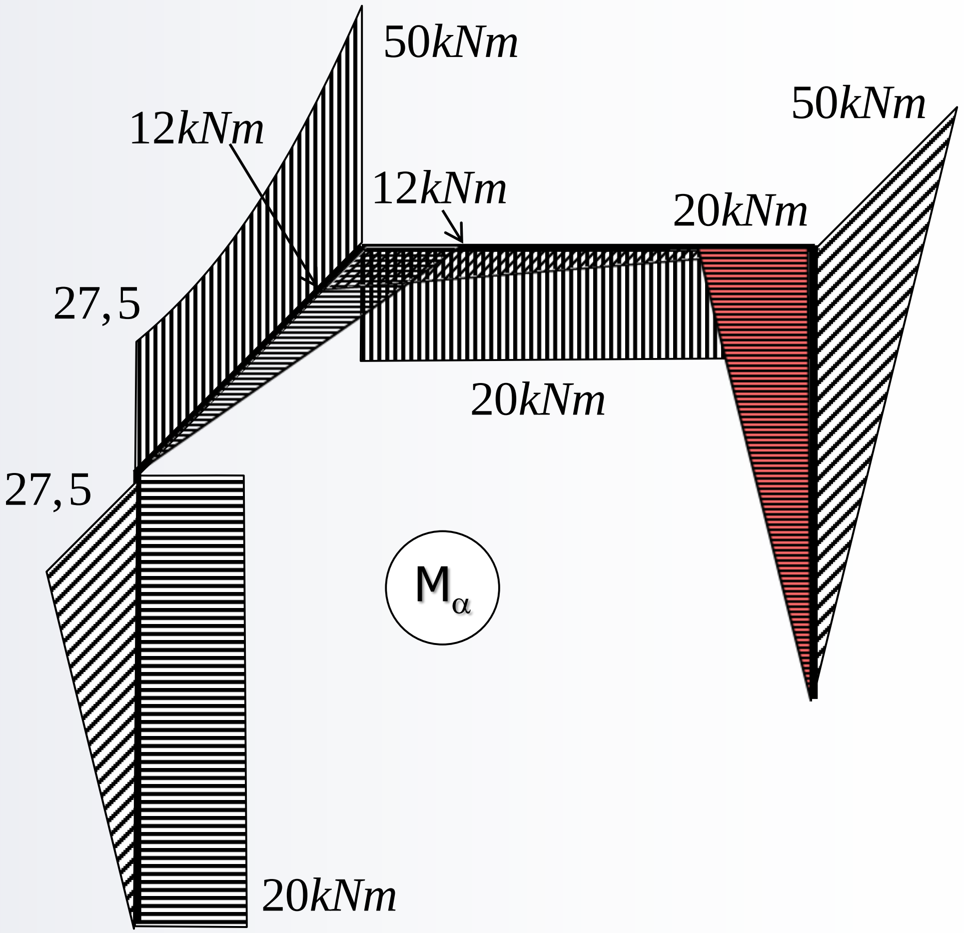

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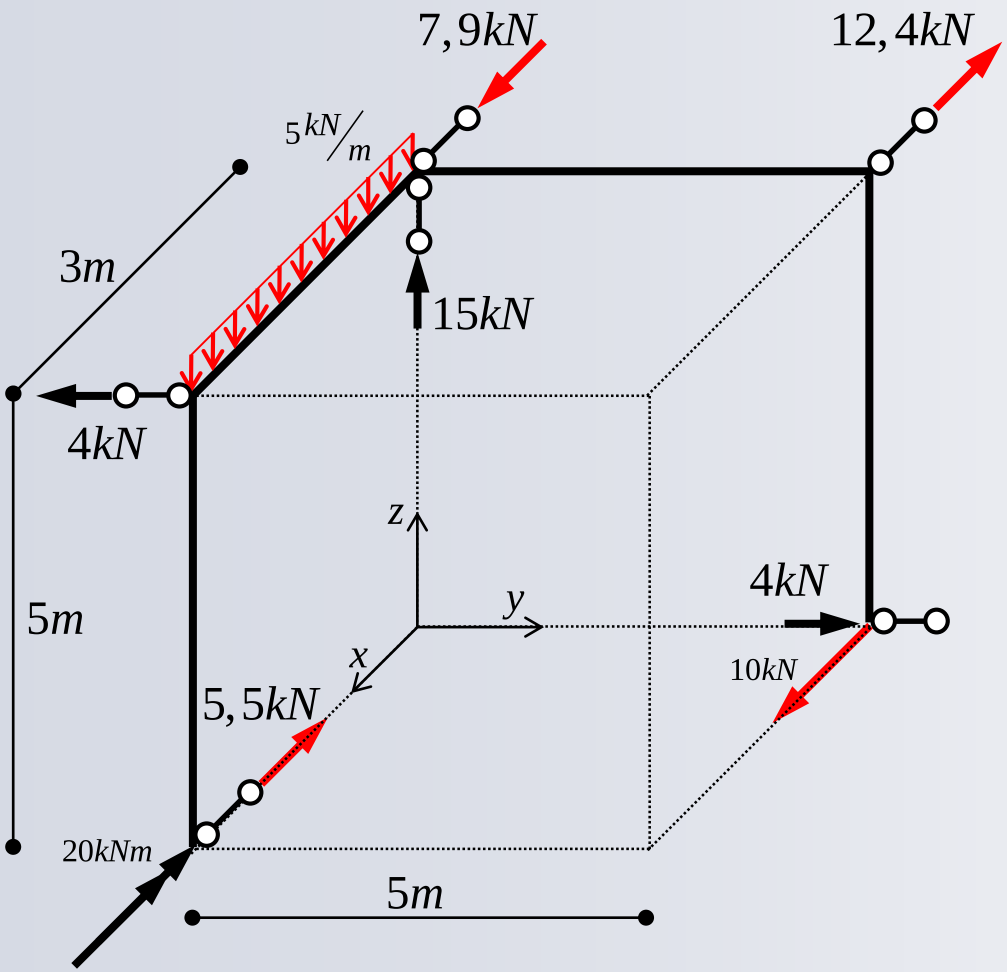
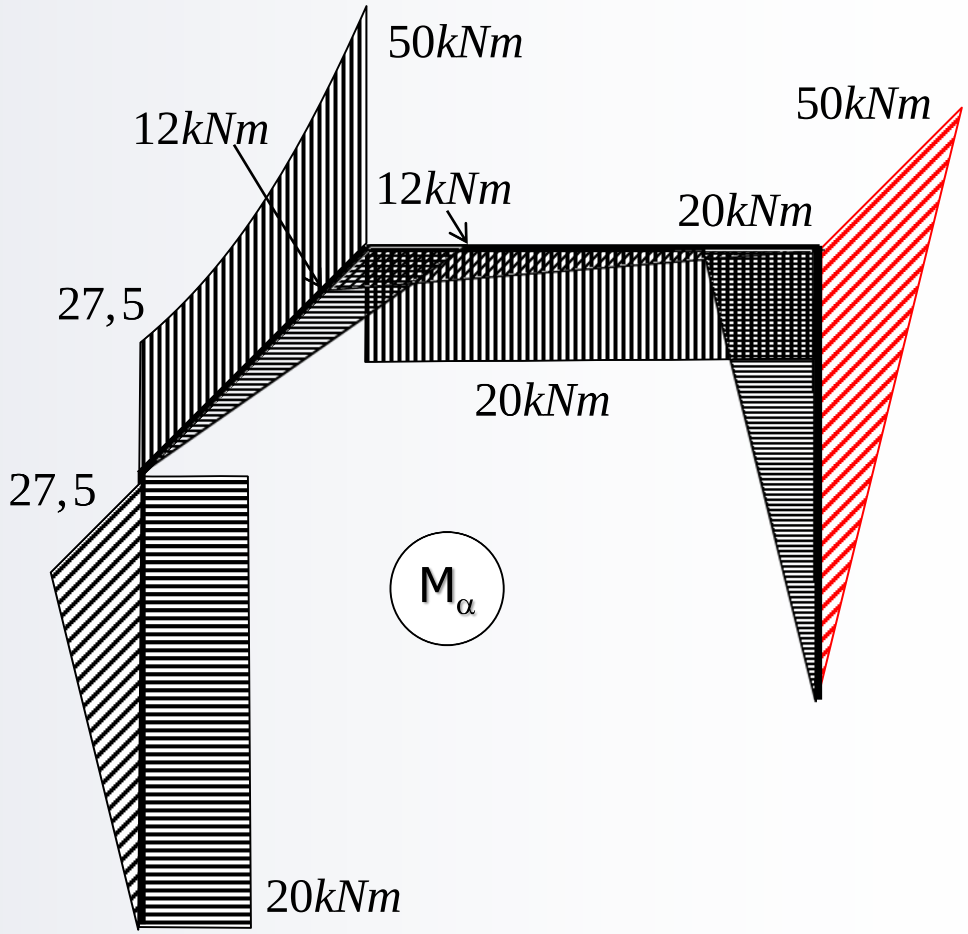
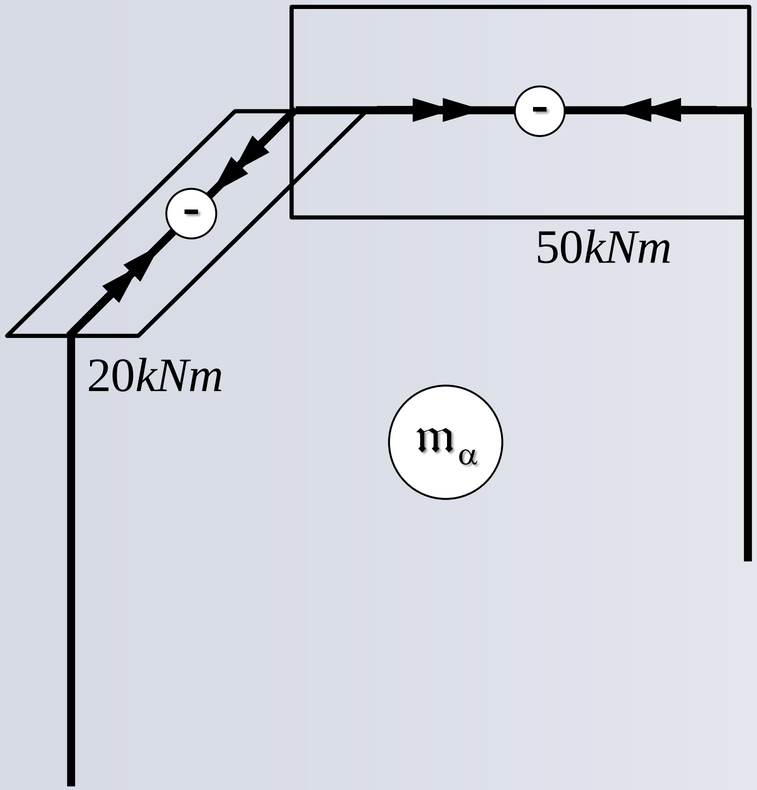
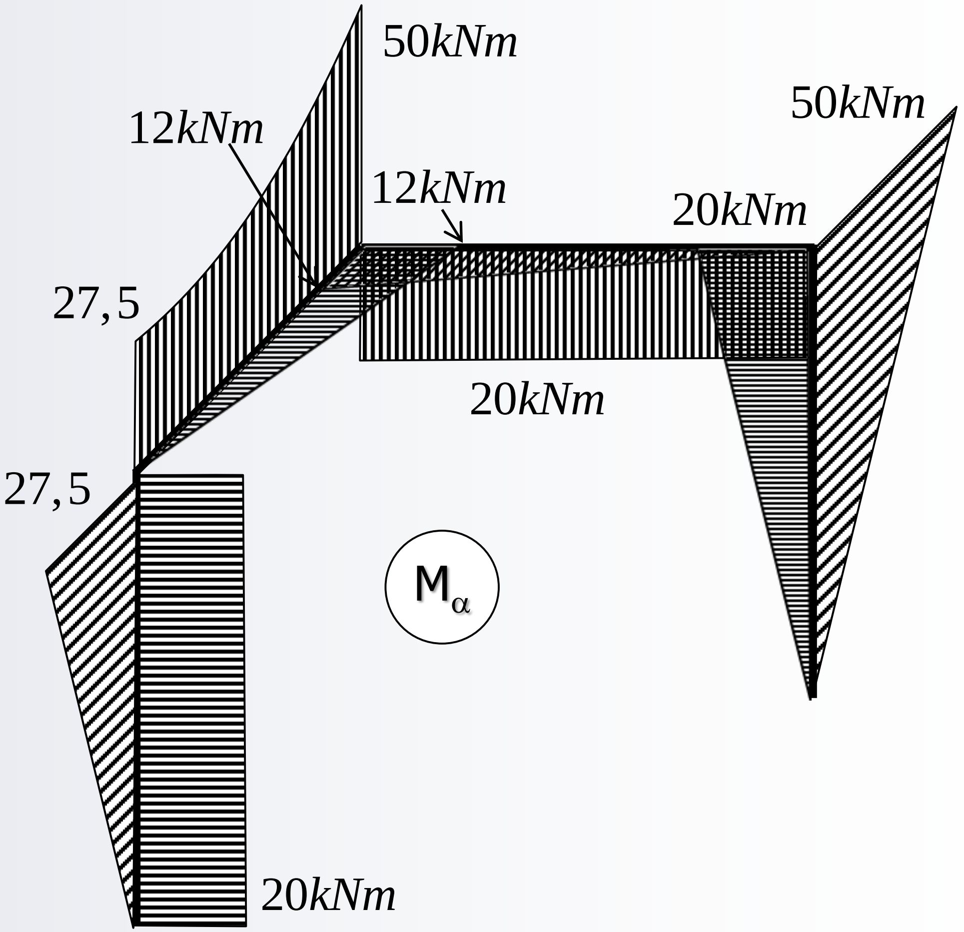
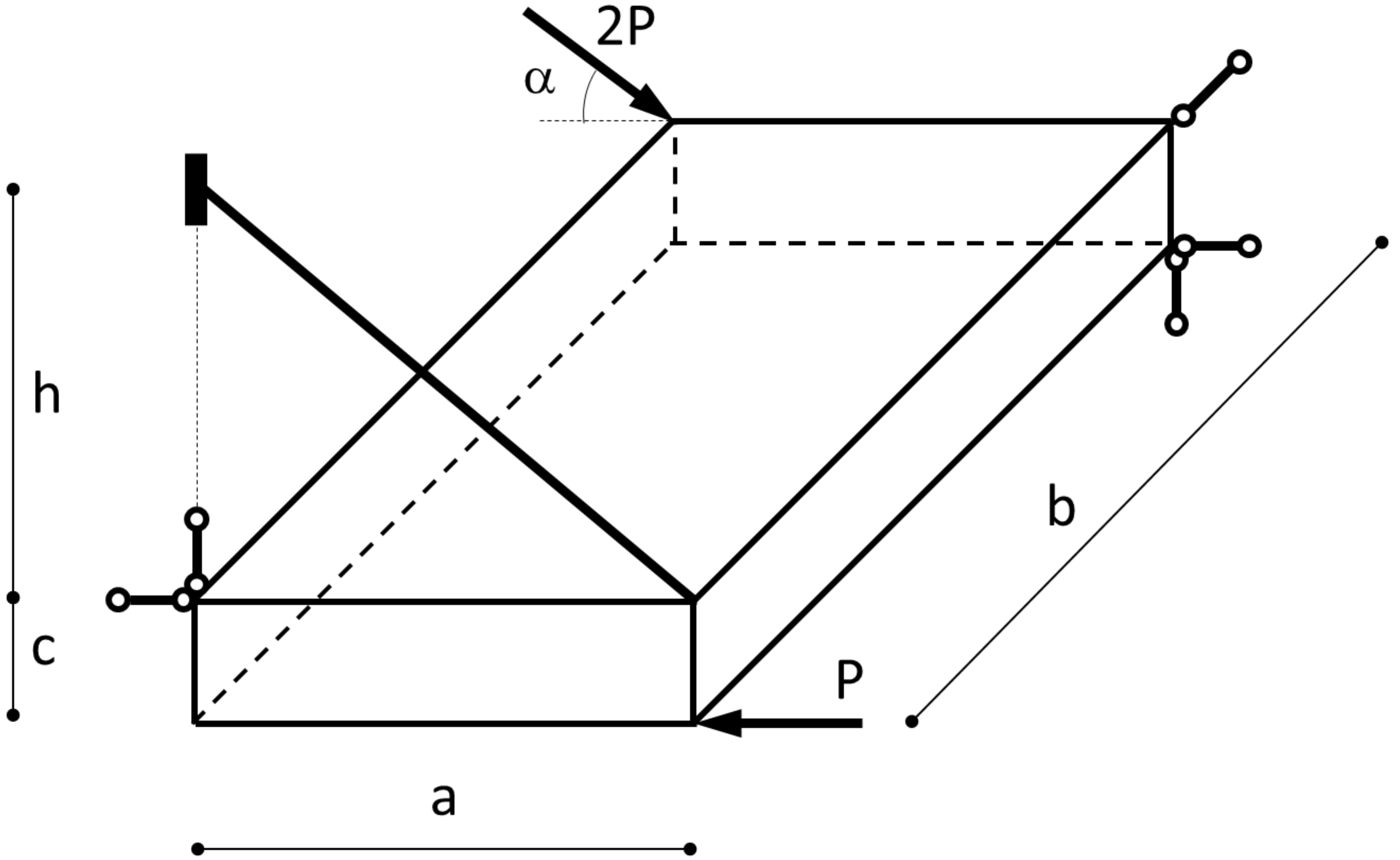


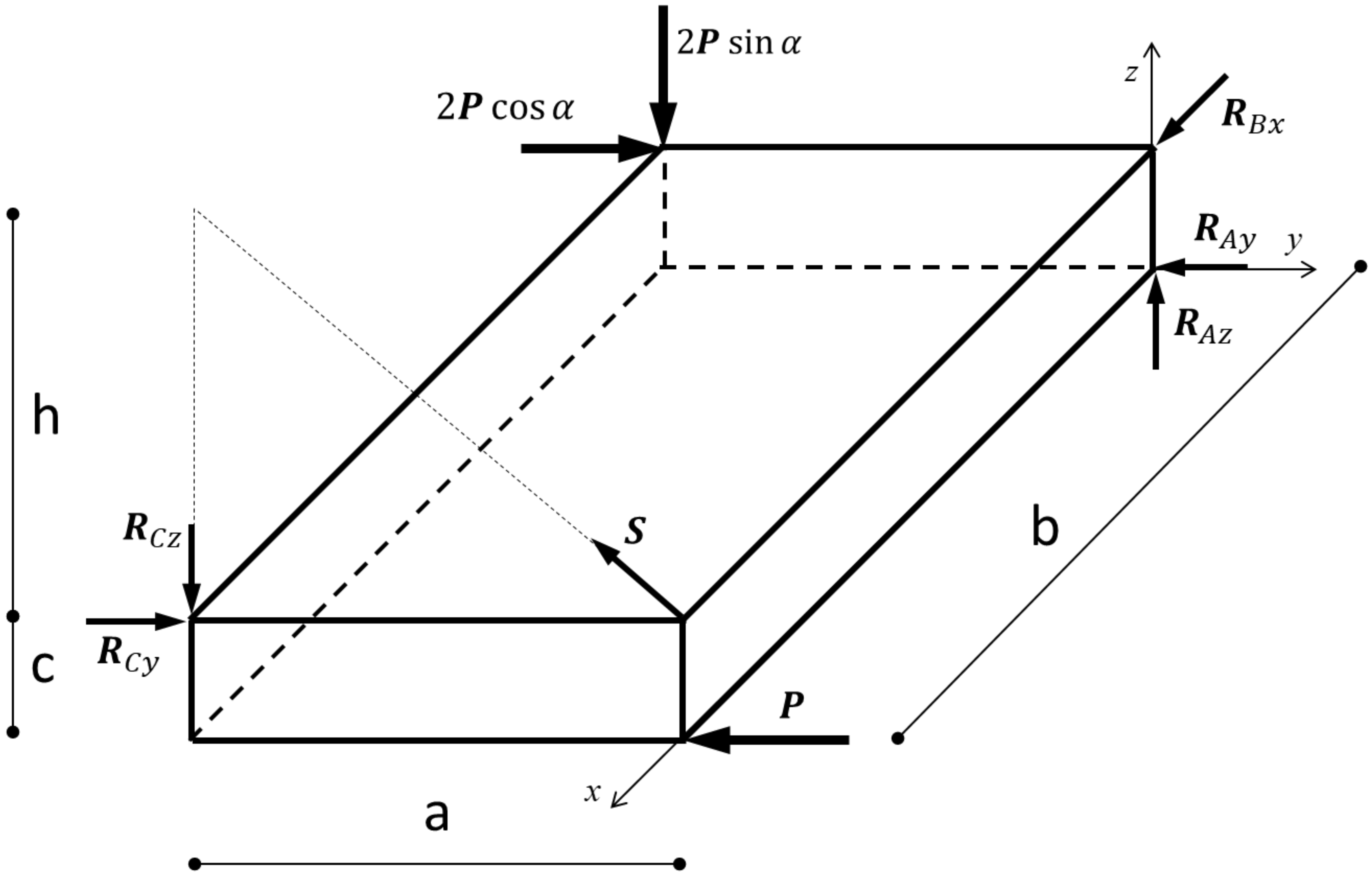


Momenty skręcające i zginające

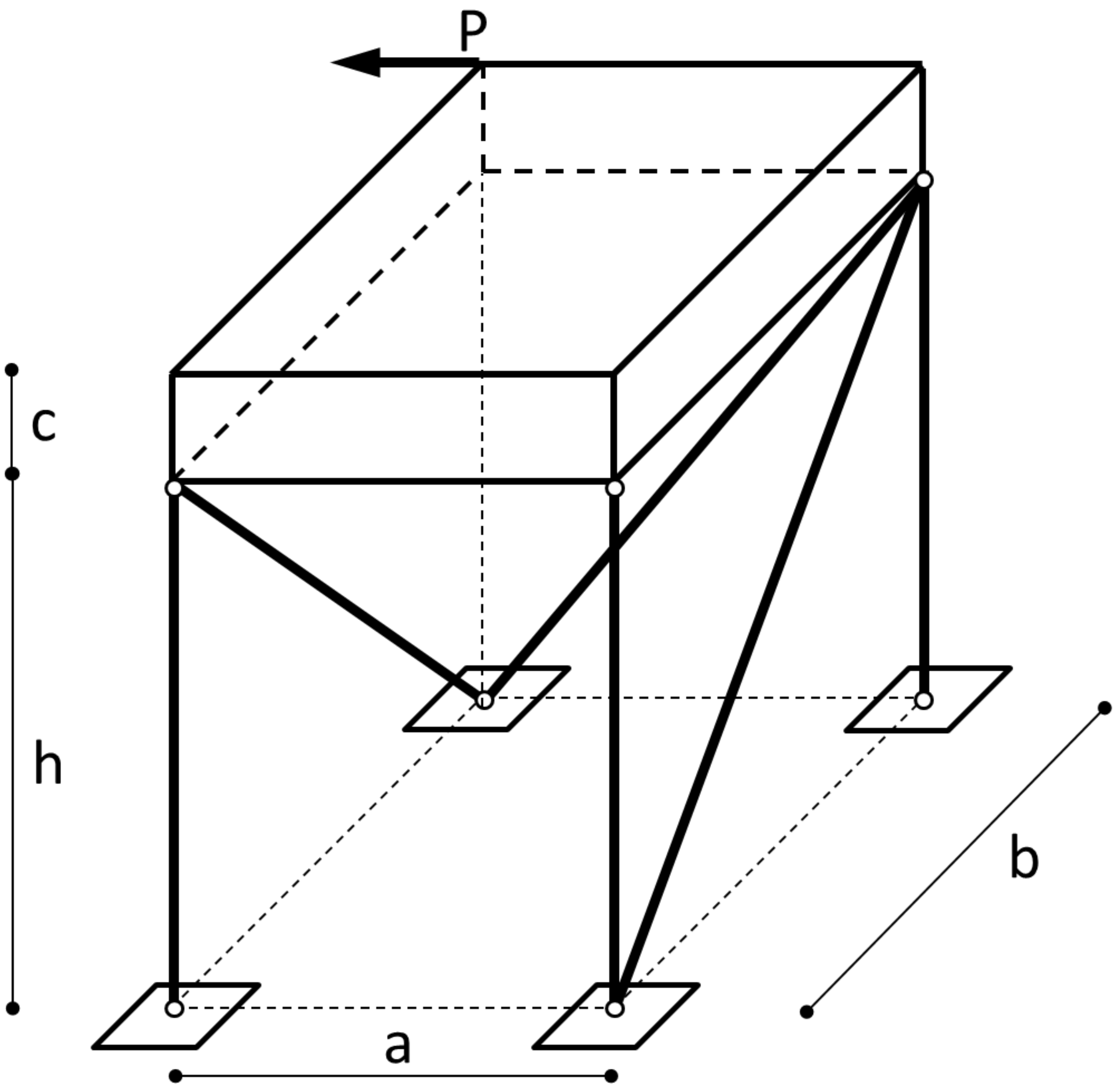


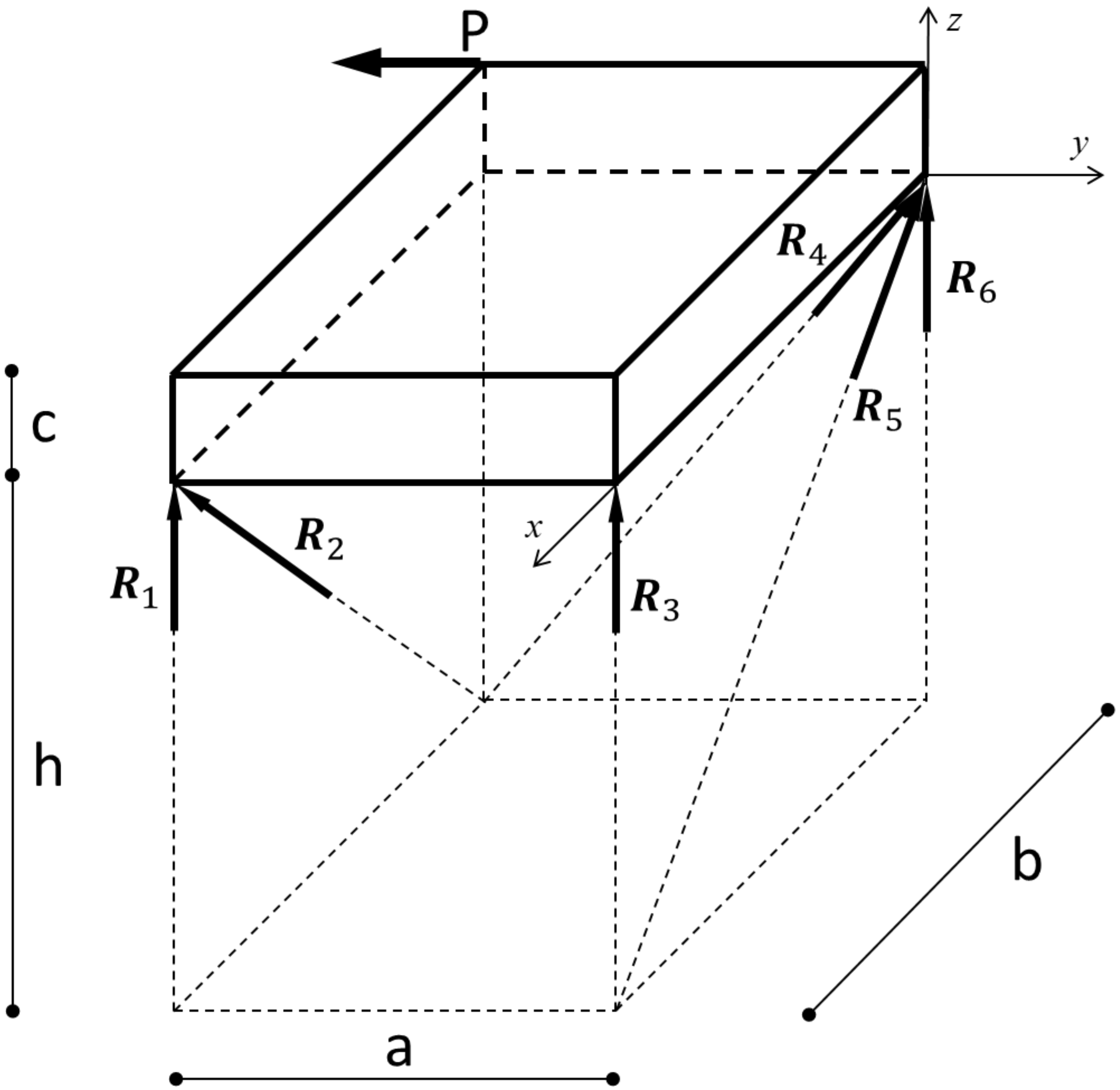


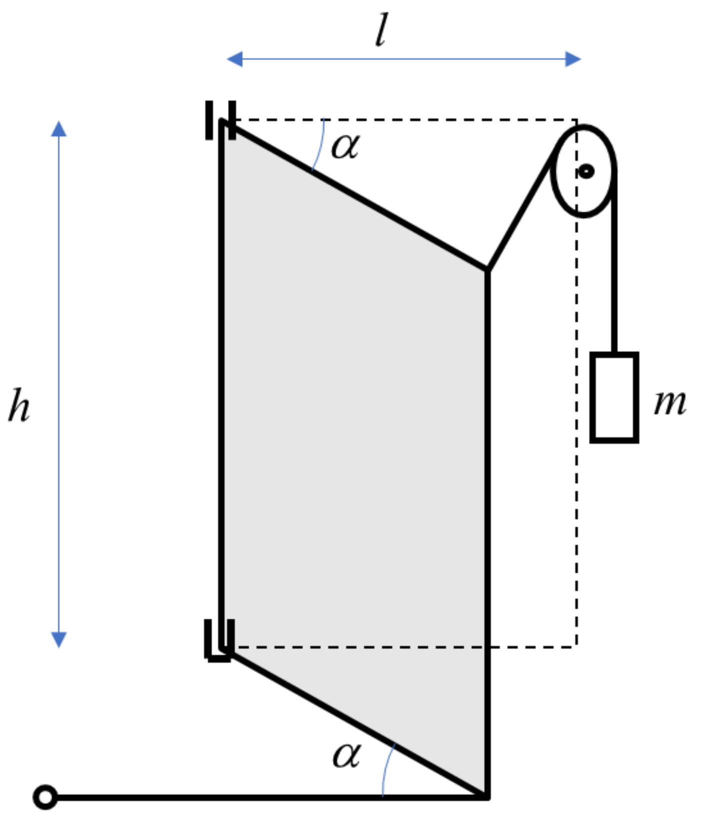

2P
α
h
c
a
b
P



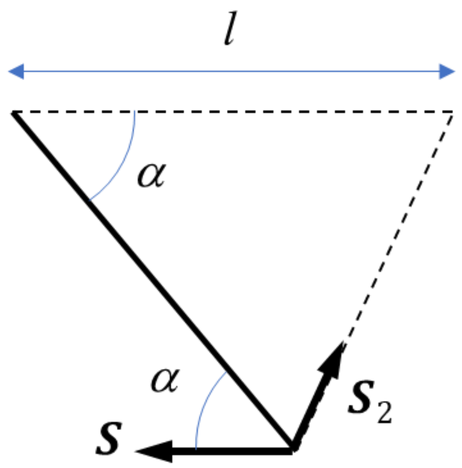
$2P \sin\alpha$
$2P \cos\alpha$
z
R_{Bx}
R_{Ay}
y
R_{Az}
h
R_{Cz}
S
b
c
R_{Cy}
P
x
a


P
c
h
b
a

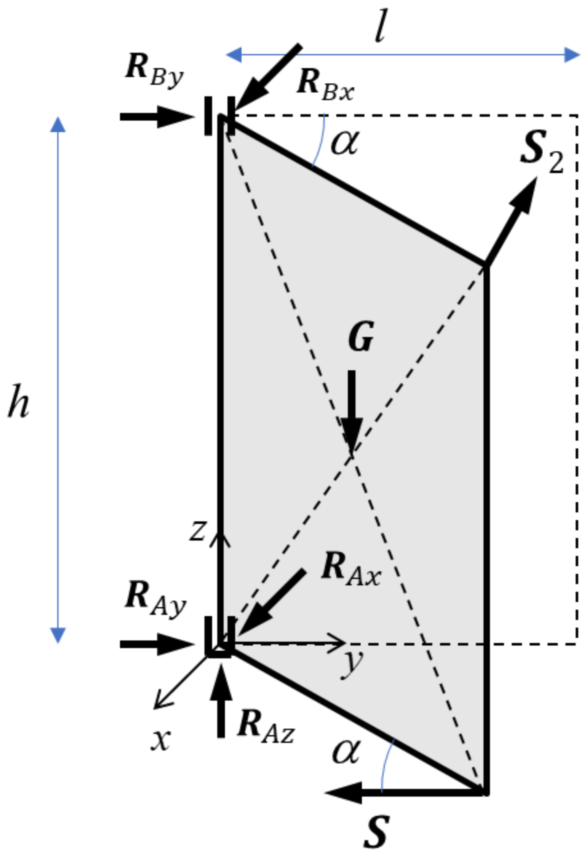

P
z
y
x
R_1
R_2
R_3
R_4
R_5
R_6
c
h
a
b



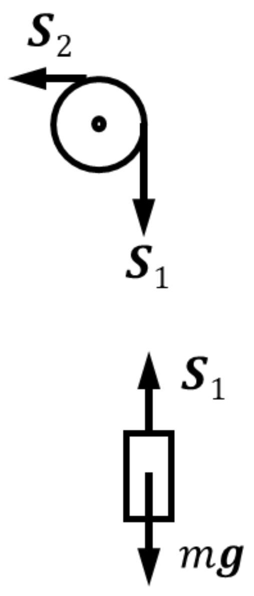
l
h
α
m
α



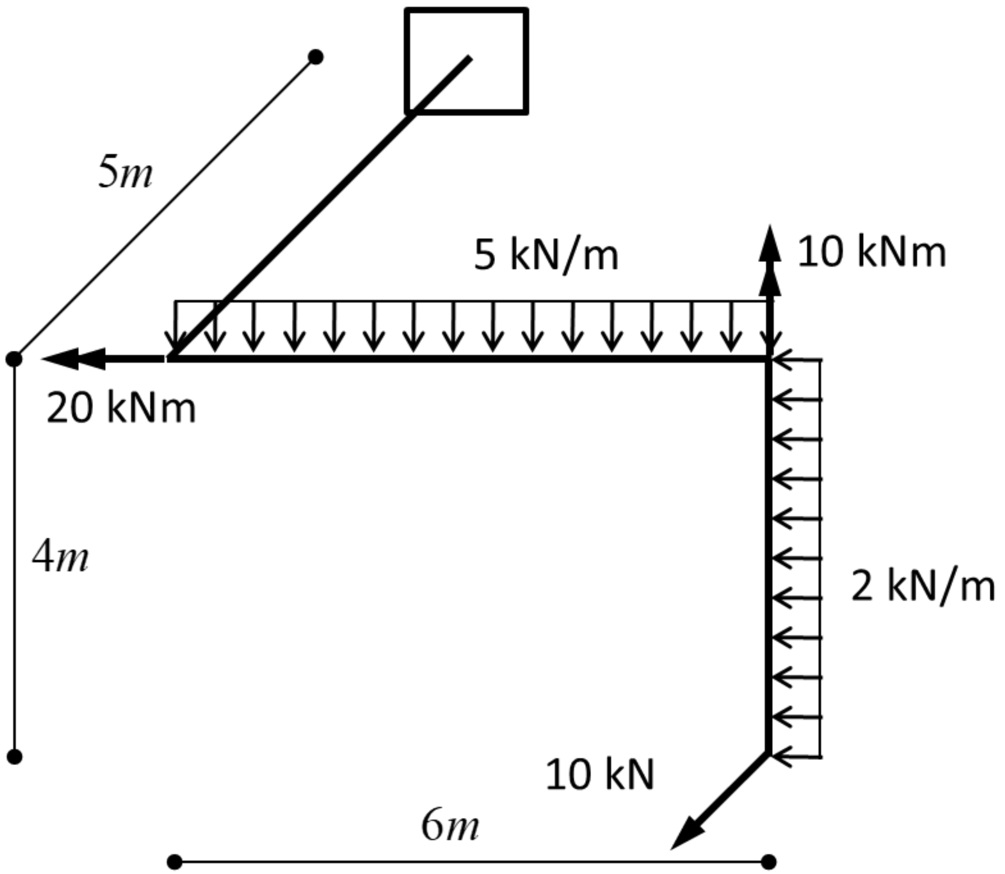
l
α
α
S
S2

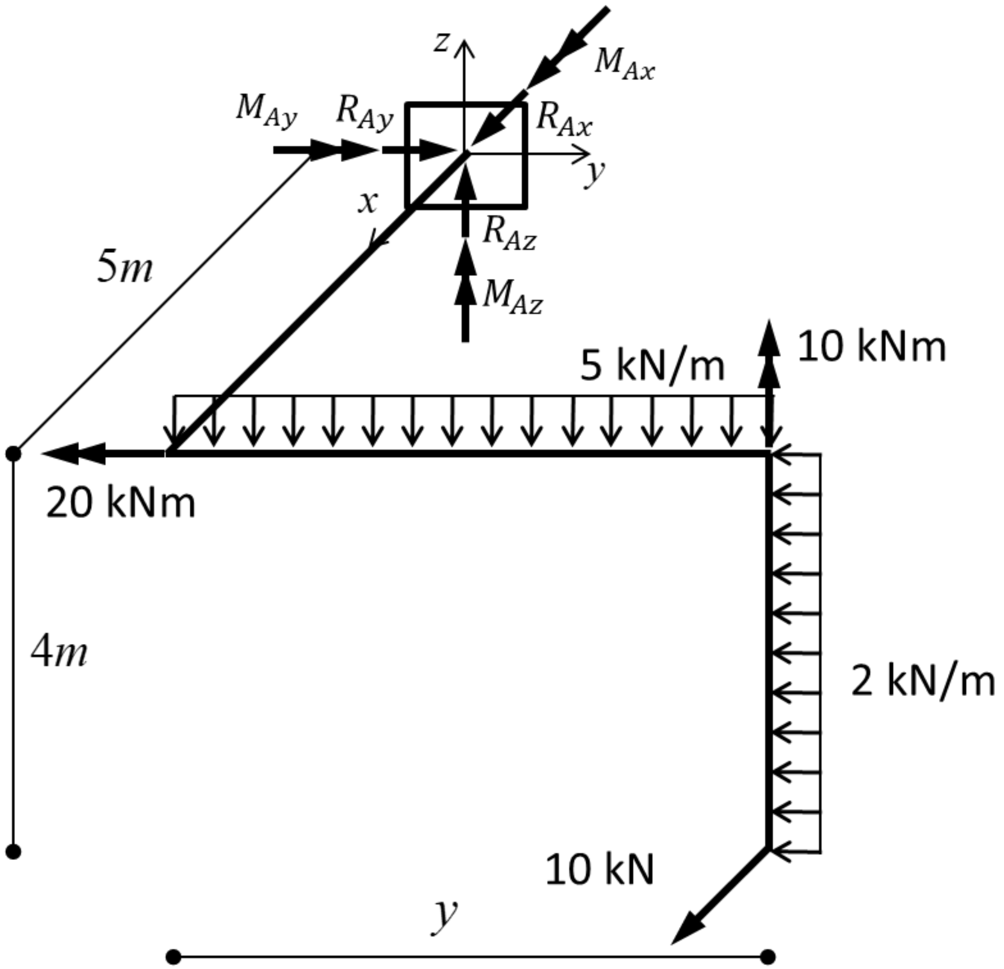


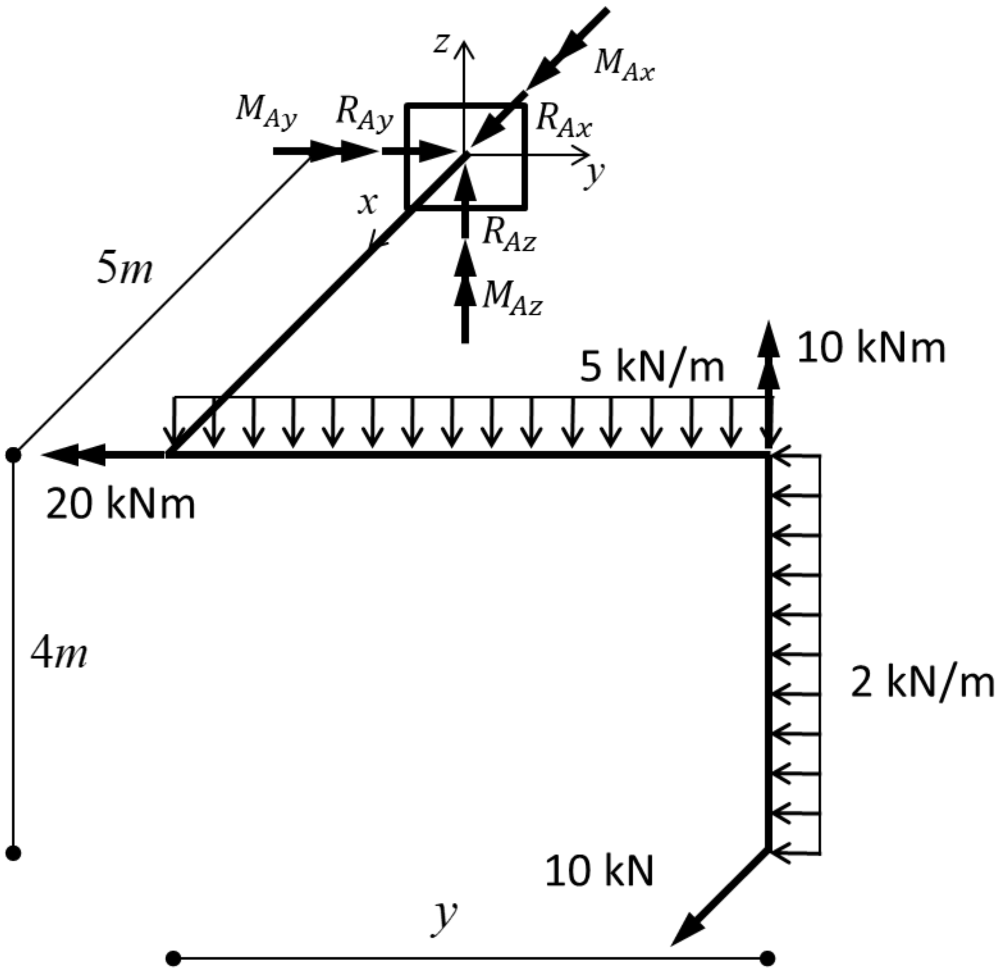
RBy
RBx
l
α
S2
h
G
z
RAx
RAy
y
x
RAz
α
S



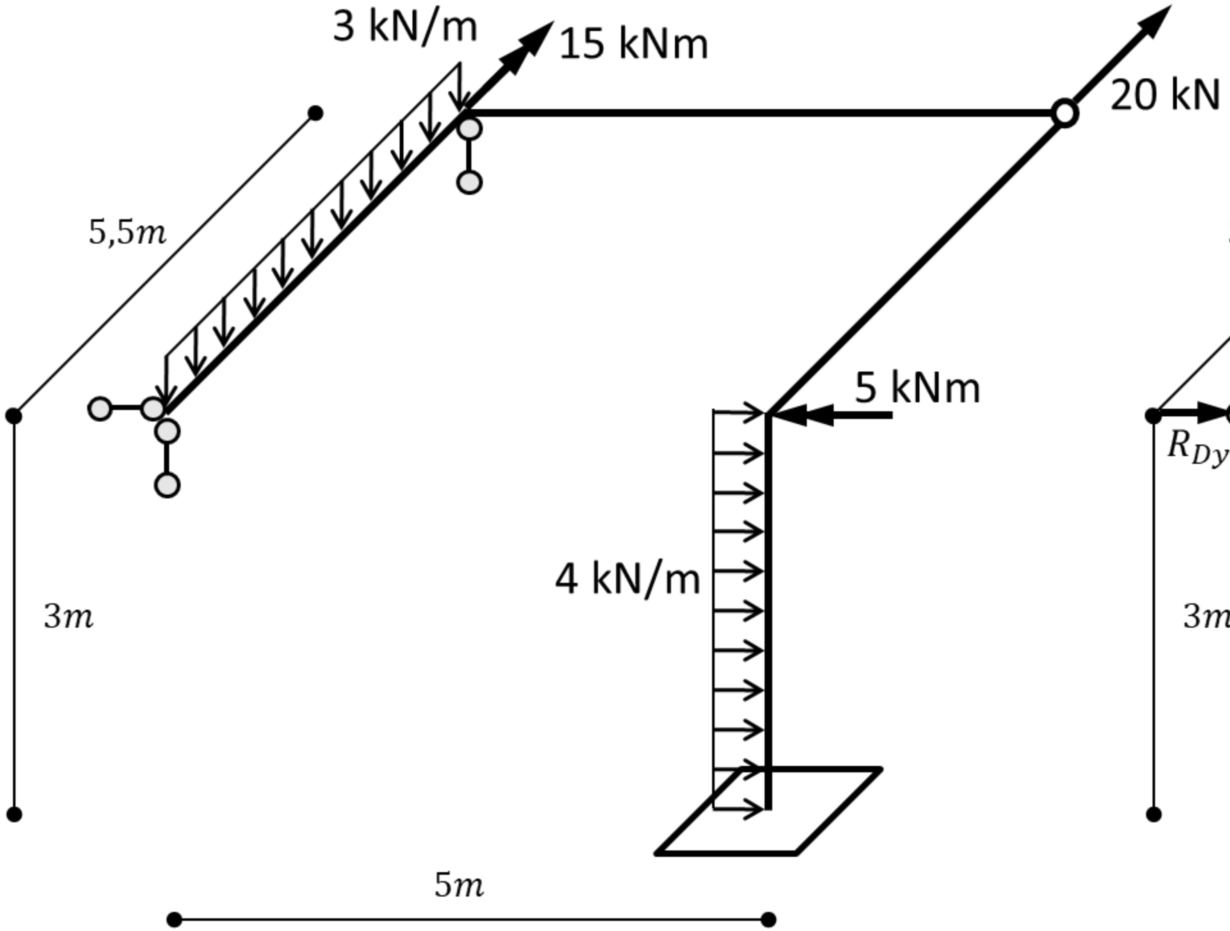
S2
S1
S1
mg


5m
5 kN/m
10 kNm
20 kNm
4m
2 kN/m
10 kN
6m

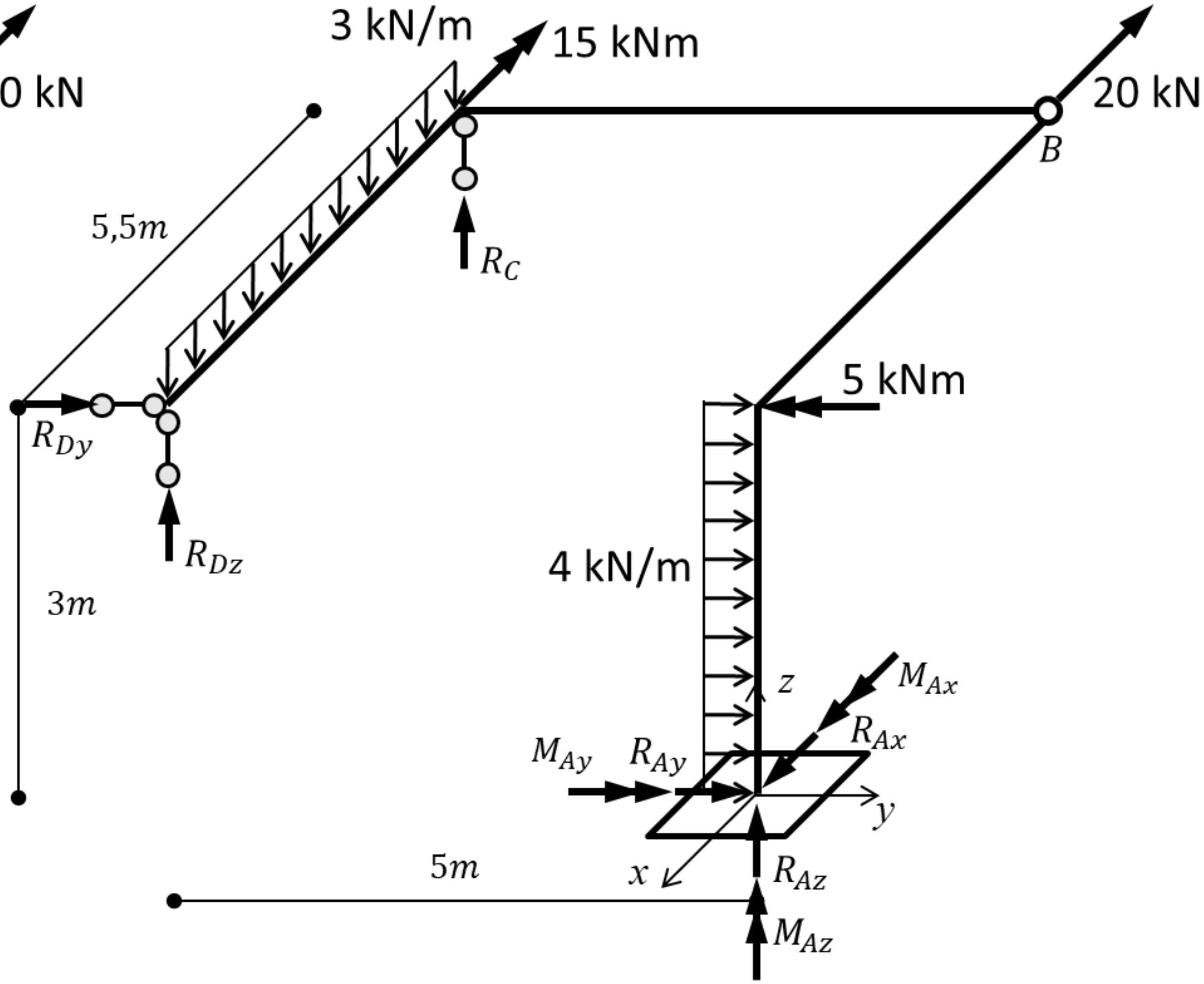

z
M_{Ax}
M_{Ay}
R_{Ay}
R_{Ax}
y
x
R_{Az}
5m
M_{Az}
10 kNm
5 kN/m
20 kNm
4m
2 kN/m
10 kN
y



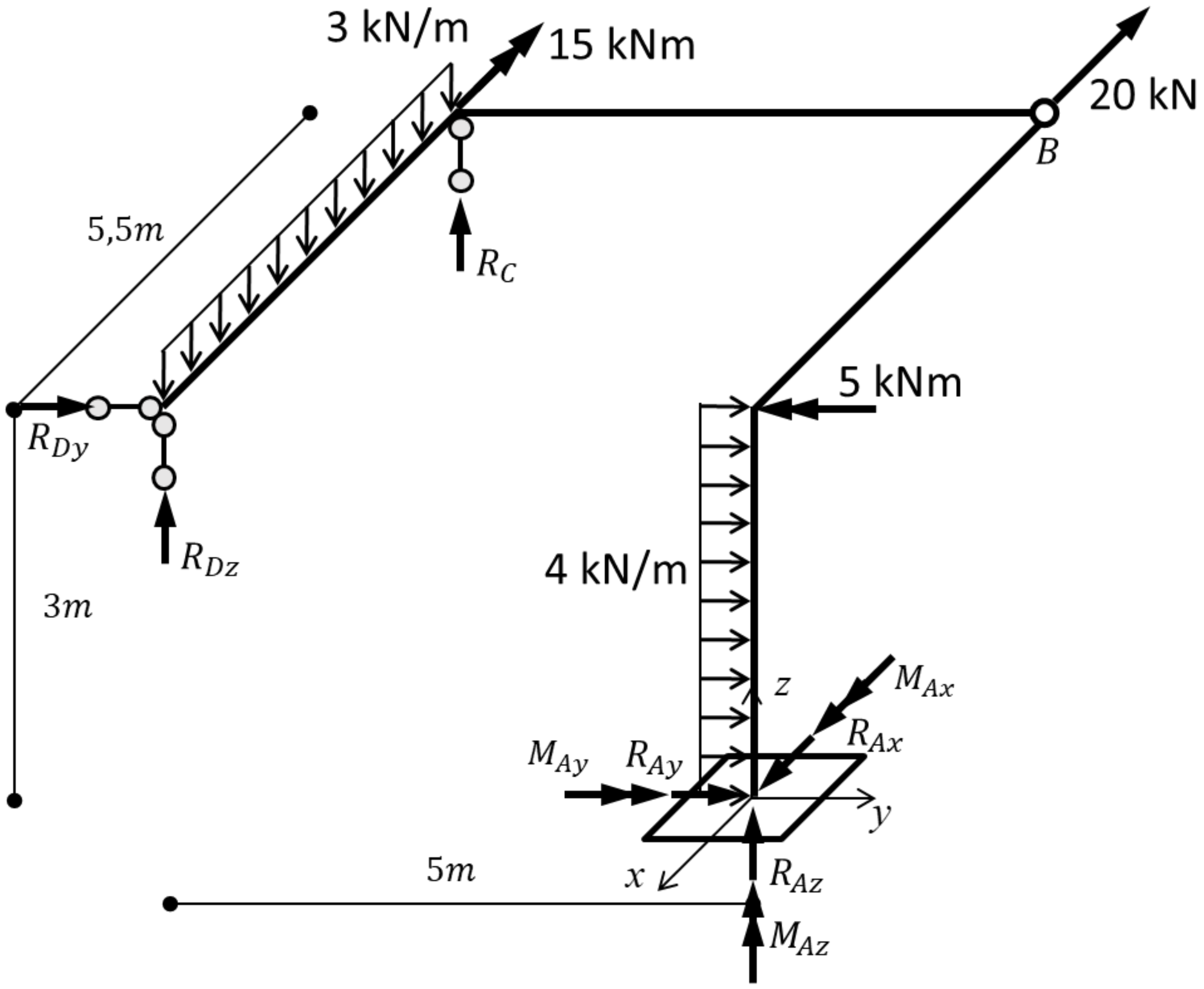
z
M_{Ax}
M_{Ay}
R_{Ay}
R_{Ax}
y
x
R_{Az}
$5m$
M_{Az}
10 kNm
5 kN/m
20 kNm
$4m$
2 kN/m
10 kN
y



3 kN/m
15 kNm
20 kN
5,5m
5 kNm
4 kN/m
3m
5m



3 kN/m
15 kNm
20 kN
B
5,5m
R_C
5 kNm
R_{Dy}
R_{Dz}
4 kN/m
3m
z
M_{Ax}
R_{Ax}
M_{Ay}
R_{Ay}
y
5m
x
R_{Az}
M_{Az}


3 kN/m
15 kNm
20 kN
B
5,5m
R_C
5 kNm
R_{Dy}
R_{Dz}
4 kN/m
3m
z
M_{Ax}
R_{Ax}
M_{Ay}
R_{Ay}
y
5m
x
R_{Az}
M_{Az}