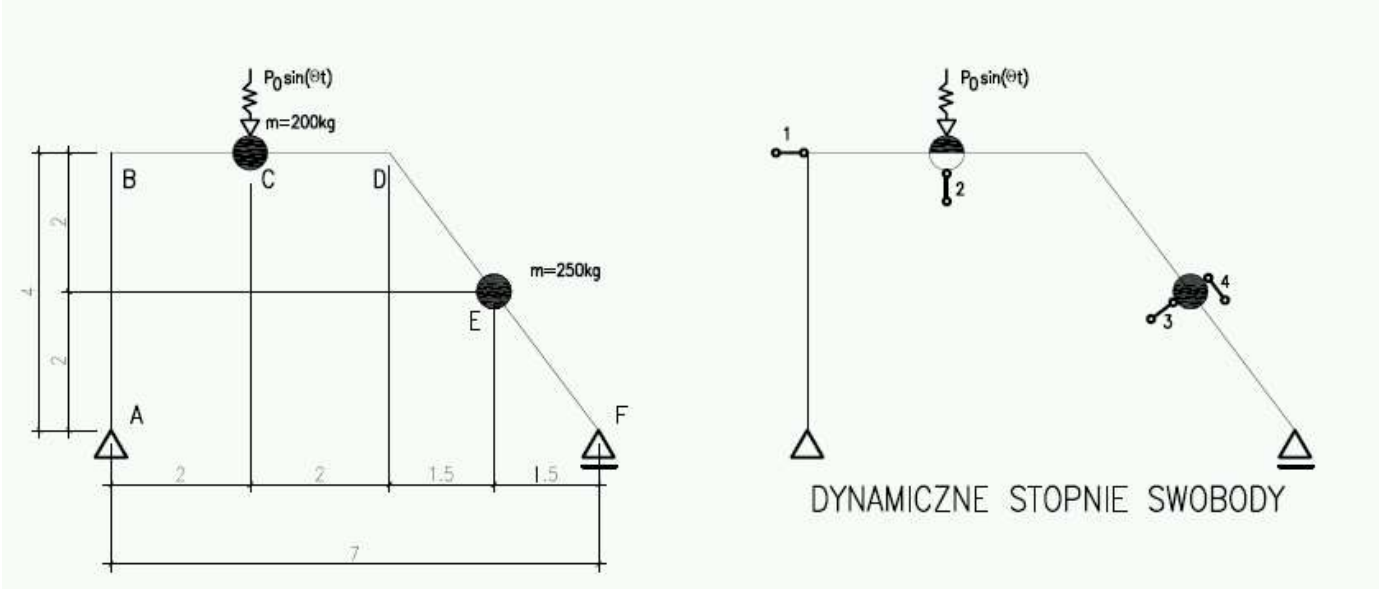
Dane materiałowe

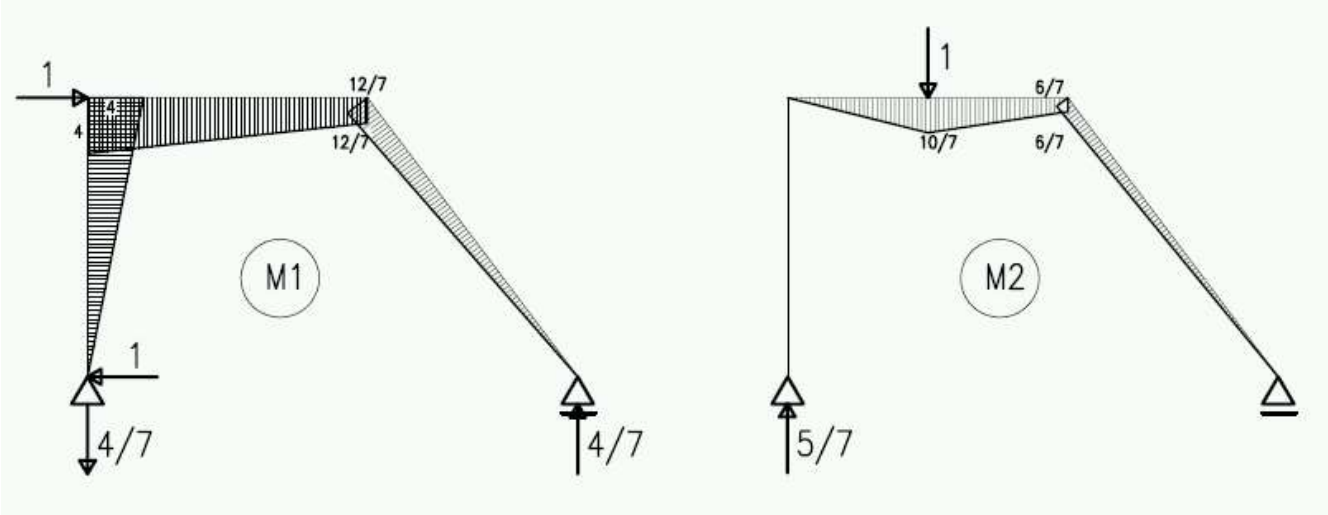
$E := 210 \cdot 10^{6} \cdot kPa$ $h := 10cm$ $b := 24cm$

$J := \frac{b \cdot h^{3}}{12} = 2 \times 10^{-5} \cdot m^{4}$ $EJ := E \cdot J = 4.2 \times 10^{3} \cdot kN \cdot m^{2}$ $m_0 := 50 \cdot kg$

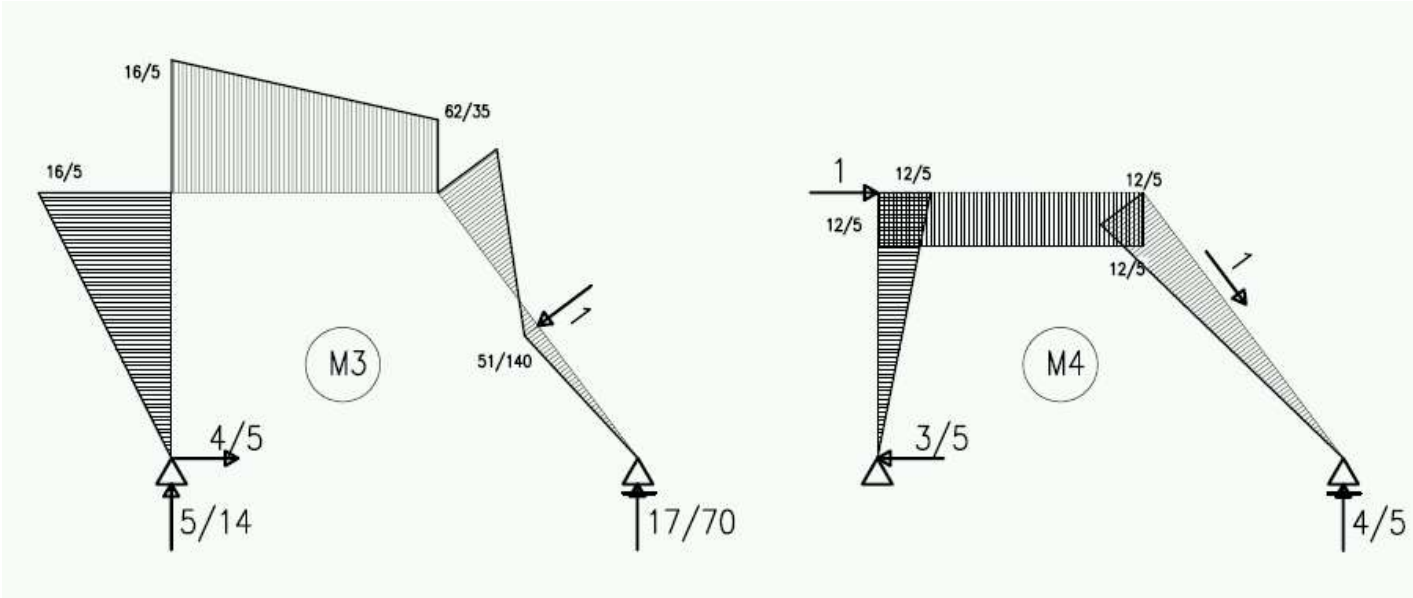
Schemat ustroju i dynamiczne stopnie swobody ustroju



Stan 1 (M1) oraz Stan 2 (M2)



Stan 3 (M3) oraz Stan 4 (M4)



Sprawdzenie delt z RM-WIN jako przemieszczenia na odpowiednich stopiach swobody

$14.435\text{mm}\cdot EJ = 60.627\cdot\text{kN}\cdot\text{m}^3$

$2.954\text{mm}\cdot EJ = 12.407\cdot\text{kN}\cdot\text{m}^3$ $\quad 1.251\text{mm}\cdot EJ = 5.254\cdot\text{kN}\cdot\text{m}^3$

$-11.654\text{mm}\cdot EJ = -48.947\cdot\text{kN}\cdot\text{m}^3$ $\quad -2.384\text{mm}\cdot EJ = -10.013\cdot\text{kN}\cdot\text{m}^3$ $\quad 9.845\text{mm}\cdot EJ = 41.349\cdot\text{kN}\cdot\text{m}^3$

$1.1211\text{cm}\cdot EJ = 47.086\cdot\text{kN}\cdot\text{m}^3$ $\quad 0.2939\text{cm}\cdot EJ = 12.344\cdot\text{kN}\cdot\text{m}^3$ $\quad -0.8914\text{cm}\cdot EJ = -37.439\cdot\text{kN}\cdot\text{m}^3$ $\quad 0.96\text{cm}\cdot EJ = 40.32\cdot\text{kN}\cdot\text{m}^3$

Obliczenia delt

$d_{11} = (M_1 \cdot M_1)$ $\quad d_{11} := \frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 4 + \frac{1}{2}4\cdot 4\left(\frac{2}{3}4 + \frac{1}{3}\frac{12}{7}\right) + \frac{1}{2}\cdot\frac{12}{7}4\left(\frac{1}{3}4 + \frac{2}{3}\frac{12}{7}\right) + \frac{1}{2}\cdot\frac{12}{7}\cdot 5\cdot\frac{2}{3}\cdot\frac{12}{7} = 60.626$

$d_{12} = (M_1 \cdot M_2)$ $\quad d_{12} := \frac{1}{2}\cdot 2\cdot\frac{10}{7}\cdot\left(\frac{1}{3}\cdot 4 + \frac{2}{3}\cdot\frac{20}{7}\right) + \frac{1}{2}\frac{10}{7}2\left(\frac{2}{3}\frac{20}{7} + \frac{1}{3}\frac{12}{7}\right) \ldots = 12.408$
$+\frac{1}{2}\cdot 2\cdot\frac{6}{7}\cdot\left(\frac{1}{3}\cdot\frac{20}{7} + \frac{2}{3}\frac{12}{7}\right) + \frac{1}{2}\cdot 5\cdot\frac{12}{7}\cdot\frac{2}{3}\cdot\frac{6}{7}$

$d_{13} = (M_1 \cdot M_3)$ $\quad d_{13} := \frac{-1}{2}4\cdot 4\cdot\frac{2}{3}\frac{16}{5} - \frac{1}{2}4\cdot 4\left(\frac{2}{3}\frac{16}{5} + \frac{1}{3}\frac{62}{35}\right) - \frac{1}{2}\frac{12}{7}\cdot 4\left(\frac{1}{3}\frac{16}{5} + \frac{2}{3}\frac{62}{35}\right) \ldots = -48.946$
$+\frac{-1}{2}\frac{62}{35}\frac{5}{2}\cdot\left(\frac{2}{3}\frac{12}{7} + \frac{1}{3}\frac{6}{7}\right) + \frac{1}{2}\frac{51}{140}\frac{5}{2}\cdot\left(\frac{1}{3}\frac{12}{7} + \frac{2}{3}\frac{6}{7}\right) + \frac{1}{2}\frac{6}{7}\frac{5}{2}\cdot\frac{2}{3}\frac{51}{140}$

$d_{13} = (M_1 \cdot M_3)$ $\quad d_{14} := \frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot\frac{12}{5} + \frac{1}{2}4\cdot 4\frac{12}{5} + \frac{1}{2}4\cdot\frac{12}{7}\frac{12}{5} + \frac{1}{2}\cdot\frac{12}{7}\cdot 5\cdot\frac{2}{3}\cdot\frac{12}{5} = 47.086$

$d_{22} = (M_2 \cdot M_2)$ $\quad d_{22} := \frac{1}{2}\cdot\frac{10}{7}\cdot 2\cdot\frac{2}{3}\cdot\frac{10}{7} + \frac{1}{2}\cdot\frac{10}{7}\cdot 2\cdot\left(\frac{2}{3}\cdot\frac{10}{7} + \frac{1}{3}\cdot\frac{6}{7}\right) + \frac{1}{2}\cdot\frac{6}{7}\cdot 2\cdot\left(\frac{1}{3}\cdot\frac{10}{7} + \frac{2}{3}\cdot\frac{6}{7}\right) + \frac{1}{2}\cdot\frac{6}{7}\cdot 5\cdot\frac{2}{3}\cdot\frac{6}{7} = 5.252$

$d_{23} = (M_2 \cdot M_3)$ $\quad d_{23} := \left[\frac{-1}{2}\frac{10}{7}\cdot 2\left(\frac{1}{3}\frac{16}{5} + \frac{2}{3}\frac{87}{35}\right) - \frac{1}{2}\frac{10}{7}\cdot 2\left(\frac{2}{3}\frac{87}{35} + \frac{1}{3}\frac{62}{35}\right) - \frac{1}{2}\frac{6}{7}\cdot 2\left(\frac{1}{3}\frac{87}{35} + \frac{2}{3}\frac{62}{35}\right)\right] \ldots = -10.016$
$+\frac{1}{2}\frac{6}{7}\cdot\frac{5}{2}\left(\frac{-2}{3}\frac{62}{35} + \frac{1}{3}\frac{51}{140}\right) + \frac{1}{2}\frac{3}{7}\frac{5}{2}\left(\frac{-1}{3}\frac{62}{35} + \frac{2}{3}\frac{51}{140}\right) + \frac{1}{2}\frac{3}{7}\frac{5}{2}\frac{2}{3}\frac{51}{140}$

$d_{24} = (M_2 \cdot M_4)$ $\quad d_{24} := \frac{1}{2}\cdot 2\cdot\frac{10}{7}\cdot\left(\frac{12}{5}\right) + \frac{1}{2}\frac{10}{7}2\left(\frac{12}{5}\right) + \frac{1}{2}\frac{6}{7}2\left(\frac{12}{5}\right) + \frac{1}{2}\cdot 5\cdot\frac{6}{7}\cdot\left(\frac{2}{3}\frac{12}{5}\right) = 12.343$

$d_{33} = (M_3 \cdot M_3)$ $\quad d_{33} := \frac{1}{2}\cdot\frac{16}{5}4\cdot\frac{2}{3}\frac{16}{5} + \frac{1}{2}\frac{16}{5}4\cdot\left(\frac{2}{3}\frac{16}{5} + \frac{1}{3}\frac{62}{35}\right) + \frac{1}{2}\frac{62}{35}4\cdot\left(\frac{1}{3}\frac{16}{5} + \frac{2}{3}\frac{62}{35}\right) \ldots = 41.347$
$+\frac{1}{2}\frac{62}{35}\frac{5}{2}\left(\frac{2}{3}\frac{62}{35} - \frac{1}{3}\frac{51}{140}\right) + \frac{1}{2}\frac{51}{140}\frac{5}{2}\left(\frac{-1}{3}\frac{62}{35} + \frac{2}{3}\frac{51}{140}\right) + \frac{1}{2}\frac{51}{140}\frac{5}{2}\frac{2}{3}\frac{51}{140}$

$d_{34} = (M_3 \cdot M_4)$ $\quad d_{34} := \frac{-1}{2}4\cdot\frac{12}{5}\cdot\frac{2}{3}\frac{16}{5} - \frac{12}{5}4\cdot\left(\frac{1}{2}\frac{16}{5} + \frac{1}{2}\frac{62}{35}\right) - \frac{1}{2}\frac{62}{35}\frac{5}{2}\cdot\left(\frac{2}{3}\frac{12}{5} + \frac{1}{3}\frac{6}{5}\right) \ldots = -37.439$
$+\frac{1}{2}\frac{51}{140}\frac{5}{2}\cdot\left(\frac{1}{3}\frac{12}{5} + \frac{2}{3}\frac{6}{5}\right) + \frac{1}{2}\frac{6}{5}\frac{5}{2}\cdot\frac{2}{3}\frac{51}{140}$

$d_{44} = (M_1 \cdot M_1)$ $\quad d_{44} := \frac{1}{2}\cdot 4\cdot\frac{12}{5}\cdot\frac{2}{3}\cdot\frac{12}{5} + \frac{12}{5}\cdot 4\frac{12}{5} + \frac{1}{2}\cdot\frac{12}{5}\cdot 5\cdot\frac{2}{3}\cdot\frac{12}{5} = 40.32$

$d_{21} := d_{12}$ $\quad d_{42} := d_{24}$

$d_{31} := d_{13}$ $\quad d_{43} := d_{34}$

$d_{41} := d_{14}$

$d_{32} := d_{23}$

Masy na poszczególnych dynamicznych stopniach swobody

masa porównawcza $m_0 := 50 \cdot kg$

$m_1 := 4 \qquad m_2 := 4 \qquad m_3 := 5 \qquad m_4 := 5$

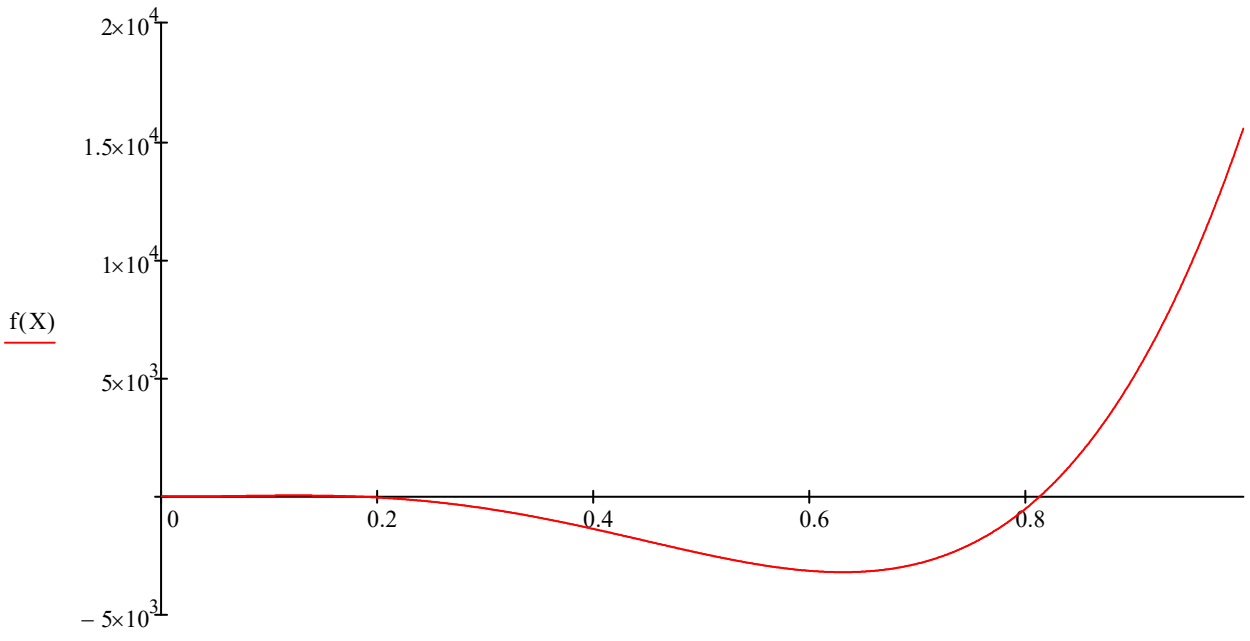
$$\begin{pmatrix} m_1 \cdot \frac{d_{11}}{EJ} - \frac{1}{\omega^2} & m_2 \cdot \frac{d_{12}}{EJ} & m_3 \cdot \frac{d_{13}}{EJ} & m_4 \cdot \frac{d_{14}}{EJ} \\ m_1 \cdot \frac{d_{21}}{EJ} & m_2 \frac{d_{22}}{EJ} - \frac{1}{\omega^2} & m_3 \cdot \frac{d_{23}}{EJ} & m_4 \cdot \frac{d_{24}}{EJ} \\ m_1 \cdot \frac{d_{31}}{EJ} & m_2 \cdot \frac{d_{32}}{EJ} & m_3 \cdot \frac{d_{33}}{EJ} - \frac{1}{\omega^2} & m_4 \cdot \frac{d_{34}}{EJ} \\ m_1 \cdot \frac{d_{41}}{EJ} & m_2 \cdot \frac{d_{42}}{EJ} & m_3 \cdot \frac{d_{43}}{EJ} & m_4 \cdot \frac{d_{44}}{EJ} - \frac{1}{\omega^2} \end{pmatrix} \cdot \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

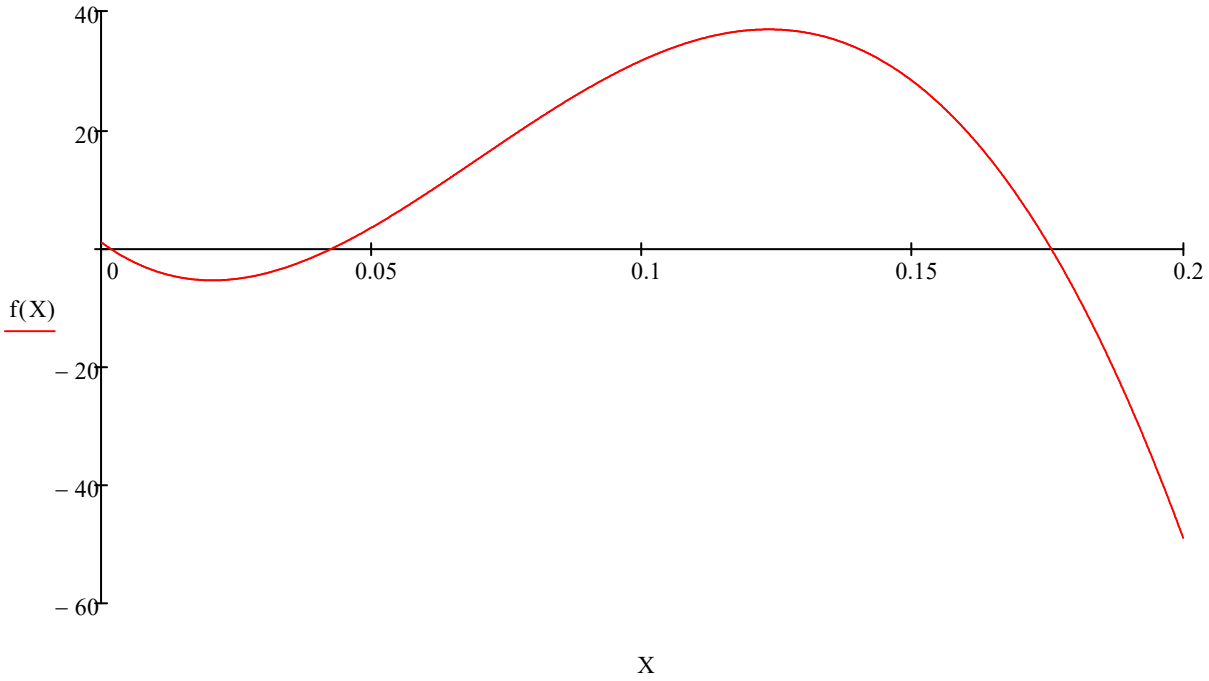
$$X = \frac{m_0 \cdot m^3 \cdot \omega}{EJ}$$

$$\begin{pmatrix} m_1 \cdot d_{11} \cdot X - 1 & m_2 \cdot d_{12} \cdot X & m_3 \cdot d_{13} \cdot X & m_4 \cdot d_{14} \cdot X \\ m_1 \cdot d_{21} \cdot X & m_2 d_{22} \cdot X - 1 & m_3 \cdot d_{23} \cdot X & m_4 \cdot d_{24} \cdot X \\ m_1 \cdot d_{31} \cdot X & m_2 \cdot d_{32} \cdot X & m_3 \cdot d_{33} \cdot X - 1 & m_4 \cdot d_{34} \cdot X \\ m_1 \cdot d_{41} \cdot X & m_2 \cdot d_{42} \cdot X & m_3 \cdot d_{43} \cdot X & m_4 \cdot d_{44} \cdot X - 1 \end{pmatrix} \cdot \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$a(X) := \begin{pmatrix} m_1 \cdot d_{11} \cdot X - 1 & m_2 \cdot d_{12} \cdot X & m_3 \cdot d_{13} \cdot X & m_4 \cdot d_{14} \cdot X \\ m_1 \cdot d_{21} \cdot X & m_2 d_{22} \cdot X - 1 & m_3 \cdot d_{23} \cdot X & m_4 \cdot d_{24} \cdot X \\ m_1 \cdot d_{31} \cdot X & m_2 \cdot d_{32} \cdot X & m_3 \cdot d_{33} \cdot X - 1 & m_4 \cdot d_{34} \cdot X \\ m_1 \cdot d_{41} \cdot X & m_2 \cdot d_{42} \cdot X & m_3 \cdot d_{43} \cdot X & m_4 \cdot d_{44} \cdot X - 1 \end{pmatrix}$$

$f(X) := -671.845663265306178 \cdot X + 19559.481575963745506 \cdot X^2 + -108215.5328798190812 \cdot X^3 + 104852.60770975207187 \cdot X^4 +$





$roz(X) := |a(X)|$

$roz(X) \rightarrow -671.845663265306178 \cdot X + 19559.481575963745506 \cdot X^2 + -108215.5328798190812 \cdot X^3 + 104852.60770975207187 \cdot X^4$

$X := 1$

Given

$|a(X)| = 0$

$X := (Find(X)) \rightarrow (0.81205504899715134103 \quad 0.17552859606136613151 \quad 0.042930689844557922351 \quad -4.77260847658712142$

$X = \left(0.812 \quad 0.176 \quad 0.043 \quad 1.559 \times 10^{-3}\right)$

$\omega_1 := \sqrt{X_{1,4}} \cdot \sqrt{E \cdot \frac{J}{m_0 \cdot m^3}}$ $\omega_1 = 11.442 \frac{1}{s}$ [rad/s] $f_1 := \frac{\omega_1}{2\pi} = 1.821 \cdot Hz$

$\omega_2 := \sqrt{X_{1,3}} \cdot \sqrt{E \cdot \frac{J}{m_0 \cdot m^3}}$ $\omega_2 = 60.051 \frac{1}{s}$ [rad/s] $f_2 := \frac{\omega_2}{2\pi} = 9.557 \cdot Hz$

$\omega_3 := \sqrt{X_{1,2}} \cdot \sqrt{E \cdot \frac{J}{m_0 \cdot m^3}}$ $\omega_3 = 121.427 \frac{1}{s}$ [rad/s] $f_3 := \frac{\omega_3}{2\pi} = 19.326 \cdot Hz$

$\omega_4 := \sqrt{X_{1,1}} \cdot \sqrt{E \cdot \frac{J}{m_0 \cdot m^3}}$ $\omega_4 = 261.175 \frac{1}{s}$ [rad/s] $f_4 := \frac{\omega_4}{2\pi} = 41.567 \cdot Hz$

$d_{11} := d_{11} \cdot \frac{m^3}{E \cdot J}$ $d_{21} := d_{21} \cdot \frac{m^3}{E \cdot J}$ $d_{31} := d_{31} \cdot \frac{m^3}{E \cdot J}$ $d_{41} := d_{41} \cdot \frac{m^3}{E \cdot J}$

$d_{12} := d_{12} \cdot \frac{m^3}{E \cdot J}$ $d_{22} := d_{22} \cdot \frac{m^3}{E \cdot J}$ $d_{32} := d_{32} \cdot \frac{m^3}{E \cdot J}$ $d_{42} := d_{42} \cdot \frac{m^3}{E \cdot J}$

$d_{13} := d_{13} \cdot \frac{m^3}{E \cdot J}$ $d_{23} := d_{23} \cdot \frac{m^3}{E \cdot J}$ $d_{33} := d_{33} \cdot \frac{m^3}{E \cdot J}$ $d_{43} := d_{43} \cdot \frac{m^3}{E \cdot J}$

$d_{14} := d_{14} \cdot \frac{m^3}{E \cdot J}$ $d_{24} := d_{24} \cdot \frac{m^3}{E \cdot J}$ $d_{34} := d_{34} \cdot \frac{m^3}{E \cdot J}$ $d_{44} := d_{44} \cdot \frac{m^3}{E \cdot J}$

Wyliczenie amplitud drgań własnych

$m_1 := m_1 \cdot m_0$ $m_2 := m_2 \cdot m_0$ $m_3 := m_3 \cdot m_0$ $m_4 := m_4 \cdot m_0$

Dla $\omega_1 = 11.442 \frac{1}{s}$ [rad/s]

$A_1 := 1$ $A_2 := 1$ $A_3 := 1$ $A_4 := 1$

Given

$$\left[m_1 \cdot d_{11} - \frac{1}{(\omega_1)^2}\right] \cdot A_1 + m_2 \cdot d_{12} \cdot A_2 + m_3 \cdot d_{13} \cdot A_3 + m_4 \cdot d_{14} \cdot A_4 = 0$$

$$m_1 \cdot d_{21} \cdot A_1 + \left(m_2 d_{22} - \frac{1}{\omega_1^2}\right) \cdot A_2 + m_3 \cdot d_{23} \cdot A_3 + m_4 \cdot d_{24} \cdot A_4 = 0$$

$$m_1 \cdot d_{31} \cdot A_1 + m_2 \cdot d_{32} \cdot A_2 + \left(m_3 d_{33} - \frac{1}{\omega_1^2}\right) \cdot A_3 + m_4 \cdot d_{34} \cdot A_4 = 0$$

$$P := \text{Find}(A_2, A_3, A_4) \rightarrow \begin{pmatrix} 0.22533943885124369444 \\ -0.81538460812406907968 \\ 0.80018938878466942192 \end{pmatrix}$$

$A_{11} := 1$

$A_{21} := P_{1,1} = 0.225$

$A_{31} := P_{2,1} = -0.815$

$A_{41} := P_{3,1} = 0.8$

$$P = \begin{pmatrix} 0.225 \\ -0.815 \\ 0.8 \end{pmatrix}$$

Dla $\omega_2 = 60.051 \frac{1}{s}$ [rad/s]

$A_1 := 1$ $A_2 := 1$ $A_3 := 1$ $A_4 := 1$

Given

$$\left[m_1 \cdot d_{11} - \frac{1}{(\omega_2)^2}\right] \cdot A_1 + m_2 \cdot d_{12} \cdot A_2 + m_3 \cdot d_{13} \cdot A_3 + m_4 \cdot d_{14} \cdot A_4 = 0$$

$$m_1 \cdot d_{21} \cdot A_1 + \left(m_2 d_{22} - \frac{1}{\omega_2^2}\right) \cdot A_2 + m_3 \cdot d_{23} \cdot A_3 + m_4 \cdot d_{24} \cdot A_4 = 0$$

$$m_1 \cdot d_{31} \cdot A_1 + m_2 \cdot d_{32} \cdot A_2 + \left(m_3 d_{33} - \frac{1}{\omega_2^2}\right) \cdot A_3 + m_4 \cdot d_{34} \cdot A_4 = 0$$

$P := \text{Find}(A_2, A_3, A_4)$

$A_{12} := 1$

$A_{22} := P_{1,1} = -2.178$

$A_{32} := P_{2,1} = -1.811$

$A_{42} := P_{3,1} = -2.354$

Dla $\omega_3 = 121.427 \frac{1}{s}$ [rad/s]

$A_1 := 1$ $A_2 := 1$ $A_3 := 1$ $A_4 := 1$

Given

$$\left[m_1 \cdot d_{11} - \frac{1}{(\omega_3)^2}\right] \cdot A_1 + m_2 \cdot d_{12} \cdot A_2 + m_3 \cdot d_{13} \cdot A_3 + m_4 \cdot d_{14} \cdot A_4 = 0$$

$$m_1 \cdot d_{21} \cdot A_1 + \left(m_2 d_{22} - \frac{1}{\omega_3^2}\right) \cdot A_2 + m_3 \cdot d_{23} \cdot A_3 + m_4 \cdot d_{24} \cdot A_4 = 0$$

$$m_1 \cdot d_{31} \cdot A_1 + m_2 \cdot d_{32} \cdot A_2 + \left(m_3 d_{33} - \frac{1}{\omega_3^2}\right) \cdot A_3 + m_4 \cdot d_{34} \cdot A_4 = 0$$

$P := \text{Find}(A_2, A_3, A_4)$

$A_{13} := 1$

$A_{23} := P_{1,1} = -1.084$

$A_{33} := P_{2,1} = 1.061$

$A_{43} := P_{3,1} = 0.326$

Dla $\omega_4 = 261.175 \frac{1}{s}$ [rad/s]

$A_1 := 1$ $A_2 := 1$ $A_3 := 1$ $A_4 := 1$

Given

$$\left[m_1 \cdot d_{11} - \frac{1}{(\omega_4)^2}\right] \cdot A_1 + m_2 \cdot d_{12} \cdot A_2 + m_3 \cdot d_{13} \cdot A_3 + m_4 \cdot d_{14} \cdot A_4 = 0$$

$$m_1 \cdot d_{21} \cdot A_1 + \left(m_2 d_{22} - \frac{1}{\omega_4^2}\right) \cdot A_2 + m_3 \cdot d_{23} \cdot A_3 + m_4 \cdot d_{24} \cdot A_4 = 0$$

$$m_1 \cdot d_{31} \cdot A_1 + m_2 \cdot d_{32} \cdot A_2 + \left(m_3 d_{33} - \frac{1}{\omega_4^2}\right) \cdot A_3 + m_4 \cdot d_{34} \cdot A_4 = 0$$

$P := \text{Find}(A_2, A_3, A_4)$

$A_{14} := 1$

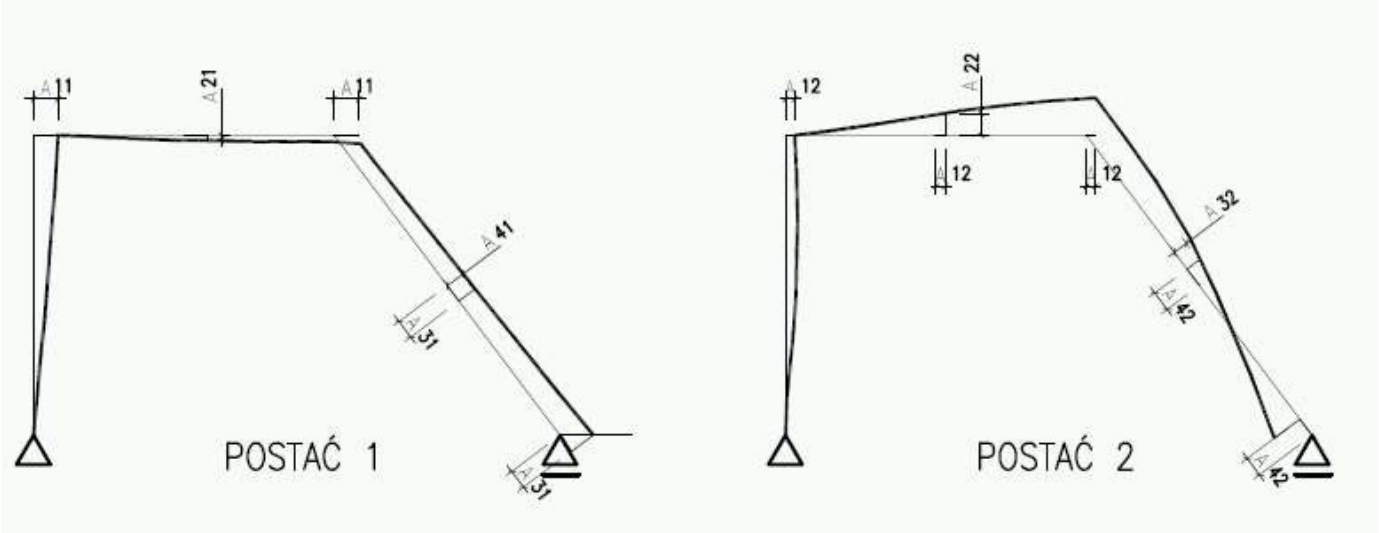
$A_{24} := P_{1,1} = 1.131$

$A_{34} := P_{2,1} = 0.424$

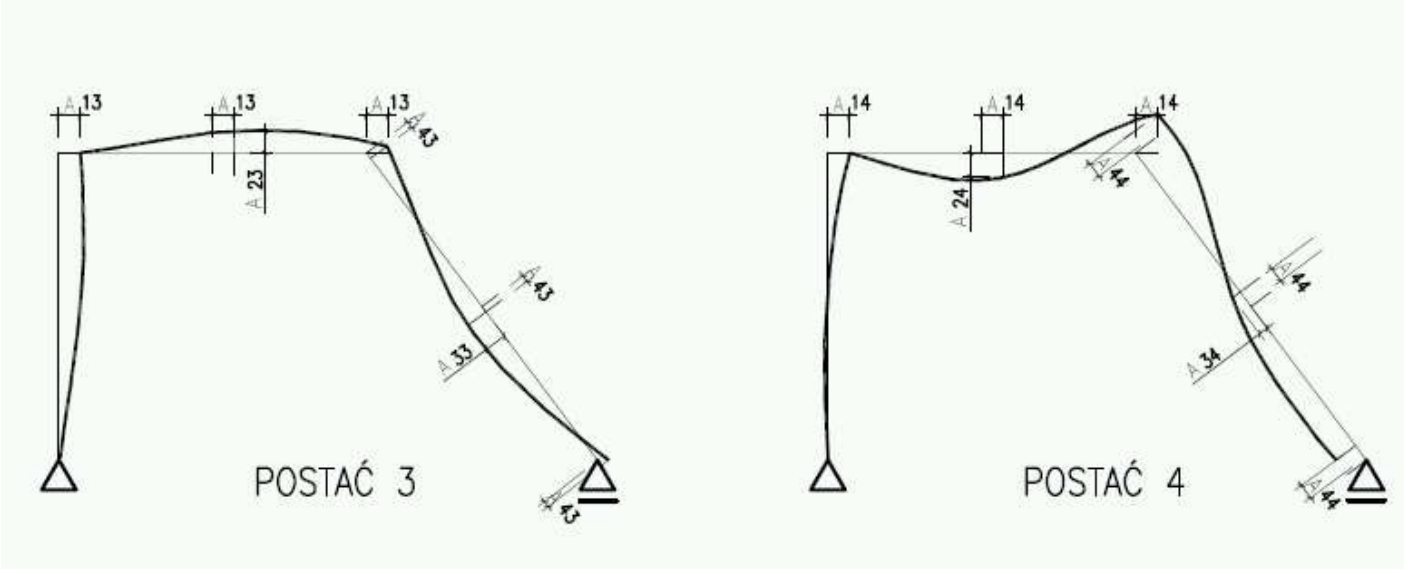
$A_{44} := P_{3,1} = -0.823$

Postacie drgań

Postać pierwsza i druga

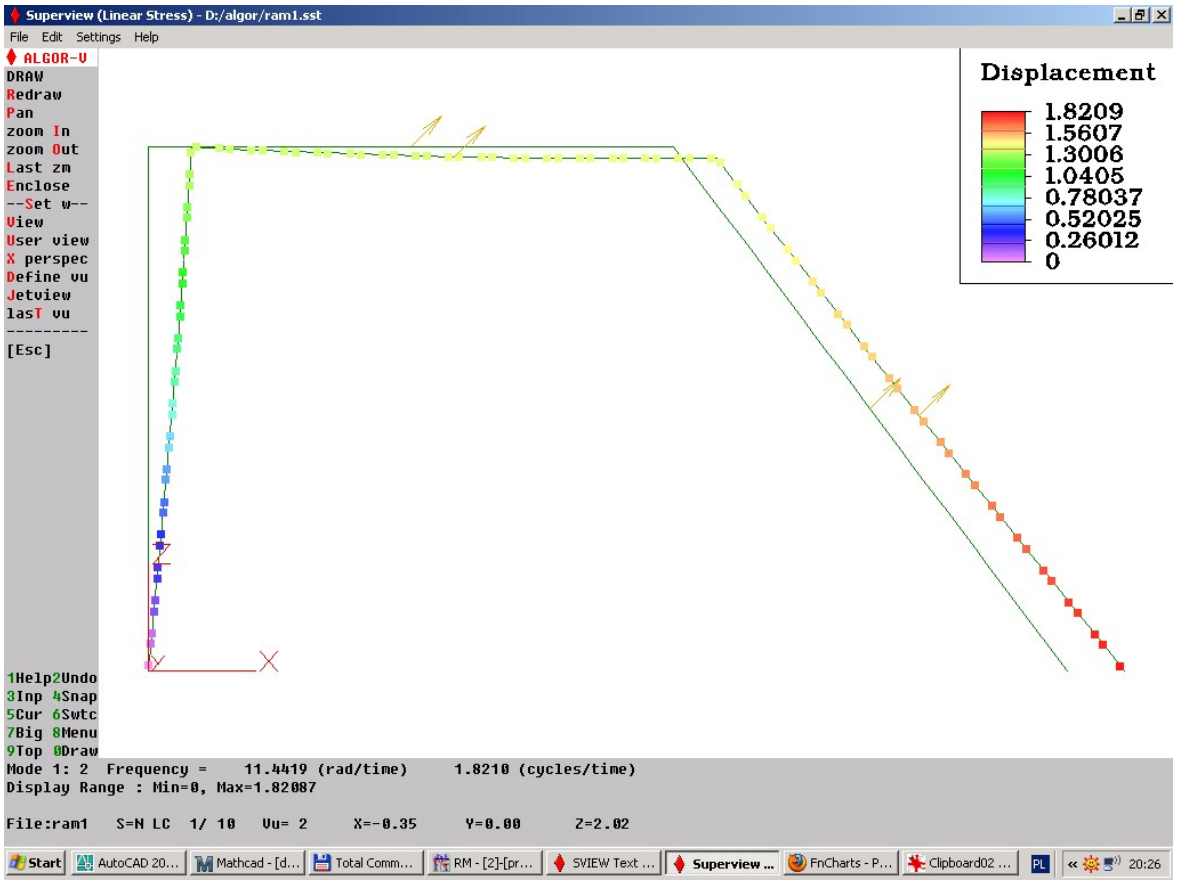


Postać trzecia oraz czwarta

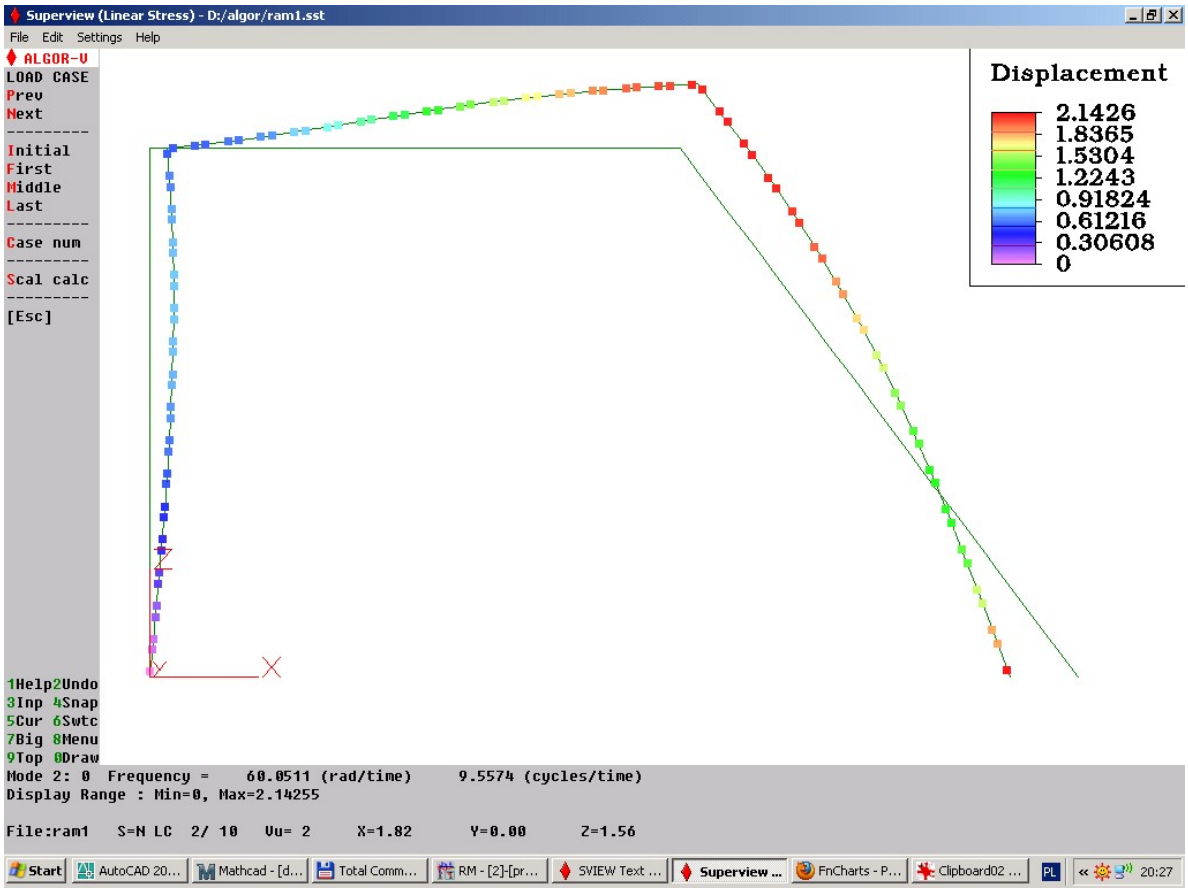


Postacie z programu ALGOR

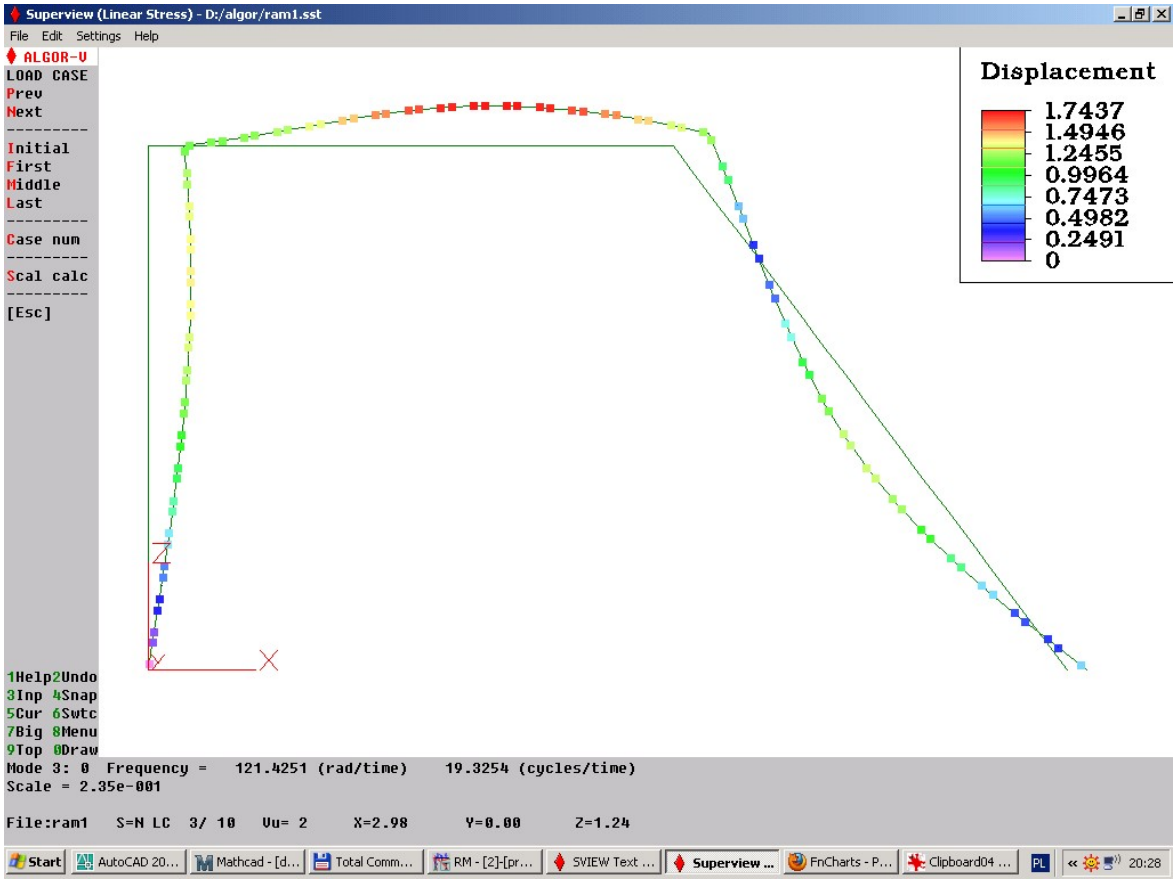
Postać pierwsza



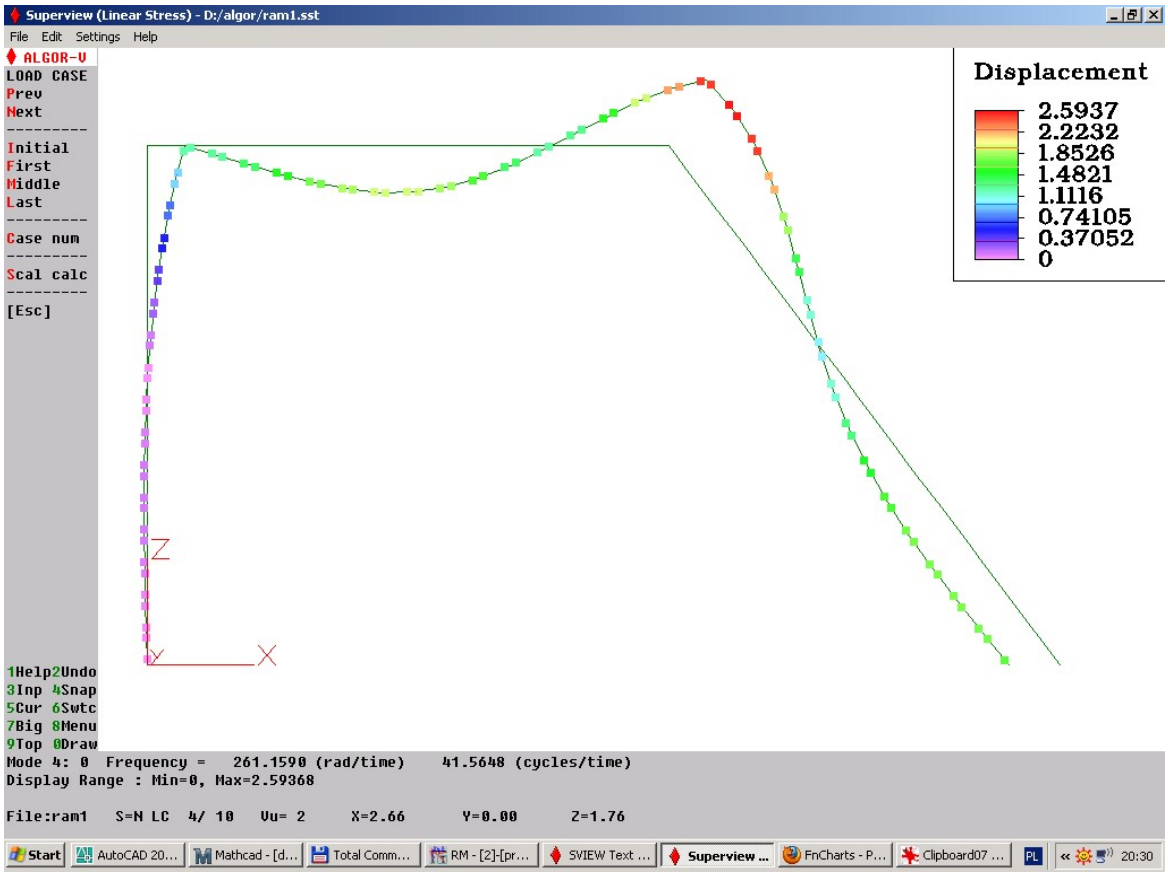
Postać druga



Postać trzecia



Postać czwarta



Warunek ortoganalności drgań

$m_1 \cdot A_{11} \cdot A_{12} + m_2 \cdot A_{21} \cdot A_{22} + m_3 \cdot A_{31} \cdot A_{32} + m_4 \cdot A_{41} \cdot A_{42} = -5.684 \times 10^{-14}\,kg$ sprawdzenie postaci 1 i 2

$m_1 \cdot A_{11} \cdot A_{13} + m_2 \cdot A_{21} \cdot A_{23} + m_3 \cdot A_{31} \cdot A_{33} + m_4 \cdot A_{41} \cdot A_{43} = -1.279 \times 10^{-13}\,kg$ sprawdzenie postaci 1 i 3

$m_1 \cdot A_{11} \cdot A_{14} + m_2 \cdot A_{21} \cdot A_{24} + m_3 \cdot A_{31} \cdot A_{34} + m_4 \cdot A_{41} \cdot A_{44} = 2.842 \times 10^{-14}\,kg$ sprawdzenie postaci 1 i 4

$m_1 \cdot A_{12} \cdot A_{13} + m_2 \cdot A_{22} \cdot A_{23} + m_3 \cdot A_{32} \cdot A_{33} + m_4 \cdot A_{42} \cdot A_{43} = 3.979 \times 10^{-12}\,kg$ sprawdzenie postaci 2 i 3

$m_1 \cdot A_{12} \cdot A_{14} + m_2 \cdot A_{22} \cdot A_{24} + m_3 \cdot A_{32} \cdot A_{34} + m_4 \cdot A_{42} \cdot A_{44} = -2.501 \times 10^{-12}\,kg$ sprawdzenie postaci 2 i 4

$m_1 \cdot A_{13} \cdot A_{14} + m_2 \cdot A_{23} \cdot A_{24} + m_3 \cdot A_{33} \cdot A_{34} + m_4 \cdot A_{43} \cdot A_{44} = -1.45 \times 10^{-12}\,kg$ sprawdzenie postaci 3 i 4

Metoda Dunkerlaya $\omega_1 = 11.442\,\frac{1}{s}$

$$\omega_D = \frac{1}{\sqrt{\sum_i \left(m_i \cdot \delta_{ii}\right)}}$$

$$\omega_D := \frac{1}{\sqrt{m_1 \cdot d_{11} + m_2 \cdot d_{22} + m_3 \cdot d_{33} + m_4 \cdot d_{44}}} \qquad \omega_D = 11.182\,\frac{1}{s}$$

Metoda Reyleigh`a $\omega_1 = 11.442\,\frac{1}{s}$

$$\omega_R = \sqrt{\frac{E_{pmax}}{E_k}}$$

$$E_{pmax} = \frac{1}{2} \cdot \left(\left(P_1 \cdot Y_1 + P_2 \cdot Y_2 + P_3 \cdot Y_3 + P_4 \cdot Y_4\right)\right) \qquad P = P_1 = P_2 = P_3$$

$$E_{kmax} = \frac{1}{2} \cdot \left(m_1 \cdot {Y_1}^2 + m_2 \cdot {Y_2}^2 + m_3 \cdot {Y_3}^2 + m_4 \cdot {Y_4}^2\right) \qquad y(t) = Y \cdot \sin \cdot \omega \cdot t$$

$P := 1$

$P_1 := P$

$P_2 := P$

$P_3 := P$

$P_4 := P$

$Y_1 := P \cdot \left(d_{11} + d_{12} + d_{13} + d_{14}\right)$ $\qquad Y_1 = 1.695 \times 10^{-5}\,\frac{s^2}{kg}$

$Y_2 := P \cdot \left(d_{21} + d_{22} + d_{23} + d_{24}\right)$ $\qquad Y_2 = 4.759 \times 10^{-6}\,\frac{s^2}{kg}$

$Y_3 := P \cdot \left(d_{31} + d_{32} + d_{33} + d_{34}\right)$ $\qquad Y_3 = -1.311 \times 10^{-5}\,\frac{s^2}{kg}$

$Y_4 := P \cdot \left(d_{41} + d_{42} + d_{43} + d_{44}\right)$ $\qquad Y_4 = 1.484 \times 10^{-5}\,\frac{s^2}{kg}$

$E_{pmax} := \frac{1}{2} \cdot \left(P_1 \cdot Y_1 + P_2 \cdot Y_2 + P_3 \cdot Y_3 + P_4 \cdot Y_4\right)$ $\qquad E_{pmax} = \frac{1}{85350}\,\frac{s^2}{kg}$

$E_k := \frac{1}{2} \cdot \left(m_1 \cdot {Y_1}^2 + m_2 \cdot {Y_2}^2 + m_3 \cdot {Y_3}^2 + m_4 \cdot {Y_4}^2\right)$ $\qquad E_k = \frac{1}{12504494}\,\frac{s^4}{kg}$

$\omega_R := \sqrt{\frac{E_{pmax}}{E_k}}$ $\qquad \omega_R = 12.104\,\frac{1}{s}$

$Y_1 := (m_1 \cdot d_{11} + m_2 \cdot d_{12} + m_3 \cdot d_{13} + m_4 \cdot d_{14})$ $Y_1 = 3.367 \times 10^{-3}\, s^2$

$Y_2 := (m_1 \cdot d_{21} + m_2 \cdot d_{22} + m_3 \cdot d_{23} + m_4 \cdot d_{24})$ $Y_2 = 9.795 \times 10^{-4}\, s^2$

$Y_3 := (m_1 \cdot d_{31} + m_2 \cdot d_{32} + m_3 \cdot d_{33} + m_4 \cdot d_{34})$ $Y_3 = -2.575 \times 10^{-3}\, s^2$

$Y_4 := (m_1 \cdot d_{41} + m_2 \cdot d_{42} + m_3 \cdot d_{43} + m_4 \cdot d_{44})$ $Y_4 = 3.001 \times 10^{-3}\, s^2$

$E_{pmax} := (m_1 \cdot Y_1 + m_2 Y_2 + m_3 \cdot Y_3 + m_4 \cdot Y_4)$ $E_{pmax} = \frac{40}{41}\, s^2 \cdot kg$

$E_k := (m_1 \cdot Y_1^{\,2} + m_2 \cdot Y_2^{\,2} + m_3 \cdot Y_3^{\,2} + m_4 \cdot Y_4^{\,2})$ $E_k = \frac{1}{157}\, s^4 \cdot kg$

$\omega_R := \sqrt{\frac{E_{pmax}}{E_k}}$ $\omega_R = 12.378 \frac{1}{s}$

Sprawdzenie zakresu czestości

$\omega_D = 11.182 \frac{1}{s}$ $\omega_1 = 11.442 \frac{1}{s}$ $\omega_R = 12.378 \frac{1}{s}$

Drgania Wymuszone

liczba obrotów silnika	$n := \frac{30}{s}$		
częstość kołowa wymuszenia	$\Theta := 2 \cdot \pi \cdot n$	$\Theta = 188.496 \frac{1}{s}$	[rad/s]
masa wirnika	$mw := 40 \cdot kg$		
mimośród	$ew := 2cm$		
amplituda drgań wymuszonych	$P0 := mw \cdot ew \cdot \Theta^2$	$P0 = 28.424 \cdot kN$	

Sprawdzenie rezonansu

1 częstość drgań własnych	$\omega_1 = 11.442 \frac{1}{s}$	$0.9 \cdot \omega_1 = 10.298 \frac{1}{s}$	$1.1 \cdot \omega_1 = 12.586 \frac{1}{s}$
2 częstość drgań własnych	$\omega_2 = 60.051 \frac{1}{s}$	$0.9 \cdot \omega_2 = 54.046 \frac{1}{s}$	$1.1 \cdot \omega_2 = 66.057 \frac{1}{s}$
3 częstość drgań własnych	$\omega_3 = 121.427 \frac{1}{s}$	$0.9 \cdot \omega_3 = 109.284 \frac{1}{s}$	$1.1 \cdot \omega_3 = 133.569 \frac{1}{s}$
4 częstość drgań własnych	$\omega_4 = 261.175 \frac{1}{s}$	$0.9 \cdot \omega_4 = 235.058 \frac{1}{s}$	$1.1 \cdot \omega_4 = 287.293 \frac{1}{s}$

częstość drgań wymuszonych $\Theta = 188.496 \frac{1}{s}$

Częstość dragań wymuszonych nie pokrywa się z żadną częstością drgań własnych.
Nie grozi rezonans.

Wyznaczenie sił bezwładności

$\Delta_{1p} := d_{21} \cdot P0 = 0.084\,m$

$\Delta_{2p} := d_{22} \cdot P0 = 0.036\,m$

$\Delta_{3p} := d_{23} \cdot P0 = -0.068\,m$

$\Delta_{4p} := d_{24} \cdot P0 = 0.084\,m$

Układ równań

$$\begin{pmatrix} d_{11} - \frac{1}{m_1 \cdot \Theta^2} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} - \frac{1}{m_2 \cdot \Theta^2} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} - \frac{1}{m_3 \cdot \Theta^2} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} - \frac{1}{m_4 \cdot \Theta^2} \end{pmatrix} \cdot \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix} + \begin{pmatrix} \Delta_{1p} \\ \Delta_{2p} \\ \Delta_{3p} \\ \Delta_{4p} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$WS := \begin{pmatrix} d_{11} - \frac{1}{m_1 \cdot \Theta^2} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} - \frac{1}{m_2 \cdot \Theta^2} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} - \frac{1}{m_3 \cdot \Theta^2} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} - \frac{1}{m_4 \cdot \Theta^2} \end{pmatrix}$$

$$WS = \begin{pmatrix} 0.014 & 2.954 \times 10^{-3} & -0.012 & 0.011 \\ 2.954 \times 10^{-3} & 1.11 \times 10^{-3} & -2.385 \times 10^{-3} & 2.939 \times 10^{-3} \\ -0.012 & -2.385 \times 10^{-3} & 9.732 \times 10^{-3} & -8.914 \times 10^{-3} \\ 0.011 & 2.939 \times 10^{-3} & -8.914 \times 10^{-3} & 9.487 \times 10^{-3} \end{pmatrix} \cdot \frac{m}{kN}$$

$$WP := -\begin{pmatrix} \Delta_{1p} \\ \Delta_{2p} \\ \Delta_{3p} \\ \Delta_{4p} \end{pmatrix} \qquad WP = \begin{pmatrix} -0.084 \\ -0.036 \\ 0.068 \\ -0.084 \end{pmatrix} m$$

Wektor rozwiązań układu

$B := WS^{-1} \cdot WP$

$$B = \begin{pmatrix} 26.32 \\ -13.052 \\ 17.472 \\ -19.447 \end{pmatrix} \cdot kN$$

Obliczenie sił statycznych

Przyśpieszenie ziemskie $g = 9.807 \frac{m}{s^2}$

Ciężar 2 masy $G2 := m_2 \cdot g = 1.961 \cdot kN$

Ciężar 3 masy $G3 := m_3 \cdot g = 2.452 \cdot kN$

$P0 = 28.424 \cdot kN$

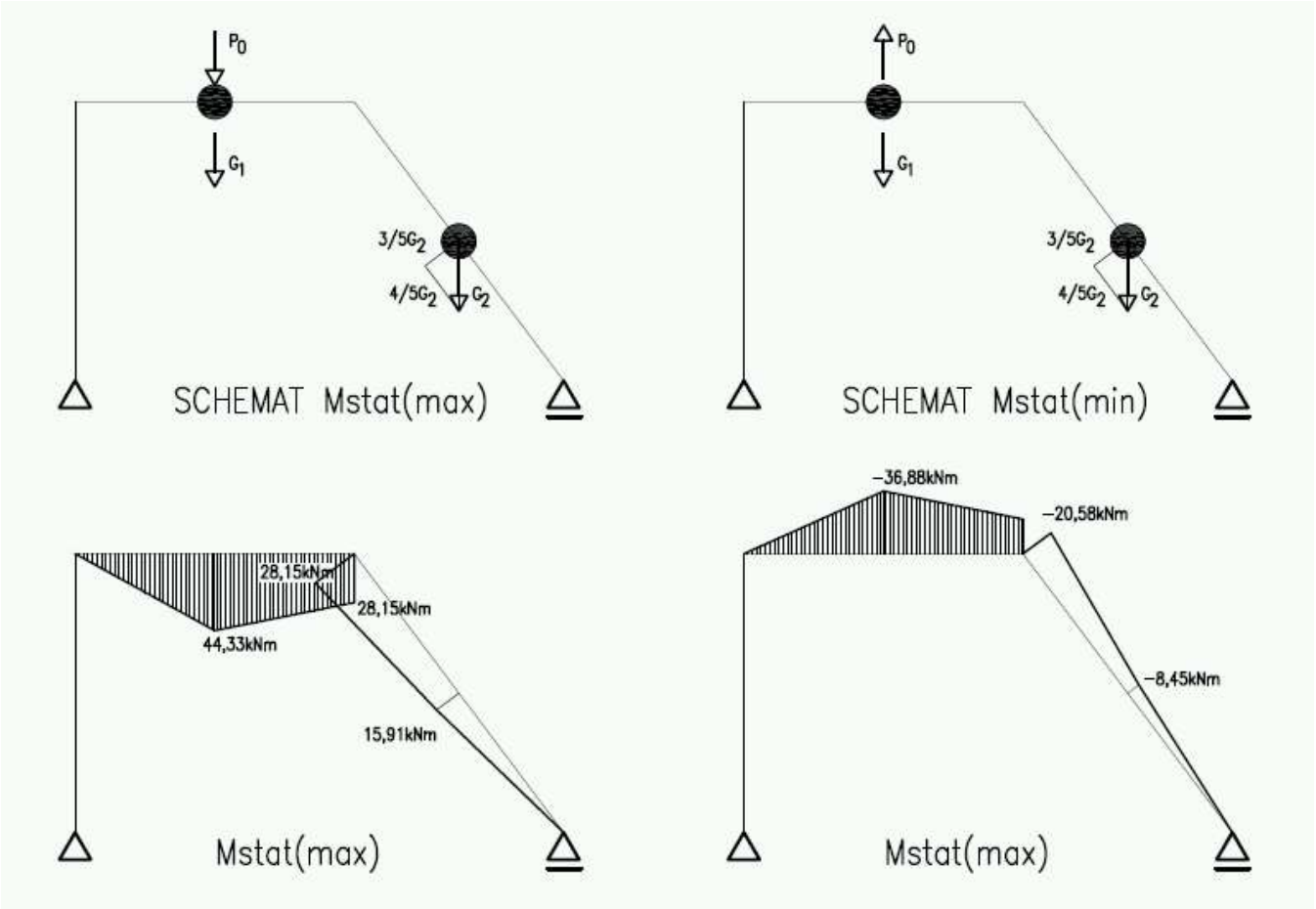
Momenty statyczne

$$Mst(max) = MB2 \cdot (P0 + G1) + MB3 \cdot \left(\frac{3}{5} G2\right) + MB4 \cdot \left(\frac{4}{5} G2\right)$$

$$Mst(min) = MB2 \cdot (-P0 + G1) + MB3 \cdot \left(\frac{3}{5} G2\right) + MB4 \cdot \left(\frac{4}{5} G2\right)$$

punkt	MB1	MB2	MB3	MB4	Mst(max)	Mst(min)
	[m]	[m]	[m]	[m]	[kNm]	[kNm]
A	0	0	0	0	0	0
B	4	0	-3,2	2,4	0	0
C	3,714286	1,428571	-2,57143	2,4	44,33351	-36,8779
D	1,714286	0,857143	-1,77143	2,4	28,14729	-20,5796
E	0,857143	0,428571	0,364286	1,2	15,91302	-8,45041
F	0	0	0	0	0	0

RYSUNEK



Momenty dynamiczne

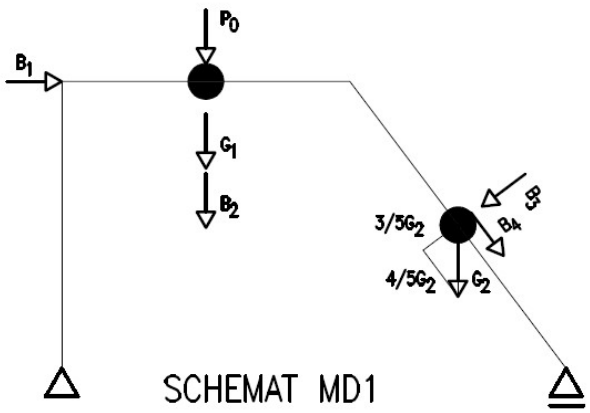
$$MD1 = MB1 \cdot B1 + MB2 \cdot (P0 + G1 + B2) + MB3 \cdot \left(\frac{3}{5}G2 + B3\right) + MB4 \cdot \left(\frac{4}{5}G2 + B4\right)$$

$$MD2 = MB1 \cdot B1 + MB2 \cdot (-P0 + G1 + B2) + MB3 \cdot \left(\frac{3}{5}G2 - B3\right) + MB4 \cdot \left(\frac{4}{5}G2 + B4\right)$$

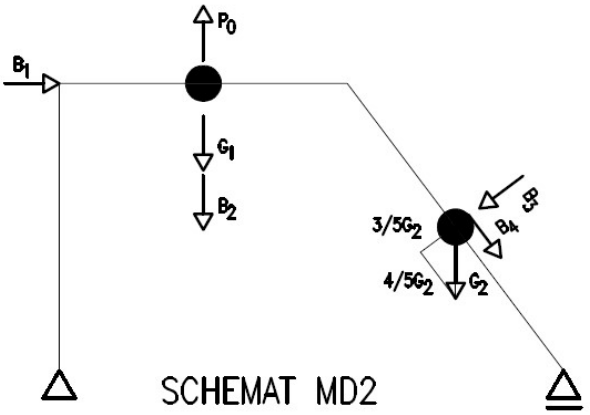
$$MD3 = MB1 \cdot -B1 + MB2 \cdot (P0 + G1 - B2) + MB3 \cdot \left(\frac{3}{5}G2 - B3\right) + MB4 \cdot \left(\frac{4}{5}G2 - B4\right)$$

$$MD4 = MB1 \cdot -B1 + MB2 \cdot (-P0 + G1 - B2) + MB3 \cdot \left(\frac{3}{5}G2 - B3\right) + MB4 \cdot \left(\frac{4}{5}G2 - B4\right)$$

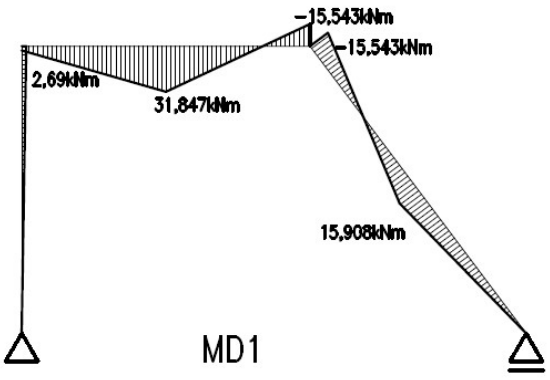
punkt	MB1	MB2	MB3	MB4	M1D	M2D	M3D	M4D	Mmax	Mmin
	[m]	[m]	[m]	[m]	[kNm]	[kNm]	[kNm]	[kNm]	[kNm]	[kNm]
A	0	0	0	0	0	0	0	0	0	0
B	4	0	-3,2	2,4	2,6968	2,6968	-2,6968	-2,6968	2,6968	-2,6968
C	3,714286	1,428571	-2,57143	2,4	31,847	-49,3644	56,82003	-24,3914	56,82003	-49,3644
D	1,714286	0,857143	-1,77143	2,4	-15,5433	-64,2702	71,83791	23,11106	71,83791	-64,2702
	0,857143	0,428571	0,364286	1,2	15,9077	-8,45573	15,91833	-8,4451	15,91833	-8,45573
RYSUNEK	0	0	0	0	0	0	0	0	0	0



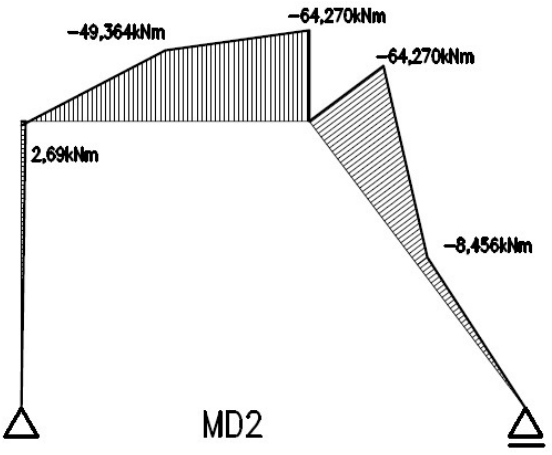
SCHEMAT MD1



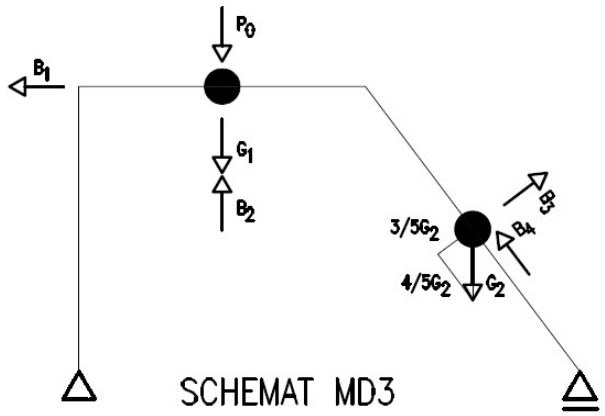
SCHEMAT MD2



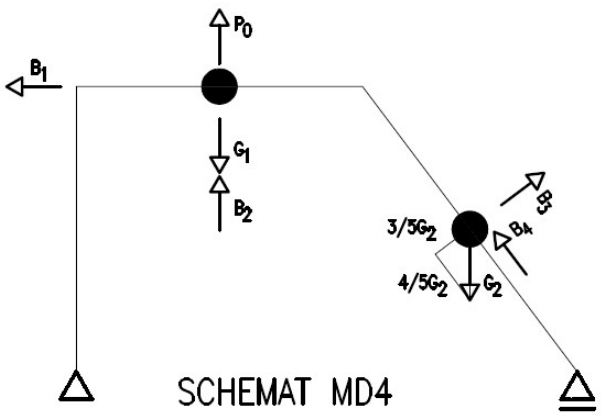
MD1



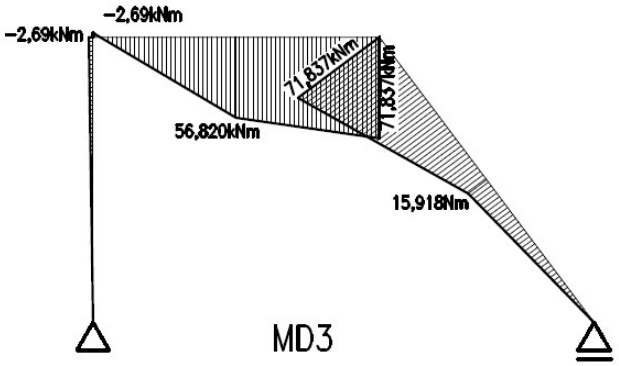
MD2



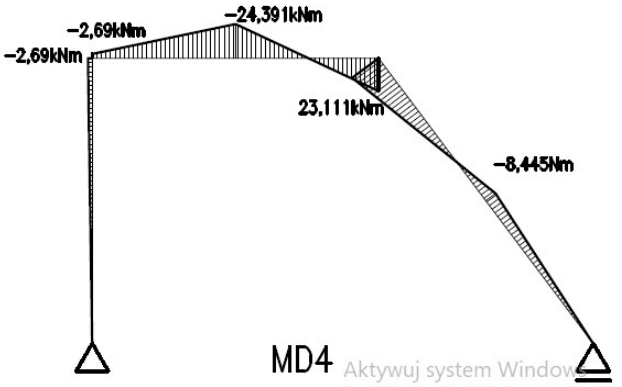
SCHEMAT MD3



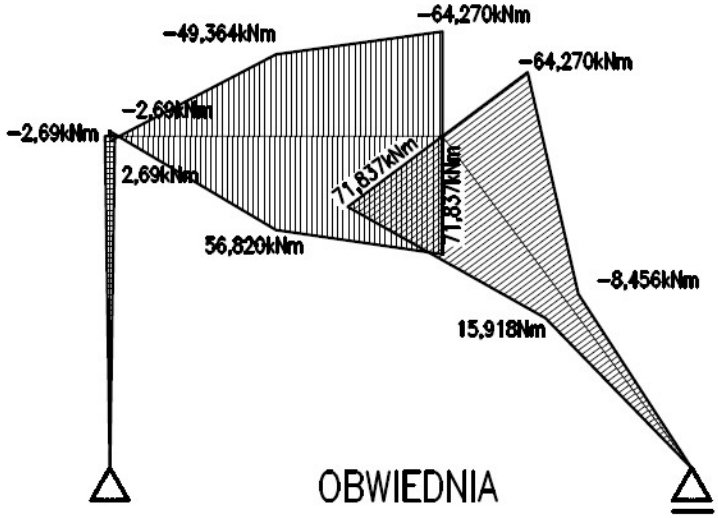
SCHEMAT MD4



MD3



MD4



OBWIEDNIA

Wymiarowanie konstrukcji

$J = 2 \times 10^{-5}\,m^4$

$Wx := \frac{b \cdot h^2}{6} = 400 \cdot cm^3$ $\quad J = 2 \times 10^3 \cdot cm^4$

$Mst := 44kN \cdot m$ $\quad MD := 81.837 \cdot kN \cdot m$

$f_d := 355 \cdot MPa$

$f_{ddyn} := 0.6 \cdot f_d = 213 \cdot MPa$

naprężenia

$\sigma_{st} := \left| \frac{Mst}{Wx} \right|$ $\quad \sigma_{st} = 110 \cdot MPa$ $\quad \sigma_{st} < f_d = 1$ warunek spełniony

$\sigma_{dyn} := \left| \frac{MD}{Wx} \right|$ $\quad \sigma_{dyn} = 204.593 \cdot MPa$ $\quad \sigma_{dyn} < f_{ddyn} = 1$ warunek spełniony

+ 1.0

27e-29i 0.0015585457197518072671 + 4.77260847658712142 27e-29i)